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Azimuthally matched interactions and azimuthal correlation of Bessel light beams

N A Khilo

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Abstract. A theoretical investigation is reported of the regime of azimuthally matched interactions in the process of conversion of the frequency of Bessel light beams. It is shown that this nonlinear interaction regime is accompanied by the establishment of correlations of the mutual azimuthal orientations of plane-wave components of Bessel beams and by an increase in the overlap integral.

1. Introduction

Recent years have seen an upsurge of interest in the characteristic features of the processes of nonlinear optical conversion of the frequency of Bessel light beams (BLBs) in the course of generation of the second [1 – 4] and third [5 – 7] harmonics and by parametric conversion of light [8 – 11]. It has been found that the main characteristic of nonlinear optical interactions of BLBs is related, in contrast to the interaction of Gaussian beams, with the feasibility of various vector interactions involving BLBs. For example, self-phase-matching is possible in harmonic generation when the spatial phase matching conditions are varied. A similar effect occurs in optical parametric oscillators.

The enhancement of the role of the vector interactions in the nonlinear optics of BLBs is clearly a consequence of the conical structure of their spatial frequency spectra. It is worth noting that the spatial spectra of BLBs are optimal in respect of the complexity of their structure (the spatial spectra of BLBs are much simpler than the spectra of Gaussian beams, but much more complex than the plane-wave spectra). This opens up new avenues for utilisation of BLBs not only as a convenient theoretical model, but also as an optical object attainable in practice. Moreover, the conical structure of the BLB spectra is the reason for their well-known diffraction-free properties.

We shall report a theoretical investigation of the vector interactions of BLBs in the specific case of second-harmonic generation. We shall determine the conditions for the operation of the regime of azimuthally matched interactions of BLBs [11] and identify the physical mechanism of the increase in the overlap integral of BLBs in these interactions.

N A Khilo Optical Problems in Informatics Division, National Academy of Sciences of Belarus, ul. Kuprevicha 1, korp. 2, 220141, Minsk, Belarus, tel. +375 (0172) 63-75-35; e-mail: nkhilo@optoinform.bas-net.by

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2. Azimuthally matched interactions of Bessel light beams

Let us assume that a fundamental-frequency radiation is a BLB of zeroth order with the radius R_B and with the field intensity

$$E_1(\rho, z) = A_1(z) j_0(q_1\rho) \exp[\mathrm{i}(k_{1z} z - \omega t)] , \tag{1}$$

and that the second-harmonic field can be represented by a superposition of modal Bessel beams [4]:

$$E_2(\rho, z) = \sum_{m=1}^{M} A_{2m}(z) j_0(q_{2m}\rho) \exp[\mathrm{i}(k_{2zm} z - 2\omega t)] , \tag{2}$$

where $A_{1,2m}(z)$ are slowly varying amplitudes; ρ is the cylindrical coordinate;

$$\begin{aligned} &k_{1z,2zm} = k_{1,2}\cos\gamma_{1,2m}; \quad q_{1,2m} = k_{1,2}\sin\gamma_{1,2m}; \\ &k_1 = (\omega/c)n_1; \quad k_2 = (2\omega/c)n_2; \end{aligned} \tag{3}$$

γ_1 and γ_{2m} are the cone angles of BLBs, i.e. half-angles at the vertices of the wave-vector cones. Expressions (1) and (2) contain the normalised Bessel functions

$$\begin{aligned} &j_0(q_{1,2m}\rho) = J_0(q_{1,2m}\rho) W_{1,2m}^{-1/2} \quad \text{for} \quad \rho < R_\mathrm{B} , \\ &j_0(q_{1,2m}\rho) = 0 \quad \text{for} \quad \rho \geqslant R_\mathrm{B} , \end{aligned} \tag{4}$$

where

$$W_{1,2m}^{1/2} = \left[2\pi \int_0^{R_\mathrm{B}} J_0^2(q_{1,2m}\rho)\rho \mathrm{d}\rho\right]^{1/2} = \sqrt{\pi} R_\mathrm{B} J_1(q_{1,2m}R_\mathrm{B}) ;$$

$J_{0,1}(x)$ are the Bessel functions of the zeroth and first orders. It follows that the functions $j_0(q_{2m}\rho)$ are normalised in the cylindrical region of the existence of a fundamental-frequency BLB, in accordance with the condition

$$2\pi \int_0^{R_\mathrm{B}} j_0^2(q_{2m}\rho)\rho \mathrm{d}\rho = 1. \tag{5}$$

Since the field at the doubled frequency is a superposition of the modes in a cylindrical region of radius R_B, the parameters $q_{2m}R_\mathrm{B}$ are zeros of the zeroth-order Bessel function and it therefore follows that $q_{2m} \approx (m - 0.25)\pi/R_\mathrm{B}$.

The nonlinear polarisations follow from Eqns (1), (2):

$$\begin{aligned} P_{1m}(\rho, z) &= \chi_1 A_{2m}(z) A_1^*(z) j_0(q_{2m}\rho) j_0(q_1\rho) \\ &\quad \times \exp[\mathrm{i}(k_{2zm} - k_{1z})z], \end{aligned} \tag{6}$$

$$P_2(\rho, z) = \chi_2 A_1^2(z) j_0^2(q_1\rho) \exp(2\mathrm{i}k_{1z}z) , \tag{7}$$

where $\chi_{1,2}$ are the nonlinear susceptibilities.

The amplitudes $A_{1,2m}(z)$ satisfy the following system of reduced equations:

$$\frac{\mathrm{d}A_1(z)}{\mathrm{d}z} = \mathrm{i}d_1 \sum_m g_m A_{2m}(z) A_1^*(z) \exp(\mathrm{i}\Delta k_{zm} z)\,,$$
$$\frac{\mathrm{d}A_{2m}(z)}{\mathrm{d}z} = \mathrm{i}d_2 g_m A_1^2(z) \exp(-\mathrm{i}\Delta k_{zm} z)\,, \tag{8}$$

where $d_{1,2}$ are the nonlinear coupling coefficients; $\Delta k_{zm} = k_{2zm} - 2k_{1z}$ are the values of the wave mismatch;

$$g_m = 2\pi \int_0^{R_\mathrm{B}} j_0^2(q_1\rho) j_0(q_{2m}\rho)\rho\mathrm{d}\rho \tag{9}$$

are the overlap integrals of the nonlinear polarisations and of the corresponding radiation fields. The normalised Bessel functions make it possible to use unique overlap integrals in the reduced equations for the three-wave interactions [4].

It follows from the system of equations (8) that second-harmonic generation generally occurs in many channels characterised by the mode indices m. The restrictions on possible frequency conversion channels are related to the dependence of the wave mismatch and overlap integrals on m. Our earlier theoretical and experimental investigation [4] demonstrated that the nature of the vector interactions actually occurring in nonlinear frequency conversion processes is governed by two maxima of the overlap integrals defined by expression (9). The first maximum corresponds to the vector interaction of oppositely oriented plane-wave components of BLBs characterised by $\Delta\varphi \approx \pi$ (Fig. 1). The second-harmonic field then has a near-Gaussian structure. The second maximum corresponds to a collinear interaction and the second-harmonic field is a BLB. We shall use the term 'transverse phase matching' for the harmonic-generation conditions corresponding to a maximum of the overlap integral. In an investigation of the generation of doubled-frequency BLBs it is necessary to satisfy transverse phase-matching conditions of the second type. Fig. 2 shows

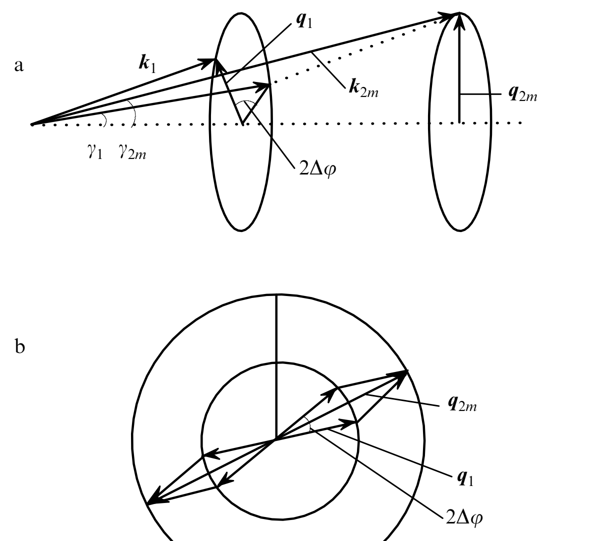

Figure 1. Schematic diagram showing the vector interactions in the generation of the second harmonic of Bessel light beams (a) and the interaction of the angular spectral components in a transverse plane (b).

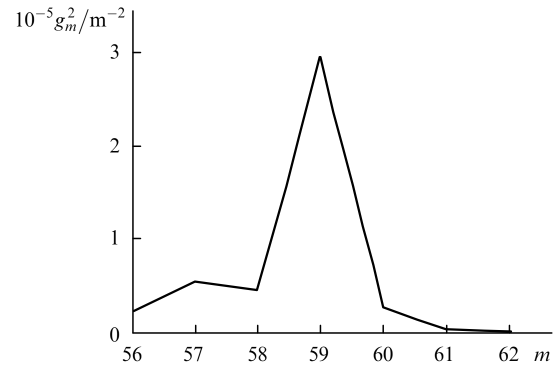

Figure 2. Dependence of the square of the overlap integral of a Bessel light beam on the mode index m in the vicinity of the collinear phase matching, plotted for $\gamma_1 = 1°$, $R_B = 0.9$ mm, and $\gamma = 1.064$ µm.

the behaviour of the function g_m^2, which governs the strength of the interaction, in the vicinity of the second-type of transverse phase matching for frequency doubling of Nd:YAG laser radiation in the form of a BLB with the cone angle $\gamma = 1°$ and with the radius 0.9 mm. We can see that, in the first approximation, we need to consider only a single transverse mode with $m = M = 59$. The maximum value of the square of the overlap integral is then $g_{59}^2 = 2.93 \times 10^5$ m^{-2}.

A calculation of the azimuthal angle $\Delta\varphi$ (Fig. 1) from $\Delta\varphi(m) = \cos^{-1}(q_{2m}/2q_1)$ gives $\Delta\varphi(M) \approx 7.6°$ for the beam parameters given above, and this corresponds to near-collinear phase matching. In addition to the mode with $m = M$, also nonmodal Bessel beams closest to that mode are generated in practice in an extracavity system. For the latter beams the longitudinal and transverse values of the mismatch are within the limits of the width of the phase-matching curve [4]. The corresponding azimuthal angles lie in a certain region near $\Delta\varphi_\mathrm{max}$, close to $\Delta\varphi = 0$. However, a reduction in the width $\Delta\theta_\mathrm{s}$ of the phase-matching curve for the collinear interaction increases the selective influence of the wave mismatch $\Delta k_z = \Delta k_z(\Delta\varphi)$ and reduces the range $\Delta\varphi_\mathrm{max}$. If $\Delta\theta_\mathrm{s} \ll \gamma_m$, we find that $\Delta\varphi_\mathrm{max} \ll 1$, i.e. only one vector interaction geometry remains. The conicity parameter of a second-harmonic BLB is then $q_2(M) \approx 2q_1$.

It follows that the investigated nonlinear interaction is effective if the cone of the collinear phase-matching directions in a crystal is characterised by an angular width considerably smaller than the cone angle of a fundamental-frequency BLB. Consequently, out of all the possible vector interactions of BLBs only those occur in practice which are phase-matched and characterised by near-zero values of $\Delta\varphi$ (Fig. 1). Such interactions were considered earlier [11] and were called 'azimuthally matched'. This name reflects also the circumstance that, in the investigated regime, only certain pairs of plane-wave components of a BLB interact effectively (Fig. 1) and these components satisfy the conditions of longitudinal and transverse spatial phase matching.

3. Azimuthal correlation of Bessel light beams

The above analysis of the process of second-harmonic generation is based on an approach generally employed in nonlinear optics of BLBs (see, for example, Refs [1–3, 6]). We shall show later that this approach, founded on expressions (6) and (7) for the nonlinear polarisation and on expression (9) which follows from them and describes the overlap integral, is incorrect when describing steady-state azimuthally matched interactions. In other words, the changeover to the azimuthally matched interactions alters the spatial structure of the nonlinear polarisation and, consequently, the structure of the beam overlap integrals.

In view of this, we shall analyse the generation of subharmonics and investigate the following factor, dependent on the transverse coordinate ρ and occurring in the expression for the nonlinear polarisation (6) at the fundamental frequency:

$$p_{1m}(\rho) = J_0(q_1\rho)J_0(q_{2m}\rho)\,. \tag{10}$$

We shall transform the above expression by introducing, in a cylindrical coordinate system, a function analogous to a plane wave in a Cartesian system:

$$\psi_m(\rho,\,\varphi) = \exp(\mathrm{i}q_m\rho\cos\varphi)\,. \tag{11}$$

This function can be called an azimuthal or angular spectral component of a BLB [11], since the angular superposition of functions described by expression (11) represents a BLB within the limits $0-2\pi$:

$$J_0(q_m\rho) = \frac{1}{2\pi}\int_0^{2\pi}\psi_m(\rho,\,\varphi)\mathrm{d}\varphi\,. \tag{12}$$

The azimuthal spectral components described by expression (11) can be used to rewrite expression (10) in the form

$$p_{1m}(\rho) = \frac{1}{2\pi}\int_0^{2\pi}p_{1m}(\rho,\,\varphi)\mathrm{d}\varphi\,, \tag{13}$$

where

$$p_{1m}(\rho,\varphi) = \frac{1}{\pi}\int_0^{\pi}\psi_{2m}(\rho,\,\varphi+\Delta\varphi)\psi_1^*(\rho,\,\varphi)\mathrm{d}(\Delta\varphi)\,. \tag{14}$$

A structural feature of the function $p_{1m}(\rho)$ makes it possible to consider this feature as the result of two consecutive averaging procedures. Initially, the product of the spectral components $\psi_{2m}(\rho,\varphi+\Delta\varphi)\,\psi_1^*(\rho,\varphi)$ is averaged over the azimuthal angle $\Delta\varphi$ [expression (14)] and then the result is averaged over the angle φ [expression (13)]. In developing this approach it should be noted that the absence of a weighting function, dependent on $\Delta\varphi$, in the integrand of expression (14) can be interpreted as a consequence of an equiprobable mutual orientation of the azimuthal components at the fundamental and doubled frequencies. However, as shown above, under conditions of selection of the type of the vector interactions the angle $\Delta\varphi$ between the plane-wave components of a fundamental-frequency BLB and the generated second harmonic is not arbitrary. This angle is determined by the phase-matching conditions and in the case of the investigated collinear phase matching it is located in the vicinity of zero.

It follows that the equiprobable mutual orientation of the azimuthal components of BLBs at the frequencies 2ω and ω is a consequence of the absence of selection of the vector interactions. Conversely, selection of the vector interactions results in unequal probability of the mutual orientations of the azimuthal components of BLBs. In the latter case, a correct mathematical description of the formation of the nonlinear polarisation can be provided by introducing a suitable weighting function $\mu_{21}(\Delta\varphi)$ in expression (14):

$$p_{1m}(\rho,\varphi) = \int_0^{\pi}\mu_{21}(\Delta\varphi)\psi_{2m}(\rho,\,\varphi+\Delta\varphi)\psi_1^*(\rho,\,\varphi)\mathrm{d}(\Delta\varphi). \tag{15}$$

The function $\mu_{21}(\Delta\varphi)$ represents the probability density of mutual orientations of pairs of plane-wave components of second-harmonic and fundamental-frequency BLBs whose orientation differs by $\Delta\varphi$. In the specific case of the nonselective interaction, we have to assume that $\mu_{21} = 1/\pi$ [expression (14)].

Introduction of the probability density function is equivalent to the assumption that, in the course of second-harmonic generation under the conditions of selection of the vector interactions, a mutual correlation appears between the azimuthal orientations of the plane-wave components of BLBs at the frequencies 2ω and ω and this happens in a certain range of angles $\delta\varphi$. A reduction in $\delta\varphi$ reduces correspondingly the angular width of the probability density function $\mu_{21}(\Delta\varphi)$. The limit when $\delta\varphi \to 0$ corresponds to the azimuthally matched interaction. Mathematically, this limit can be described on the assumption that

$$\mu_{21}(\Delta\varphi) = \delta(\Delta\varphi)\,, \tag{16}$$

where $\delta(x)$ is the delta function. Selecting in expression (15) a resonant second-harmonic mode characterised by $m = M$ and then integrating it, we obtain

$$p_{1M}(\rho,\,\varphi) = \psi_{2M}(\rho,\varphi)\psi_1^*(\rho,\,\varphi)\,. \tag{17}$$

The above expression corresponds to a certain 'pure' state in which its average value represents a product of the spectral components. It should be pointed out that expressions (16) and (17) describe the special case of the collinear interaction. In general, the conditions for longitudinal and transverse phase matching may correspond to some vector interaction and we then have $\mu_{21}(\Delta\varphi) = \delta(\Delta\varphi - \Delta\varphi_0)$.

Substituting expression (17) in expression (13) and then integrating, we find that

$$p_{1M}(\rho) \propto J_0(q_1\rho)\,. \tag{18}$$

Consequently, the nonlinear polarisation at the fundamental frequency is proportional to the first degree of the Bessel function and not to the product of the functions, as is true of expression (6). Hence, it follows that a fundamental-frequency BLB generated by the nonlinear polarisation field and described by expression (18) has the maximum overlap integral $g_M = W_{2M}^{-1/2}$. However, an estimate of this quan-tity based on the parameters employed above (Fig. 2) gives $g_M \approx 1.07\times10^4\ \mathrm{m}^{-2}$, which is approximately 20 times greater than the maximum overlap integral g_M for the uncorrelated beams.

4. Conclusions

We have thus demonstrated that the formation of azimuthally correlated Bessel beams is possible in three-wave mixing processes. It is necessary to ensure then the regime of azimuthally matched interactions, i.e. selection of the vector interactions of one type as a result of angular narrowing of the curves representing transverse and longitudinal phase matching. It is found that a nonlinear polarisation field, formed by azimuthally correlated BLBs, is also a Bessel beam and not a quadratic combination of the Bessel functions, as is true of the uncorrelated beams. The overlap integral then assumes its maximum value because of the identity of the spatial structures of the polarisation and of the field generated by the polarisation. A mutual correlation of the azimuthal components of two beams is established in the course of second-harmonic generation and autocorrelation occurs in the generation of subharmonics. Consequently, the process $2\omega \rightarrow \omega + \omega$ is accompanied by the appearance of autocorrelation in an initially uncorrelated fundamental-frequency Bessel beam. As the interaction length increases, the relative contribution of the correlated component becomes greater and the beam overlap integral increases correspondingly. It follows that the optimal scheme for observing the correlation effect is an extracavity configuration. Similar correlation effects occur also in third-harmonic generation and in parametric processes.

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