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Ring gas lasers with magneto-optical control for laser gyroscopy (invited paper)

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Abstract. The main physical principles of the operation of ring gas lasers in the laser-gyroscope regime are examined. The influence of nonreciprocal effects on the operational parameters of ring gas lasers and the methods of controlling, with the aid of the nonreciprocal magneto-optical Zeeman effect, the parameters of these lasers used in gyroscopes are discussed.

1. Introduction

The study of ring gas lasers (RGLs) [1 – 6] with magneto-optical control based on the Zeeman or Faraday effect is of great interest from both theoretical and practical points of view. The theoretical aspect of this interest involves the securing of detailed information about the dynamics of the operation of RGLs and about the influence on the latter of nonreciprocal effects, whereas the practical importance of the investigations is very great primarily for laser gyroscopy [7 – 14].

This communication describes a study of the fundamental physical principles of the operation of RGLs with magneto-optical control in the laser-gyroscope regime, in which the influence of nonreciprocal effects on the operational parameters of the RGLs is most strikingly manifested. The principal physical models of a laser gyroscope based on an RGL as well as methods for controlling the operational parameters of RGLs in the laser-gyroscope regime with the aid of nonreciprocal magneto-optical effects are discussed in this communication.

2. Main principles of the operation of a ring laser during the measurement of the angular rate of rotation

There exist numerous different optophysical ring-laser systems — gas, solid-state, and semiconductor lasers emitting one, two, and many modes, differing in the configuration of the cavity, the methods and devices for controlling the lasing parameters, the ways in which the active medium is pumped, etc. [8 – 13]. Cw He – Ne gas lasers which have an adequate gain, a high stability, and a low power consumption and are reliable, compact, and mechanically strong proved best for use in laser gyroscopy.

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The ability of bidirectional RGLs to split the cavity-eigenmode frequencies for counterpropagating waves following the introduction of a nonreciprocity into the cavity, for example during the rotation of the cavity in inertial space, is used to measure the angular rates of rotation [8 – 13, 15]. The simplest optical system of a laser gyroscope sensor is presented in Fig. 1. The output frequency (beat frequency) $\Delta\nu$, equal to the difference between the frequencies of counterpropagating waves, is proportional to the angular rate of rotation Ω:

$$\Delta\nu = M\Omega \, , \tag{1}$$

where $M = 4S/\lambda L$ is the scale factor determined by the ring-laser parameters; S and L are the area and perimeter of the ring cavity, respectively; λ is the lasing wavelength. For laser gyroscopy, it is significant that $M \gg 1$.

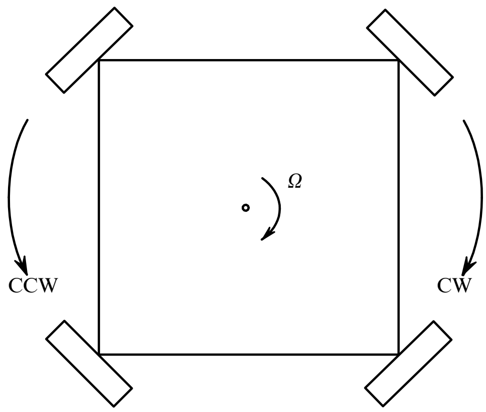

Figure 1. Four-mirror ring cavity rotating with an angular velocity Ω.

We shall now carry out certain numerical estimates. For a ring He – Ne laser ($\lambda = 0.632$ μm) with a square-shaped four-mirror cavity having a 4 cm side, the scale factor is $M \approx 6.32 \times 10^4$ (the inverse of the parameter M is called the 'measure of output pulse'). In the measurement of the angular rate of rotation of the Earth ($\Omega = 15°\ \mathrm{h}^{-1}$), $\Delta\nu \approx 4.6$ Hz.

Naturally, direct measurement of the optical frequency is difficult, so that an optical mixer (Fig. 2) is used to measure the beat frequency. It is made in the form of a prism in which the counterpropagating waves create an interference pattern recorded by a detector. In order to record the interference pattern with a specified distance between the fringes (1 – 3 mm), the angle α at the prism vertex must differ from a right angle by an angle Θ of the order of $10'' – 20''$. This device consists of a double photodetector, the two photosensitive areas of which are made in the form of parallel strips placed in the

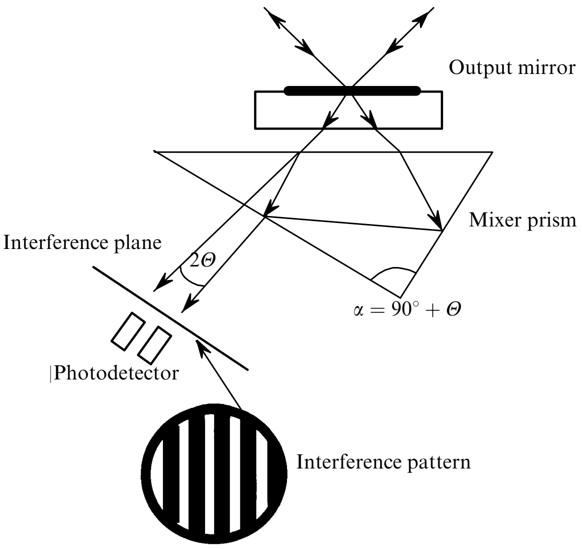

Figure 2. Optical mixer of a laser gyroscope.

plane of the interference pattern and displaced relative to one another by a quarter of the interference-pattern period.

In an ideal laser gyroscope at rest ($\Omega = 0$), the counterpropagating wave frequencies are equal ($\Delta\nu = 0$) and the interference pattern is stationary. When the laser rotates about an axis perpendicular to the plane of the configuration of the cavity (the so-called sensitivity axis), the interference pattern moves at a rate proportional to $\Delta\nu$. As a result of this, sinusoidal signals phase-shifted by a quarter of the period appear at the two photodetector outputs. The direction of rotation may be determined from the sign of the phase shift.

The sinusoidal signals from the photodetector output are converted with the aid of electronic circuits into a sequence of pulses, the number of which is counted by a bidirectional counter. The output signal of the device consists of the number of pulses during the period of the measurement (naturally taking into account a possible change in the direction of rotation during this time). Thus, a laser gyroscope is an integrating device measuring the integral angle of rotation of the laser cavity in inertial space during the data retrieval time. The data retrieval time is usually in the range 10 – 100 Hz.

The frequency characteristic (1) is linear only for an ideal laser gyroscope in which there are no unintended or unmonitored phase shifts and nonreciprocal phenomena leading to so-called false rotation. Real laser gyroscopes are characterised by significantly nonlinear frequency characteristics. The distortions of the frequency characteristics are due to the following causes: (1) displacement of the zero of the characteristic (Langmuir drift, thermal gradients in the walls of the discharge channels, mode repulsion, external magnetic fields, inhomogeneity of the refractive index along the active-channel cross section, etc.); (2) the locking of the counterpropagating wave frequencies (the presence of a dead zone near the zero owing to the counterpropagating wave coupling via backscattering on the mirrors and inhomogeneities of the cavity); (3) a change in the scale factor (for example, a change in the slope of the frequency characteristic owing to peculiarities in the dispersion properties of the active medium leading to a dependence of the degree of pulling of the lasing frequency towards the centre of the gain line on this frequency). There are also a number of other less important errors leading to the nonlinearity or distortion of the frequency characteristic. Experimental and theoretical studies have shown that the key problem in the construction of reliable laser gyroscopes is the locking of the counterpropagating wave frequencies.

In order to eliminate the locking of the counterpropagating wave frequencies, it appears attractive to shift the operating point away from the zero, i.e. from the dead zone, by the linear section of the frequency characteristic. Such a shift may be attained, for example, by unidirectional (or bidirectional) rotation of the laser gyroscope at a constant angular velocity (called the 'frequency biasing'), which permits measurements along the linear section of the frequency characteristic (near Ω_b, Fig. 3) [10 – 14, 16].

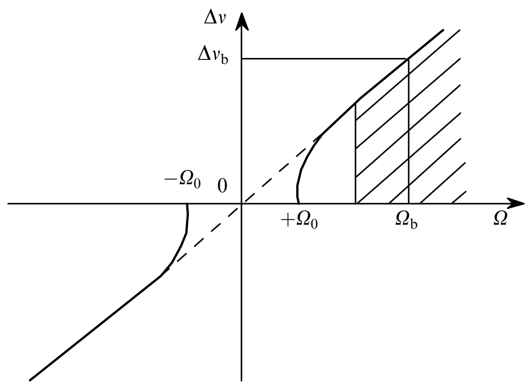

Figure 3. Frequency characteristic of a laser gyroscope (the shaded region corresponds to the linear frequency characteristic).

The method of eliminating the frequency locking by the periodic rocking of the laser-gyroscope cavity (called the mechanical biasing or vibrating bias [16 – 18]) has come to be widely employed. In order to shift the operating point to the linear section or to eliminate the locking zone, nonmechanical nonreciprocal effects are frequently used in practical laser gyroscopy, including the Faraday or Zeeman effects (the Faraday or Zeeman frequency biasing [19 – 26]). A constant shift is not then used in practice owing to the strong influence of its instability on the parameters governing the precision of the laser gyroscope; mainly bidirectional (periodic) shifts of the operating point with a change in their sign are used to subtract the corresponding errors (the magneto-optical analogue of a vibrating bias).

We shall consider the main aspect of the practical implementation of laser gyroscopes based on RGLs. Such laser gyroscopes may be conventionally divided into two main types: laser gyroscopes with a linear wave polarisation, a planar cavity configuration, and a mechanical method for the elimination of the counterpropagating frequency locking phenomenon and laser gyroscopes with a circular wave polarisation, a nonplanar cavity configuration, and a magneto-optical (based on the Zeeman and Faraday effects) method for the elimination of the influence of frequency locking. There is a possible third intermediate type — laser gyroscopes with a planar configuration, a circular wave polarisation (created by a special reciprocal device), and a magneto-optical (Faraday) method for the elimination of frequency locking. Mainly laser gyroscopes of the second type will be considered below.

Four-mirror nonplanar ring cavities are used in laser gyroscopes with magneto-optical control (Fig. 4). They ensure the stable existence of counterpropagating waves with circu-

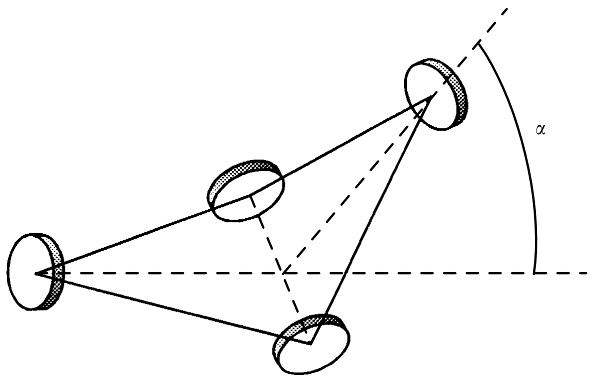

Figure 4. Optical system of a nonplanar four-mirror ring cavity (α is the angle of tilt).

lar polarisation of the radiation. These cavities, first proposed in 1967 by Soviet investigators [20], are characterised by the lifting of the degeneracy (typical of cavities with a planar configuration) of the frequencies of the cavity eigenmodes with the opposite direction of rotation of the plane of polarisation. The eigenfrequency spectrum of a cavity with a nonplanar configuration is calculated on the basis of the familiar matrix equation

$$M\boldsymbol{E} = \gamma\boldsymbol{E} , \tag{2}$$

where

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

is the matrix describing the effects of all the cavity components on the optical wave performing a circular round trip through the cavity; $\boldsymbol{E}$ is the electric vector of the optical wave; γ are the eigenvalues of the matrix M.

The physical significance of the parameter γ is simple — it describes the change in the amplitude and the shift in the phase of the optical wave during its round trip through the cavity. For the cavity eigenfrequencies, the phase shift should be a multiple of 2π. The matrix equation (2) may be converted into a system of two equations for projections of the vector $\boldsymbol{E}(E_x, E_y)$ onto the x and y coordinate axes (the z axis coincides with the direction of the round trip through the cavity), the solution of which has the form

$$\gamma_{1,2} = \tfrac{1}{2}\left\{\mathrm{Tr}\, M \pm [(\mathrm{Tr}\, M)^2 - 4\det M]^{1/2}\right\} , \tag{3}$$

$$\left|\frac{E_x}{E_y}\right|_{1,2} = \frac{M_{12}}{\gamma_{1,2} - M_{11}} . \tag{4}$$

For cavities with a nonplanar configuration, the square-root expression is always negative, $[(\mathrm{Tr}\, M)^2 - 4\det M] < 0$, which implies the existence of a phase-shift difference and hence of a frequency difference between modes with different polarisation states (as a result of the plus and minus signs before the square-root term).

In the absence of nonreciprocal effects, the round-trip matrices of cold (not containing an active medium) cavities with a nonplanar configuration and ideal mirrors are identical for the clockwise (CW) and anticlockwise (CCW) directions of the round trip and assume the following form:

$$M_{ccw} = M_{cw} = \begin{bmatrix} \cos\rho_\Sigma & -\sin\rho_\Sigma \\ \sin\rho_\Sigma & \cos\rho_\Sigma \end{bmatrix} = S(\rho_\Sigma) , \tag{5}$$

where ρ_Σ is the total angle of rotation of the image of the optical wave in the round trip through the cavity. Having substituted Eqn (5) in Eqns (3) and (4), we obtain $\gamma_{1,2} = \exp(\pm \mathrm{i}\rho_\Sigma)$ and $|E_x/E_y| = \exp(\pm \mathrm{i}\pi/2) = \pm\mathrm{i}$. Thus, in cavities with a nonplanar configuration, the normal types of vibrations have right and left circular polarisations regardless of the angle of tilt of the cavity perimeter and the phase shift between waves with right and left circular polarisations is equal to twice the angle of rotation of the coordinate system $2\rho_\Sigma$ in the round trip through the cavity.

Depending on the angle of tilt of the perimeter α (Fig. 4) and hence on ρ_Σ, the shift of the resonance frequencies of the cavity changes for the left and right circular polarisations, i.e. the cavity spectrum is modified. By altering the angle α, it is possible to obtain a cavity with the required frequency spectrum. Thus, the nonplanar cavity configuration performs two functions in the general case: first, it ensures the generation of waves with circular polarisation and, second, the degeneracy of the frequencies is lifted in it, i.e. the mutual splitting of the wave frequencies with different (left and right) circular polarisations (in a planar configuration there is no such splitting) is ensured.

As a result, four waves constitute a generalised eigenmode of a cavity with a nonplanar configuration, two of the waves propagating clockwise along the cavity perimeter and the other two propagating anticlockwise; the waves in each pair have different circular polarisations (left and right) and hence different frequencies (this is called mutual splitting). We emphasise particularly that the frequencies of counterpropagating waves with identical directions of rotation of the polarisations are identical in pairs, as a result of which each pair of counterpropagating waves for the cavity at rest (CW and CCW) has the same frequencies and the same directions of circular polarisation (one pair has both left circular polarisations and the other has both right circular polarisations).

The above eigenmodes of a ring cavity refer to a cold cavity. The laser medium (in the given instance an He – Ne gas mixture excited by a direct discharge or by RF pumping) is characterised by a gain line profile centred near $\lambda =$ 632.8 nm with a characteristic width (at the 0.5 level) of $\sim$1500 MHz. Only the eigenmodes of the cold cavity which fall within the gain band of the active medium and for which the gain exceeds the losses (taking into account the pulling of the mode frequencies to the centre of the gain line, the influence of the isotopic shift, etc.) may participate in lasing.

Two-frequency and four-frequency laser gyroscopes (in terms of the number of frequencies participating in lasing) are distinguished in laser gyroscopy. We may note incidentally that any nonreciprocity in the cavity (rotation of the cavity around the sensitivity axis or the introduction into the cavity of a Faraday or Zeeman nonreciprocity, etc.) leads to an additional (nonreciprocal) splitting of the counterpropgating wave frequencies.

In the case of two-frequency laser gyroscopes, the main problem is the attainment of the maximum possible separation of the frequencies of waves propagating in one direction with orthogonal polarisations, so that only one of them enters the gain line profile. This is possible for the angle of tilt of the perimeter $\alpha = 32°$. The total angle of rotation of the coordinate system in the round trip through the cavity is then $\rho_\Sigma = 90°$ and the eigenfrequency spectrum of such a cavity with right and left circular polarisations becomes equidistant (Fig. 5), whereas the distance between the eigenmodes proves to be $c/2L$, so that it amounts to $\sim$890 MHz for $L = 16$ cm.

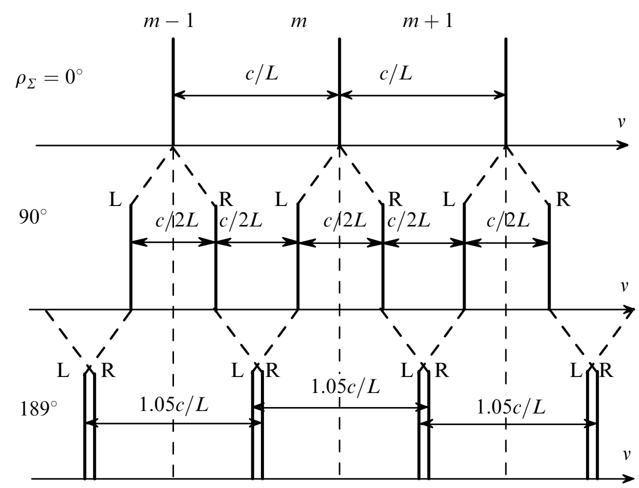

Figure 5. The eigenfrequency spectra of ring cavities as a function of the total angle of rotation ρ_Σ for a single direction of the round trip through the cavity in the case of a planar configuration (a) and also for two-frequency (b) and four-frequency (c) laser gyroscopes with a nonplanar configuration (L and R correspond to the left and right circular wave polarisations).

Thus, only one pair of the counterpropagating modes of a cold cavity may enter the centre of the gain band of the active medium (1500 MHz) and may 'attain lasing'; we may recall that this pair has the same direction of circular polarisation and is frequency degenerate in the case of a cavity at rest.

In the case of four-frequency laser gyroscopes, it is necessary to optimise the frequency differences between modes with right and left circular polarisations for the same direction, i.e. to increase still further the angle of tilt of the perimeter, so that, for example, the right-polarised mode with the number m should approach closely the left-polarised mode with the number $m+1$ (but should not coincide with it!) for the same direction of propagation (see Fig. 5, which illustrates the eigenfrequency spectrum of a cold ring cavity with $\alpha \approx 57^\circ$ and $\rho_\Sigma = 189^\circ$ in one direction of the round trip). In such a cavity, there are two pairs of counterpropagating waves with the opposite (left and right) circular polarisations and different frequencies.

Counterpropagating waves with the same polarisation are frequency-degenerate in the case of a cavity at rest for a mode with a single number. Both pairs of counterpropagating waves enter the centre of the gain band of the active medium. We may note in addition that, since the active gas in an He–Ne RGL is neon, having the two stable isotopes ^{20}Ne and ^{22}Ne, a mixture containing equal amounts of both isotopes is usually employed in a four-frequency sensor. The resulting gain line is broadened in this case (compared with a single-isotope gas), which ensures a more stable operation of the four-frequency laser gyroscope.

The four-frequency regime in a laser gyroscope makes it possible to lower to an appreciable degree the effect of external influences (for example, of an external magnetic field) on the parameters of the laser gyroscope, which may be regarded in a certain sense as two two-frequency laser gyroscopes in a single cavity. As a result of the appropriate treatment of data, it is possible to minimise (subtract) the above influence and the measurement of the angular rate of rotation is then additive.

Naturally, the real modes of a cold cavity must be represented not as ideal δ-functions, as shown in Fig. 5, but in the form of spectral lines with a definite width, which depends on the losses in the cavity. The characteristic mode width for a high-quality cavity is 0.1–1 MHz.

3. Interaction of counterpropagating waves in real RGLs

Despite their very high quality, the mirrors and other intracavity components of laser gyroscopes nevertheless scatter laser radiation. As a result of this, a forward CW wave is scattered and induces its phase in the counterpropagating CCW wave; a counterpropagating CCW wave induces its phase in the forward CW wave through scattering in exactly the same way. Hence, both counterpropagating waves are frequency-locked and the laser gyroscope becomes insensitive to low rates of rotation; this in fact constitutes the phenomenon of the synchronisation or locking of counterpropagating waves.

Thus, even negligible scattering leads to the total loss of sensitivity by the laser gyroscope to angular rotation in a certain range of angular velocities, called the locking range (Fig. 3). We may note that this real frequency characteristic gradually attains the ideal linear relationship with increase in Ω.

Several procedures have been proposed in order to eliminate the counterpropagating wave frequency locking. One of these is self evident — it involves the reduction or total elimination of scattering by dielectric mirrors and other optical components of the laser-gyroscope cavity. However, one should note that virtually all the possibilities have been either exhausted in this procedure or will soon be exhausted in the immediate future. Thus, the reflection coefficient of laser mirrors is nowadays 99.995%–99.9995% and each '9' after the decimal point is attained at the expense of enormous technological efforts and financial cost.

The essential physical nature of the locking effect can be elucidated as follows. The generation of two counterpropagating waves in an RGL is analogous to the coupled oscillations of two weakly coupled oscillating systems. The counterpropagating waves are coupled (there is energy exchange) in RGLs via the backscattering of light on the cavity mirrors and other intracavity components. The scattering is into a solid angle of 4π sr, but only the part of the scattered radiation which enters into the solid angle of the active transverse mode (in laser gyroscopes, lasing occurs as a rule only in the main transverse mode TEM_{00}) participates in the energy exchange between counterpropagating waves.

The coefficient r of light scattering into a mode, considered in terms of the amplitude of the electric field of an optical wave applicable to one mirror, may be estimated from the formula

$$r = r_s \frac{\lambda}{\pi D}, \qquad (6)$$

where D is the diameter of the waist of the cavity caustic; r_s is the integral coefficient of light scattering, in terms of amplitude, into the solid angle 4π sr. For typical He–Ne RGLs ($D = 0.1$ cm, $r_s^2 = 10^{-4}$–10^{-5}), the coefficient of scattering r^2 into a mode, in terms of power, is only 10^{-12}–10^{-13}. However, even this low coefficient exerts a significant influence on the dynamics of lasing in RGLs and laser gyroscopes based on them.

3.1 Counterpropagating-wave lasing frequencies in RGLs in the presence of coupling

Stable or unstable lasing of counterpropagating waves in RGLs depends on the characteristics of the active medium and the cavity. We shall consider what is in practice the most interesting case, namely the stable lasing of two counterpropagating waves, which is achieved by eliminating the competitive interaction of counterpropagating waves in the active medium.

The competition between counterpropagating waves arises owing to their interaction on the same excited atoms of the active medium. For example, in a single-isotope gas mixture of an He – Ne laser with an He and Ne partial pressure ratio of $\sim 10:1$, the uniform width within the Doppler gain profile is $\Delta\nu_0 \sim 100$ MHz when the total width of the profile is ~ 1700 MHz.

If the mode of the cavity is close to its centre (with a detuning not greater than 0.5 $\Delta\nu_0$), a strong competition between the counterpropagating waves is observed. In order to suppress it, one may use a mixture of the ^{20}Ne and ^{22}Ne isotopes, the centres of the active transitions of which are displaced relative to one another by 875 MHz. As a result of this, when the cavity mode is tuned to the centre of the overall Doppler line, the 'wings' of the partial ^{20}Ne and ^{22}Ne profiles, where competition is virtually ruled out, will participate in the amplification of the counterpropagating-wave frequencies.

There are also other methods of suppressing the competition between counterpropagating waves, in particular the use of the circular polarisation of the radiation and of the Zeeman splitting of the gain profiles for counterpropagating waves with the opposite directions of rotation of the polarisation vectors.

In the presence of phase nonreciprocity for counterpropagating waves in the RGL cavity (for example, on rotation of the laser), the profiles of the cavity modes of the CW and CCW waves are split. In the absence of energy coupling of the waves and competitive effects, the lasing frequencies of the CW and CCW waves followed the centres of the cavity modes, i.e. the lasing frequencies are also split (to the same extent proportional to the phase nonreciprocity of the cavity).

However, since in real lasers there is always energy coupling of the counterpropagating waves, this ideal physical model of the splitting of the counterpropagating-wave frequencies becomes significantly more complex.

In the absence of competition between counterpropagating waves, when there is stable bidirectional lasing in RGLs, the difference between the wave frequencies may be described by the approximate equation [cf. Eqn (1)]

$$\Delta\nu = \frac{\mathrm{d}\Psi}{\mathrm{d}t} = M\Omega_{\mathrm{n}} - M\left(\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\cos\beta\right)^{1/2} \times \sin(\Psi + \beta) \,, \tag{7}$$

where Ψ is the difference between the phases of the counterpropagating waves; Ω_{n} is the total phase nonreciprocity of the cavity (the angular velocity of the rotation of the laser gyroscope also enters into Ω_{n}); $\sigma_{1,2} = r_{1,2}cE_{2,1}/LE_{1,2}$ are the amplitudes of the coupling coefficients of the counterpropagating waves; $r_{1,2}$ are the relative coefficients of the total backscattering of the waves into the solid angle of the cavity mode; $E_{1,2}$ are the amplitudes of the counterpropagating-wave fields; $\beta = \beta_1 - \beta_2$ is the difference between the backscattering phases of the counterpropagating waves. The locking range is defined by the parameter

$$\Omega_0 = \left(\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\cos\beta\right)^{1/2} \,. \tag{8}$$

When account is taken of Eqn (8), Eqn (7) may be represented in the form

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = M\Omega_{\mathrm{n}} - M\Omega_0\sin(\Psi + \beta) \,. \tag{9}$$

The coefficients $r_{1,2}$, which enter into Eqns (7) – (9), are determined by the interference of all the waves scattered in the cavity. Depending on their mutual phases, the coefficients $r_{1,2}$ may vary within wide limits (theoretically from zero to the sum of their specific values). The phase difference β is determined by the nature of the scattering centres on the mirror surfaces (if these centres constitute permittivity inhomogeneities, then $\beta = 0$; if scattering is only by absorption inhomogeneities, then $\beta = \pi$).

In real lasers, the mirrors exhibit both absorption and scattering on permittivity inhomogeneities, so that $\beta \neq 0$; a nonzero locking range of Ω_0 is also always present in them. By increasing the quality of the laser-gyroscope mirrors, it is possible to reduce the backscattering by the latter and thereby decrease the locking range. Another effective possibility of diminishing the locking range is selection of the mutual phases of the waves scattered on the mirrors such that the total intensity of all the scattering waves is minimal; this can be achieved by the precise regulation of the distances between the mirrors [12].

However, even in the most favourable case, Ω_0 proves to be fairly large (for example, compared with the rate of rotation of the Earth). Indeed, assuming that the amplitudes of the counterpropagating waves and the total coefficients of their backscattering are equal, which is close to the real situation ($r_1 = r_2 = r$, $\sigma_1 = \sigma_2 = \sigma$), while $\beta = \pi/2$, we obtain an approximate formula for the estimation of the width of the locking range:

$$\Delta\nu_0 = \frac{\Omega_0}{2\pi} \approx \sigma\sqrt{2} = \frac{rc\sqrt{2}}{2\pi L} \,. \tag{10}$$

In integrated scattering from all four mirrors $r = (2 - 0.66) \times 10^{-6}$ for a laser with $L = 16$ cm, we obtain $\Delta\nu_0 = (1.7 - 0.5) \times 10^3$ Hz, which exceeds by more than two orders of magnitude the counterpropagating-wave frequency splitting induced by the rotation of the Earth (4.6 Hz). We may note that the possibilities for reducing the locking range by purely technological procedures have now been virtually exhausted, so that it is necessary to seek other ways of eliminating its influence.

Depending on Ω, Eqn (9) has two different solutions. For $\Omega < \Omega_0$, i.e. within the locking range, we evidently have $\mathrm{d}\Psi/\mathrm{d}t = \Delta\nu = 0$. For $\Omega > \Omega_0$, i.e. outside the locking range, the solution of Eqn (9) assumes the form [16]

$$\Delta\nu = M\left(\Omega^2 - \Omega_0^2\right)^{1/2} \tag{11}$$

(Fig. 3). In real laser gyroscopes, Ω may depend not only on the rotation but also on other, in the given instance harmful, nonreciprocal effects (for example on the Fizeau effect induced by the migration of the gaseous medium in the active channel of the laser gyroscope, the Langmuir effect, the Faraday effect in optical anisotropic units of the laser gyroscope, etc. [12, 16]). Evidently, all these nonreciprocal effects, leading to the false 'rotation' of the laser gyroscope, have a negative influence on the precision of the measurements of the angular motions of the laser gyroscope.

3.2 The use of magneto-optical nonreciprocal effects for the linearisation of the frequency characteristic of a laser gyroscope

As already mentioned, in order to shift the operating point of a laser gyroscope to the linear section of the frequency characteristic, it is necessary to introduce into the RGL cavity a controlled phase nonreciprocity Ω_b (Fig. 3) created either by the mechanical rotation of the laser gyroscope at a constant velocity (bidirectional rotation by an angle specified with a high degree of precision is used in practice), or by the mechanical vibration of the laser-gyroscope cavity (vibrating bias), or by a magneto-optical device based on the Zeeman and Faraday effects.

The use of the Zeeman effect [1] is extremely attractive. Its essential feature is, briefly, that when a permanent longitudinal magnetic field is applied, the Doppler gain profile of the active medium is split into two components symmetrically displaced relative to the centre of the initial unsplit profile (Fig. 6). The shift is defined by the formula [1]

$$\Delta\nu_{\mathrm{D}}^{(\pm)} = \pm\frac{g\beta H}{\hbar} \ll \Delta\nu_{\mathrm{D}} , \tag{12}$$

where g is the splitting factor; β is the Bohr magneton; $\hbar$ is the Planck constant; $\Delta\nu_{\mathrm{D}}$ is the width of the unsplit profile. A high-frequency profile with the central frequency $\Delta\nu_{\mathrm{D}}^{(+)} = \nu_0 + g\beta H/\hbar$ then amplifies only the wave with right circular polarisation, whereas the lower-frequency profile $\Delta\nu_{\mathrm{D}}^{(-)} = \nu_0 - g\beta H/\hbar$ amplifies only the wave with left circular polarisation. If the system is viewed along the wave vectors of the waves, then the pair of waves with the same circular polarisation is amplified, for example, only waves with left or right polarisation, depending on which pair of waves entered the initial unsplit gain profile.

Thus, if the frequency of the cavity modes of counterpropagating waves with right circular polarisation was tuned to the centre of the unsplit line in a two-frequency laser gyroscope whereas the vector $\boldsymbol{H}$ is directed clockwise along the profile channel (Fig. 1), then the CW wave will be amplified by the split profile with the frequency $\Delta\nu_{\mathrm{D}}^{(+)}$. On the other hand, the CCW wave will be amplified by the profile with the frequency $\Delta\nu_{\mathrm{D}}^{(-)}$.

Hence, as a result of the superposition of the longitudinal magnetic field on the active medium of the RGL the gain line profile was split into two profiles, so that the frequency of the cavity mode of both waves (CW and CCW) ceased to correspond to the centre (maximum) of the gain line. As a consequence of the linear pulling of the cavity-mode frequencies to the gain line centre [1], the initially frequency-degenerate cavity mode line profile corresponding to the CW and CCW waves is also split into two individual profiles for the CW and CCW waves (Fig. 6).

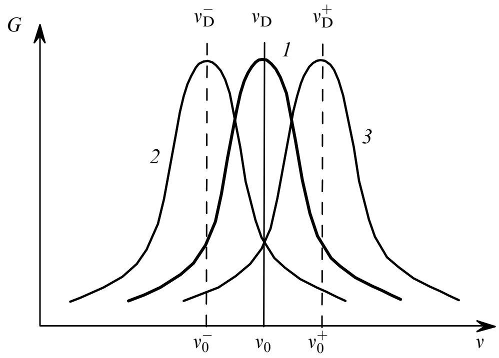

Figure 6. The gain profile of an active gas mixture in the absence of a magnetic field (1) and low-frequency (2) and high-frequency (3) profiles arising from the imposition of a longitudinal magnetic field.

If the initial tuning of the frequency of the CW and CCW modes to the maximum of the unsplit profile (for $H = 0$) has been performed, for example, for a pair of counterpropagating waves with right circular polarisation, then, after the imposition of the field, the cavity CW mode shifts towards the centre of the high-frequency profile ($\nu_D^{(+)}$), whereas the CCW mode shifts towards the centre of the low-frequency profile ($\nu_{\mathrm{D}}^{(-)}$). The splitting of the cavity modes is defined by the formula

$$\Delta\nu_{\mathrm{b}} = 2\delta\nu_{\mathrm{D}}\frac{\Delta\nu_{\mathrm{r}}}{\Delta\nu_{\mathrm{D}}}\frac{G_0}{k} \ll \Delta\nu_{\mathrm{D}}^{(\pm)} , \tag{13}$$

where $\delta\nu_{\mathrm{D}} = 2\Delta\nu_{\mathrm{D}}^{(\pm)} = 2g\beta H/\hbar$ is the total splitting of the gain profiles; $\Delta\nu_{\mathrm{r}}$ is the width of the cavity mode; G_0 is the maximum of the gain line profile; k is the loss coefficient.

For $\Delta\nu_{\mathrm{b}} \gg \Delta\nu_0$, it is possible to neglect the locking effects and the corresponding shift of the counterpropagating wave frequencies, so that the frequencies of the actually generated CW and CCW waves follow the centres of their cavity modes. For this reason, the splitting of lasing frequencies is virtually equal to $\Delta\nu_{\mathrm{b}}$ defined by formula (13). Thus, as a consequence of the imposition of a longitudinal magnetic field, a difference between the lasing frequencies of the counterpropagating waves arises in the laser gyroscope at rest. The lasing-frequency difference plays the role of a 'frequency biasing' which displaces the operating point onto the linear section of the frequency characteristic of the laser gyroscope.

Assuming that $\Delta\nu_{\mathrm{D}} = 1500$ MHz, $\Delta\nu_{\mathrm{r}} = 1$ MHz, $G_0/k \approx 2$; and $H = 20$ Oe, we obtain $\delta\nu_{\mathrm{D}} \approx 72$ MHz; $\Delta\nu_{\mathrm{b}} = 192$ kHz $\ll \Delta\nu_{\mathrm{D}}$. Since, as stated earlier, the locking band is $\Delta\nu_0 = (0.5-1.7)\times 10^3$ Hz, the condition $\Delta\nu_0 \gg \Delta\nu_{\mathrm{b}}$ holds, which ensures a high degree of linearity of the section of the laser-gyroscope frequency characteristic near the new operating point $\Delta\nu_{\mathrm{b}}$, which can now be adopted as the zero point of the readings.

We may note that, together with the Zeeman effect, it is possible to employ also the magneto-optical Faraday effect. However, there are serious technical difficulties in the implementation of magneto-optical methods for the elimination of the influence of the locking effect. They are associated with the impossibility of implementing the requirements as regards the stability of the corresponding parameters. The laser gyroscope used in practice must afford on average a precision of the determination of the angular velocity to within at least $\delta\Omega \sim 0.01-1^\circ$ h^{-1}. For the example of the laser gyroscope ($L \approx 16$ cm) examined in the present study, this requires a long-term stability of the biasing frequency at the level

$$\delta(\Delta\nu_{\mathrm{b}}) \leqslant M\delta\Omega \approx 0.003 - 0.3 \text{ Hz} ,$$

which corresponds to a relative stability of the frequency biasing of $\sim 1.5\times(10^{-8}-10^{-6})$. The long-term stabilities of all the laser-gyroscope parameters entering into formulas (12) and (13), including also the magnetic field, should be of the same order of magnitude, which cannot be achieved in practice.

3.3 Parametric modulation in RGLs with magneto-optical control

In 1968–69, Soviet investigators used for the first time an alternating-sign magneto-optical biasing (based on the Zeeman effect), which reduces to the alternating-sign periodic modulation of the parameters of the active medium of the RGL by an alternating magnetic field. As can be readily seen from Fig. 6, owing to such parametric modulation the high-frequency and low-frequency gain profiles periodically change places, inducing thereby a periodic change in the sign of the frequency biasing $\Delta\nu_b$ [see formulas (12) and (13)]. The modulation frequency f_b is usually in the range 200–1000 Hz.

By an appropriate treatment of the output signal, modulated by the frequency f_b, it is possible to eliminate (subtract) its alternating-sign component, leaving only the signal due to the useful nonreciprocal effect (i.e. rotation). Such subtraction is achieved in one modulation period, i.e. during a time 1–5 ms. The stability of the biasing frequency must be ensured precisely during this time, which is technically relatively easy, whereas in the case of a laser gyroscope with a constant magnetic displacement such stability must be ensured throughout the entire period of the operation of the laser gyroscope.

The parametric modulation of the RGL parameters by a periodic magnetic field leads to new physical effects, primarily to the appearance of parametric resonances [5, 18, 27–29], which in the case of real laser gyroscopes are manifested as dynamic synchronisation (locking) ranges. The static locking range with the width $2\Delta\nu_0$ then virtually vanishes. The above locking ranges are distributed on the frequency characteristic of the laser gyroscope with a frequency interval f_b (Fig. 7) and the number of such ranges on the section $[0;\ \Delta\nu_b]$ may be several hundreds. It is easily seen that, with all the advantages of the alternating-sign magneto-optical biasing, the presence of many locking ranges leads to a significant nonlinearity of the frequency characteristic. Methods for an appreciable decrease in the width of the ranges exist but their description is outside the scope of the present communication.

In the presence of parametric modulation, Eqn (9) must be rewritten in the form

$$\frac{d\Psi}{dt} = M\Omega - M\Omega_0 \sin(\Psi + \beta) + \Delta\nu_b F(t) \ , \qquad (14)$$

where $F(t)$ is a periodic alternating-sign function describing the nature of the modulation. Such an equation has been analysed in studies [30, 31] where it was shown that the widths of the locking ranges $\delta\nu_i$ (i is the number of the range) are linked, to a first approximation, to the width of the static locking range $\Delta\nu_0$ by the relationship

$$(\Delta\nu_0)^2 = \sum_i (\delta\nu)_i^2 \qquad (15)$$

and are determined by the form of the time variation of the alternating-sign magnetic field (sine, square wave, etc.) and also by $\Delta\nu_b$ and $\Delta\nu_0$. For $\Delta\nu_b = 0$ ($H = 0$), the dynamic locking ranges vanish, whereas the static locking range is fully restored.

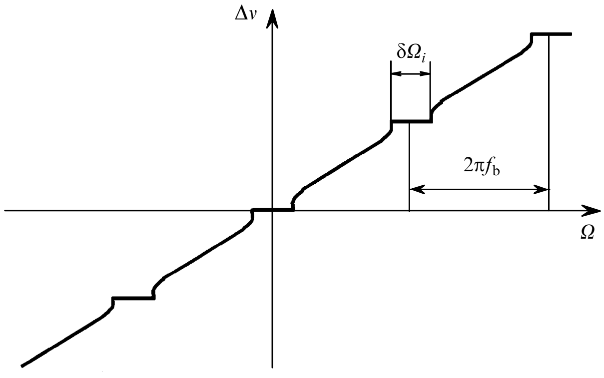

Figure 7. Frequency characteristic of a real laser gyroscope with parametric modulation of the active-medium parameters by an alternating-sign periodic magnetic field with a frequency f_b. The dynamic locking ranges can be seen.

4. Experimental results

A series of different types of Russian-produced two- and four-frequency laser-gyroscopic sensors with magneto-optical control based on the Zeeman effect (ZLK series – Zeeman ring lasers) have now been created. The external form of the two-frequency sensors of the ZLK series is illustrated in Fig. 8. The ring cavity is formed by four mirrors, three of which are planar while the fourth is spherical. Two mirrors, located on a single diagonal, are attached to precision piezoelectric drives for the monitoring and correction of the cavity perimeter. The ZLK-series sensors differ in the perimeter (from 120 to 200 mm) and their all-solid cavities are made from a Sital — a material with an ultralow coefficient of thermal expansion. The mirrors of the cavity are attached to its casing by the optical-contact method. For this purpose, the basal planes of the casing and of the surfaces of the mirror substrates are subjected to superpolishing. The gas discharge was struck in all four cavity arms, in which the current flowed in opposite direction in pairs. A permalloy shield diminishes the influence of external magnetic fields. In order to increase the precision of the output data, provision was made for the algorithm correction of the zero drift in the electronic supply and control units.

The main parameters of the series-ZLK gyroscopic sensors are presented in Table 1, from which it follows that in the steady-state (after heating) regime the stability of the displacement of the zero is $0.03-0.05^\circ\ h^{-1}$, which is comparable with the parameters of the precision of laser gyroscopes with a planar configuration, linear polarisation, and a

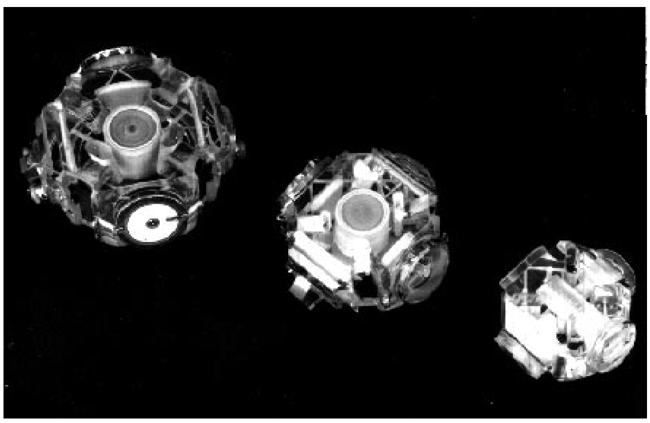

Figure 8. Photographs of ZLK series sensors. From left to right: ZLK-20, ZLK-16, ZLK-12 (the numerals denote the cavity perimeter).

Table 1. Parameters of laser gyroscopes based on He – Ne RGLs.

Number of frequencies	ZLK	Regime	Cavity length	Scale factor/pulse^{-1}	Stability of zero displacement/$^\circ$ h^{-1}	Reproducibility of zero displacement/$^\circ$ h^{-1}	Reproducibility of scaling coefficient/million^{-1}	Random drift/$^\circ$ h$^{-1/2}$
2	ZLK-12	Steady-state	12.8	4	0.1	0.3	–	0.02
	ZLK-16		16.0	3.3	0.05	0.15	–	0.01
	ZLK-20		20.0	2.7	0.05	0.15	–	0.01
	ZLK-12	Transitional	12.8	4	–	5 – 10	200	–
	ZLK-16		16.0	3.3	–	1 – 3	100	–
	ZLK-20		20.0	2.7	–	1 – 2	100	–
4	ZLK-28	Steady-state	28.5	3.0	0.03	0.1	–	0.005
		Transitional	28.5	3.0	–	0.5	50	–

mechanical vibrating bias. In a nonsteady-state regime (immediately after switching on), the reproducibility of the displacement of the zero is sharply impaired, but in this regime too the precision parameters are found to be quite satisfactory for many practical applications; the reduced precision in the transitional regime is compensated by the short (of the order of several seconds) time in which they become ready for use.

The possibilities of compact RGL-based laser gyroscopes with magneto-optical control are at present far from exhausted. A decrease in the intracavity losses and scattering, attainable primarily by the improvement of the quality of the cavity mirrors, a decrease in the locking range, an increase in the precision of the construction and adjustment of the cavity, algorithm correction for the reproducible dependences of the zero drift on temperature and other external conditions, etc., would make it possible in the immediate future to raise the stability of the zero displacement in the steady state to $0.01 - 0.03^\circ$ h^{-1} with a random drift up to 0.003° h$^{-1/2}$ [26], which would significantly expand the range of applications of the laser gyroscopes under consideration.

5. Conclusions

Ring gas lasers with magneto-optical control and a nonplanar cavity configuration make it possible to construct effective and compact laser gyroscopes which are resistant to external influences and have precision parameters sufficient for many practical applications such as integration of satellite-navigation systems for civil aviation, the monitoring and control of movement in automobile and railway transport, the navigation of small and medium-size vessels, the construction of robots, etc.

In the present communication, an attempt was made to elucidate the current state of the problems of the employment of RGLs with magneto-optical control in laser gyroscopy. Naturally, a whole series of extremely important questions were disregarded by the authors. They include a detailed analysis of the frequency characteristics of a real laser gyroscope, taking into account all the accompanying physical reciprocal and nonreciprocal effects, the role of the nonlinearity of the scale factor and the methods for the elimination of its influence, allowance for the state of the surfaces of precision optical components and units in laser gyroscopes (roughness, microprofile, correlation lengths, the presence of a defective layer and various surface states in the latter, etc.), the physics of strong optical contact, the physics of a discharge in a gaseous medium and of its interaction with the surfaces of active channels and mirrors, the precision metrology of ultrareflecting mirrors and high-quality laser cavities, etc.

Unfortunately it is noteworthy that the participation of investigators from the Russian Academy of Sciences and Higher Educational Establishments in the solution of these frequently fundamental problems of laser gyroscopy is still clearly inadequate and the authors' sincere hope is that the publication of the present communication will help draw attention to these problems.

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