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The influence of frequency nonreciprocity on the emission dynamics of solid-state ring lasers (review)

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Abstract. The results of theoretical and experimental studies of characteristics of solid-state ring lasers (SSRLs) operating in different regimes of lasing in the presence of optical nonreciprocity in the laser cavity are analysed in the context of possible applications of such systems in laser gyroscopy. The influence of frequency nonreciprocity Ω on the existence and stability of stationary self-modulation lasing regimes and the characteristics of laser radiation in these regimes is considered. We discuss different methods for detecting Ω, which are based on the measurement of self-modulation and beat frequencies, relaxation frequencies and the relevant decay rates, the ratio of mean intensities of counterpropagating waves, the modulation depth of the intensities of these waves, etc.

1. Introduction

Solid-state ring lasers (SSRLs) attract considerable attention from researchers working in the area of laser physics. Recent progress in the development of diode-pumped slab solid-state lasers (chip lasers) has demonstrated that such lasers offer much promise in the context of technological applications.

A solid-state ring laser with a homogeneously broadened gain line is a complex nonlinear system, where the competition and coupling of counterpropagating waves may give rise to specific regimes of uni- and bidirectional lasing that are not observed in linear lasers. A characteristic feature of the emission dynamics in SSRLs is the high sensitivity of laser emission to the frequency nonreciprocity of the laser cavity. This allows high-precision studies of various nonreciprocal effects to be performed [1].

From the viewpoint of applications it is of considerable interest to explore the possibility of using SSRLs for measuring the angular rotation velocity ω_{rot}, which is particularly important for laser gyroscopy. The frequency nonreciprocity Ω of a laser cavity (which is equal to the difference in cavity eigenfrequencies for counterpropagating waves, $\omega_{1\mathrm{c}} - \omega_{2\mathrm{c}}$) is due to the rotation of this cavity and is related to the angular rotation velocity ω_{rot} by the following formula:

$$\frac{\Omega}{2\pi} = \frac{4S\omega_{\mathrm{rot}}}{\lambda L} = M\omega_{\mathrm{rot}} ,$$

where L and S are the perimeter length and the contour area of the cavity, and λ is the lasing wavelength. Note that, for real ring lasers, the scaling coefficient $M = 10^2 - 10^5$.

The nonlinear dynamics of SSRLs is highly sensitive to the number of axial and transverse oscillation modes involved in lasing and the nature of interaction between these modes. The number of axial modes involved in lasing and the interaction between these modes in the regime of free-running lasing differ considerably from the number of such modes and the interaction between them in the regime of mode locking, which arises when laser parameters are modulated with the intermode-beat frequency. The dynamics of SSRLs generating light beams in the fundamental transverse mode for each direction is now well understood [2 – 5].

The purpose of this paper is to analyse the influence of the frequency nonreciprocity of a ring cavity on the dynamics of lasing and emission characteristics of a continuous-wave (cw) SSRL in free-running lasing regimes.

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2. Schemes of SSRLs and the methods of controlling lasing regimes

Recently, a number of ring laser schemes varying in design (lasers consisting of discrete elements [6 – 8] and slab [9 – 12] and semislab ring lasers [13, 14]) and the methods used to excite the active medium (flashlamp pump or monochromatic pump using gas or diode lasers) have been studied.

The stability of laser parameters is an important characteristic, which determines whether a detailed analysis of the nonlinear dynamics of SSRLs can be performed. The instability of coupling of counterpropagating waves is one of the main factors limiting the stability of laser parameters [15].

Whereas technical perturbations do not allow one to achieve highly stable coupling in flashlamp-pumped SSRLs, slab lasers and diode pump open the way to elimination of this difficulty. However, one should also take into account external spurious coupling due to the backreflection of radiation from both optical elements of the excitation system and the elements of the receiving channel, which influences the amplitude and the phase of coupling of counterpropagating waves in an SSRL [15]. These effects become noticeable when the output coupler has a high transmission coefficient (higher than 1%).

The instability of the modulus and the phase of coupling coefficients limits the stability of the amplitude, frequency, and polarisation of both the lasing process and the self-modulation and parametric oscillations in an SSRL. Note that, because of small sizes of SSRLs ($L < 3$ cm), the frequency separation between the longitudinal cavity modes is large ($c/L \sim 10$ GHz), which influences the emission dynamics of SSRLs.

Diode-pumped slab lasers are an optimal choice for detailed studies of the nonlinear dynamics of SSRLs and for many technical applications. This laser scheme reduces technical perturbations to a minimum and ensures satisfactory thermal stabilisation (because of the small size of the lasers and reasonably low heat release).

The overwhelming majority of SSRLs employ Nd : YAG crystals, characterised by favourable operational characteristics (high thermal conductivity, optical uniformity, low lasing threshold, etc.).

Lasing regimes in SSRLs are usually switched by changing the complex coupling coefficients of counterpropagating waves. Therefore, by simply varying the coupling of counterpropagating waves or by introducing frequency or amplitude nonreciprocity, one can implement virtually all the lasing regimes in the same SSRL. Usually, coupling coefficients in SSRLs consisting of discrete elements can be easily changed through small variations in the length of the cavity contour.

The existing schemes of slab (semislab) ring chip lasers allow the working cavity contour to be shifted slightly within the slab. Thus, by adjusting the direction of the pump beam, one can vary the coupling of counterpropagating waves (through the change in the character of backscattering of counterpropagating waves from microinhomogeneities in the active me-dium, slab faces, and cavity mirrors). Another method of switching the lasing regimes employs an additional extra-cavity feedback [16, 17]. Finally, the coupling between counterpropagating waves can be changed under conditions of nonuniform heating or mechanical deformations of the active element (note that uniform heating does not change this coupling).

3. The main experimental results

3.1 The influence of frequency nonreciprocity on stationary regimes of lasing

A number of studies have been devoted to the experimental investigation of the emission dynamics of SSRLs (e. g., see Refs [18 – 28]). These studies have shown that a strong coupling between counterpropagating waves in an SSRL gives rise to a standing-wave regime in an SSRL, which is characteristic of cross-mode locking in counterpropagating waves. Regardless of the frequency nonreciprocity of the cavity, counterpropagating waves have equal intensities and frequencies in this regime. A strong coupling between counterpropagating waves may occur, for example, in lasers consisting of discrete elements, because of the reflection of light from the ends of the active element.

Along with the standing-wave regime, another stationary regime may occur in SSRLs, implying mode locking in counterpropagating waves when these waves have equal frequencies and different time-independent intensities. The ratio of the intensities of counterpropagating waves in this regime depends not only on the coupling coefficients of counterpropagating waves, but also on the amplitude and/or frequency nonreciprocity of the laser cavity [29].

The running-wave regime is a particular case of mode locking in counterpropagating waves. In this regime, most of the energy stored in the cavity is emitted in one direction, whereas radiation emitted in the opposite direction results from scattering due to intracavity optical inhomogeneities. However, because of the importance of this regime for applied problems related to single-frequency lasing, the running-wave regime is sometimes considered as a separate regime rather than the limiting case of mode locking in counterpropagating waves.

Within a certain range of parameters (absolute values and phases of coupling coefficients, the excess of the pump over the threshold, frequency detuning from the gain-line centre, amplitude and frequency cavity nonreciprocities, etc.), self-modulation regimes of the first and second types, the beat regime, and various nonstationary regimes, including the regime of dynamic chaos, may exist in SSRLs.

The running-wave regime can be implemented by inserting some amplitude nonreciprocal element into the laser cavity [1]. As demonstrated in Refs [30 – 32], single-frequency unidirectional lasing may arise when a magnetic field is applied directly to the active element.

Smyshlyaev et al. [33] and Nilsson et al. [34] have shown that the use of a nonplanar cavity, ensuring the relative rotation of polarisation planes, is favourable for the implementation of single-frequency unidirectional lasing. Experiments [35 – 40] were devoted to the investigation of diode-pumped slab SSRLs operating in the running-wave regime arising in a system where the counterpropagating waves are characterised by different magnitudes of losses in an active element placed in a magnetic field. A record stability of the lasing frequency has been achieved with such lasers [36, 37].

Unidirectional single-frequency lasing can be also obtained with the use of acousto-optical nonreciprocal elements [1]. Acousto-optical nonreciprocity may be introduced into either into the active element or into an additional element inserted into the SSRL cavity. Unidirectional lasing may arise in an SSRL with an additional backreflector, leading to inequality of the absolute values and the phases of coupling coefficients and giving rise to an amplitude nonreciprocity in the laser cavity [41].

Klochan et al. [42] have demonstrated that unidirectional lasing can be implemented due to frequency nonreciprocity of the laser cavity. Unidirectional lasing can also be obtained in the presence of self-illumination waves [43].

Although the running-wave regime seems to be very simple, the dynamics of SSRL lasing in this regime is rathercomplicated. The problem is that, even when one of the counterpropagating waves is suppressed strongly, the second wave cannot be suppressed completely, which gives rise to additional noise in the lasing channel of the fundamental wave and specific features in the dynamics of lasing in such a laser [44].

Experimental studies have shown that an SSRL in the running-wave regime is characterised by three relaxation frequencies [4, 45]. In the first-order approximation, the main relaxation frequency can be described satisfactorily by the formula

$$\omega_{\mathrm{r0}} = \left(\frac{\omega\eta}{QT_1}\right)^{1/2}, \qquad (1)$$

which corresponds to conventional in-phase relaxation oscillation of the radiation power (ω/Q is the cavity band width, T_1 is the relaxation time of population inversion, and η is the excess of the pump power over the threshold).

In a cavity without amplitude and frequency nonreciprocities, the second and third relaxation frequencies coincide with each other and are given by [45]

$$\omega_{r1,\,r2} = \left(\frac{\omega\eta}{2QT_1}\right)^{1/2}. \qquad (2)$$

As demonstrated by experimental studies, a frequency nonreciprocity splits the low-frequency component in the spectrum of relaxation frequencies into two spectral components, whose frequencies depend on Ω.

3.2 Self-modulation regimes of lasing in the presence of frequency nonreciprocity

The results of investigations show that various self-modulation regimes may occur in SSRLs, including:

— in-phase and antiphase self-modulation regimes of the first type;

— self-modulation regimes of the second type (with low-frequency periodic switching of the lasing direction);

— quasi-periodic self-modulation regimes;

— regimes of dynamic chaos;

— beat regimes, when the counterpropagating waves have different frequencies.

The waveform of oscillations in self-modulation regimes of the first type is similar to a sine function. In the absence of nonreciprocities in the laser cavity, the frequency of self-modulation oscillations is determined by the coupling coefficients of counterpropagating waves and may range from tens of kilohertz up to several megahertz. This regime is usually characterised by a single-frequency emission spectrum in longitudinal modes. In self-modulation regimes of the second type, the frequency of self-modulation oscillations ω_{m} depends on the frequency nonreciprocity Ω. This dependence is satisfactorily described (in the first-order approximation) by the formula

$$\omega_{\mathrm{m}} = \left(\omega_{\mathrm{m0}}^2 + \Omega^2\right)^{1/2}, \qquad (3)$$

where ω_{m0} is the frequency of self-modulation oscillations in the absence of frequency nonreciprocity.

Several features of the self-modulation regime of the first type are similar to the features characteristic of the beat regime, which was studied thoroughly for gas ring lasers [46]. The following features are typical of both of these regimes. As the frequency nonreciprocity Ω grows, the frequency of self-modulation oscillations ω_{m} (as well as the beat frequency) increases in both regimes, approaching $M\omega_{\mathrm{rot}}$ (M is the scaling coefficient, which depends on the geometric size of the ring cavity) as $\Omega \to \infty$. The intensities of counterpropagating waves are modulated under these conditions by the self-modulation (beat) frequency, and the modulation depth decreases with growth in the frequency nonreciprocity. These effects can be used to measure the frequency nonreciprocity.

However, there are also considerable differences between the self-modulation and beat regimes. First, no frequency-locking area exists in the self-modulation regime of the first type for $\Omega \to 0$. Second, when the frequency nonreciprocity is sufficiently strong, equalisation of the mean intensities of the counterpropagating waves is observed in the beat regime, whereas in the self-modulation regime one of the waves is suppressed and the mode locking of running waves is observed.

Klimenko et al. [47] and Zolotoverkh et al. [48] have studied the influence of a periodic modulation of SSRL parameters (the pump and intracavity losses) at frequencies $\omega_{\mathrm{p}} \ll c/L$ on the dynamics of lasing. The frequency of self-modulation oscillations was mode-locked by the modulating signal. The conditions necessary for the appearance of bistable quasi-periodic regimes were also determined. The intensities of counterpropagating waves in these regimes are modulated at the frequencies ω_{m} and ω_{p} and at combination frequencies $\omega_{\mathrm{m}} \pm n\omega_{\mathrm{p}}$ ($n = 0, 1, 2, \ldots$).

Zolotoverkh et al. [48] have demonstrated experimentally that the band width of self-modulation oscillations decreases in the case when these oscillations are mode-locked by a modulation signal. The influence of frequency nonreciprocity on the periodic regimes of pulsed lasing arising in a laser with modulated parameters was also investigated in Ref. [48]. These studies have shown that the depth of antiphase pulse modulation increases with increasing Ω. In principle, this makes it possible to measure the nonreciprocity from the modulation depth of the pulse envelope. Yet another finding from Ref. [48] is that the regimes of dynamic chaos may be observed in the ranges of parametric resonances, where ω_{p} is a multiple of $\omega_{\mathrm{r}i}$ ($i = 0, 1, 2$) [49, 50].

Several important circumstances have been revealed in the investigation of relaxation oscillations in SSRLs operating in the self-modulation regime of the first type. Whereas the regime of unidirectional lasing is characterised by three relaxation frequencies, the self-modulation regime of the first type gives two relaxation frequencies. The main relaxation frequency ω_{r0}, which corresponds to in-phase modulation, is determined by Eqn (1), similar to the running-wave regime. The low-frequency components in the spectrum of relaxation oscillations display different dependences on Ω in the self-modulation and running-wave regimes [23, 51].

Along with sinusoidal self-modulation lasing regimes, SSRLs are characterised by specific self-modulation regimes (self-modulation regimes of the second type) featuring single-mode lasing with a periodic switching of lasing direction with a frequency of the order of several kilohertz accompanied by a transient process at the relaxation frequency [5, 51, 52]. Note that this regime is characteristic of lasers with relatively large cavity perimeters. To our knowledge, up to now this regime has not been observed for chip lasers with cavity perimeters $L < 3$ cm. As the frequency nonreciprocity increases in the self-modulation regime of the second type, one of the counterpropagating waves is suppressed and the system is switched to the regime of unidirectional lasing.

As demonstrated in Ref. [53], the difference in the Q factors of a ring cavity for counterpropagating waves may be responsible not only for the difference in the intensities of these waves, but also for the complication of the behaviour of these intensities as functions of the frequency nonreciprocity of the laser cavity.

The difference in the coupling coefficients of counterpropagating waves arising in schemes with an additional extra-cavity mirror changes the frequency of self-modulation oscillations and, under certain conditions, switches the system from antiphase oscillations to in-phase oscillations [25].

Zolotoverkh et al. [54] investigated the dynamics of lasing in a bidirectional ring laser with an additional backreflector for different parameters of such a laser (the excess η of the pump power over the threshold, the absolute values $m_{1,2}$ and the phases $\vartheta_{1,2}$ of the total coupling coefficients of counterpropagating waves, the relation between the frequencies of relaxation and self-modulation oscillations, and the values of these frequencies). These studies have demonstrated the possibility of tuning the frequency of antiphase oscillations within a broad range (100 – 400 kHz) by varying the coupling coefficient of the counterpropagating waves.

The revealed mutual influence of self-modulation and relaxation frequencies may give rise in certain cases (e.g., with $\omega_{\mathrm{m}} = 2\omega_{\mathrm{r0}}$) to a parametric resonance and parametric frequency locking. The width of the range where these effects are observed may reach ~ 30 kHz. It was shown that the self-modulation regime of the first type becomes unstable and period-doubling bifurcations arise in the regions of parametric resonances. Under certain conditions, dynamic chaos was also observed (in particular, the necessary condition for its existence is the condition $m_1 \neq m_2$).

Experimental studies of dynamic chaos revealed the existence of the regimes of mode-locked and non-mode-locked chaos [55, 56]. It was demonstrated that chaotic oscillations may be either totally mode-locked, with the intensities of counterpropagating waves being equal to each other, or mode-locked in a general sense, with the intensities of counterpropagating waves varying chaotically in time and the intensity difference being a periodic function of time [56]. It was also shown that, as Ω increases, the system is switched from total mode locking to generalised mode locking.

3.3 The beat regime in SSRLs

Experimental studies show that, by weakening the competition between counterpropagating waves in an SSRL, one can implement a beat regime similar to the beat regime in gas ring lasers. The difference between these regimes in solid-state and gas lasers is associated with the appearance of moving gratings of population inversion in SSRLs, which give rise to frequency shifts of counterpropagating waves reflected from these gratings.

Although it is not very difficult to implement self-modulation regimes of lasing and the standing-wave regime, the beat regime, which is of considerable practical interest, is usually unstable in the case of a homogeneously broadened gain line (e.g., in SSRLs), and special measures are usually required for its implementation. The most widespread methods employed to obtain this regime are based on:

— the use of intracavity nonlinear elements (for harmonic generation, stimulated Raman scattering, and nonlinear absorption) [57];
— the use of nonreciprocal elements with a feedback loop [58];
— the use of phase conjugation [59];
— the use of schemes with self-illumination waves [43, 60].

The influence of intracavity second-harmonic generation on the characteristics of SSRLs was investigated by Dotsenko et al. [57]. Intracavity radiation conversion into the second harmonic is equivalent to the introduction of additional nonlinear losses proportional to the intensities of counterpropagating waves. Experimental studies have shown that, for small conversion coefficients K, the frequency nonreciprocity results in a suppression of one of the competing counterpropagating waves, whereas the increase in K equalises the inten-sities of the counterpropagating waves and gives rise to the appearance of the beat regime.

Intracavity nonlinear absorption can be employed in a similar way. However, the implementation of this approach in practice is impeded by the appearance of a spiking instability [61]. The beat regime in SSRLs with a feedback loop was studied experimentally by Dotsenko et al. [58], who demonstrated that the difference in the magnitudes of losses for counterpropagating waves arising due to the presence of the feedback loop ensures a stable beat regime.

Ring chip lasers have been investigated over a broad range of temperatures (77 – 320 K). The specific features of lasing in such systems at liquid nitrogen temperature are not only the shift of the lasing frequency due to the temperature dependences of cavity parameters and the shift of the gain line, but also the switching of the lasing wavelength of a garnet laser from line A ($\lambda = 1.064$ μm) to line B ($\lambda = 1.061$ μm) [62]. Low-temperature lasing in a chip laser is also characterised by narrowing of the spectrum of self-modulation oscillations.

4. Characteristics of SSRLs in the presence of frequency nonreciprocity: theoretical analysis

4.1 1 The basic equations of laser dynamics and parameters of SSRLs

In spite of the complexity of physical processes occuring in solid-state ring lasers, there are well-developed methods for the theoretical analysis of the nonlinear dynamics of such lasers. Analytical investigations and numerical modelling of SSRL dynamics provide an adequate understanding of the main lasing regimes and the characteristics of output laser radiation (the spectrum of laser radiation, intensities of counterpropagating waves, phase relations for these waves, relaxation and self-modulation oscillations, interaction of these oscillations, etc.). The most widespread approach employs the semiclassical theory of ring lasers, which is based on the Maxwell equations for the field inside the cavity and a set of quantum-mechanical equations for the density matrix of active atoms. To solve specific problems within the framework of the semiclassical theory, many simplifying assumptions are used. Although these assumptions limit the generality of theoretical consideration, they allow us to describe the most important features of the problem [2].

Most of the studies devoted to the theoretical analysis of the nonlinear dynamics of SSRLs employ the following assumptions:

— only one mode of SSRL oscillations is assumed to be emitted in each direction;

— the plane-wave approximation is employed;

— the spatial nonuniformity of both pump and population inversion in the transverse (with respect to the cavity axis) direction is ignored;

— diffraction effects are assumed to be negligibly small;

— polarisation of light waves is assumed to be linear, and counterpropagating waves have the same polarisation.

Within the framework of this approximation, the radiation field in a ring cavity can be represented as a sum of two waves propagating in opposite directions along the cavity axis z:

$$\boldsymbol{E}(z,t) = \mathrm{Re}\left\{\sum_{1,2} \boldsymbol{e}_{1,2}\tilde{E}_{1,2}(t)\exp[\mathrm{i}(\omega t \pm kz)]\right\}, \qquad (4)$$

where $\tilde{E}_{1,2}(t) = E_{1,2}\exp \mathrm{i}\varphi_{1,2}$, $E_{1,2}$, and $\varphi_{1,2}$ are the complex amplitudes, moduli, and phases of the fields in counterpropagating waves, respectively, and $\boldsymbol{e}_1 = \boldsymbol{e}_2$ are the unit vectors. The set of equations governing the lasing dynamics in a single-mode SSRL are written as [2, 3]

$$\frac{\mathrm{d}\tilde{E}_{1,2}}{\mathrm{d}t} = -\frac{\omega}{2Q_{1,2}}\tilde{E}_{1,2} \pm \mathrm{i}\frac{\Omega}{2}\tilde{E}_{1,2} + \frac{\mathrm{i}}{2}\tilde{m}_{1,2}\tilde{E}_{2,1}$$

$$+\frac{\sigma l}{2T}(1-\mathrm{i}\delta)\big(N_0\tilde{E}_{1,2} + N_{\mp}\tilde{E}_{2,1}\big), \qquad (5)$$

$$T_1\frac{\mathrm{d}N_0}{\mathrm{d}T} = N_{\mathrm{th}}(1+\eta) - N_0\Big[1 + a\Big(|E_1|^2 + |E_2|^2\Big)\Big]$$

$$- N_+ aE_1E_2^* - N_- aE_1^*E_2\ , \qquad (6)$$

$$T_1\frac{\mathrm{d}N_+}{\mathrm{d}T} = -N_+\Big[1 + a\Big(|E_1|^2 + |E_2|^2\Big)\Big] - N_0 aE_1^*E_2\ , \qquad (7)$$

where N_{th} is the threshold population inversion; $Q_{1,2}$ are the Q factors of the cavity for counterpropagating waves; $T = L/c$ is half the cavity round-trip time; T_1 is the longitudinal relaxation time; l is the length of the active element; δ is the normalised detuning of the lasing frequency from the centre of the gain line; L is the perimeter of the ring cavity; $\sigma = \sigma_0(1+\delta^2)$ is the cross section of the laser transition; a is the saturation parameter; and

$$N_0 = \frac{1}{l}\int_0^l N\mathrm{d}z, \qquad N_{\pm} = \frac{1}{l}\int_0^l N\mathrm{e}^{\pm \mathrm{i}2kz}\mathrm{d}z \qquad (8)$$

are the spatial harmonics of the population inversion N.

The lasing dynamics in an SSRL is determined by the following main parameters:

— complex coefficients $\tilde{m}_{1,2} = m_{1,2}\exp(\pm\mathrm{i}\vartheta_{1,2})$ describing the coupling of counterpropagating waves through backscattering;

— the normalised detuning of the lasing frequency from the centre of the gain line, $\delta = (\omega - \omega_0)/\Delta\omega_{\mathrm{g}}$ ($\Delta\omega_{\mathrm{g}}$ is the width of the gain line);

— the excess η of the pump power over the threshold;

— the amplitude nonreciprocity of the cavity, $\Delta = 2(Q_1 - Q_2)/(Q_1 + Q_2)$;

— the frequency (phase) nonreciprocity of the cavity, $\Omega = \omega_{1\mathrm{c}} - \omega_{2\mathrm{c}}$ ($\omega_{1\mathrm{c}}$ and $\omega_{2\mathrm{c}}$ are the eigenfrequencies of the cavity for counterpropagating waves).

In what follows, we provide a detailed analysis of the influence of the last factor on the dynamics of SSRL emission.

In the case of an homogeneously broadened gain line, counterpropagating waves in a laser cavity acquire the energy from the same group of active ions, which results in a competition of the counterpropagating waves. Nonlinear energy coupling of counterpropagating waves through gratings $N_{\pm}$, arising due to spatial modulation of the population difference of active ions (i. e., coupling through amplitude and phase gratings induced in the active medium), is an important factor influencing the interaction of counterpropagating waves in SSRLs.

Owing to the mutual diffraction of counterpropagating waves through these gratings, the gains for opposite directions differ: the stronger wave has a higher gain, which results in suppression of the competing weak wave [3, 5]. The backscattering of radiation from cavity elements under these conditions has a stabilising effect, equalising the intensities of counterpropagating waves [5]. If the coupling of counterpropagating waves through backscattering is sufficiently strong, regimes of cross mode locking in counterpropagating waves (in particular, the standing-wave regime) become stable in SSRLs.

Eqns (5) – (7) provide an adequate description of the nonlinear dynamics of SSRLs and allow the conditions of existence and stability to be analysed for all experimentally observed regimes of lasing. These equations make it possible to find the output radiation characteristics as functions of the frequency (and amplitude) nonreciprocities of the cavity, which is of considerable importance for practical applications.

Analysis of these equations shows that an optical nonreciprocity in a laser cavity not only may quantitatively change the output radiation characteristics [such as the difference in the frequencies of counterpropagating waves, the intensities $I_{1,2}$ of these waves, and the frequencies of relaxation ($\omega_{\mathrm{r}0}$, $\omega_{\mathrm{r}1}$, and $\omega_{\mathrm{r}2}$) and self-modulation (ω_{m}) oscillations], but may also sometimes change the dynamics of lasing by switching a laser from one regime to another [2, 5].

In recent studies [23, 63 – 67], the dynamics of SSRLs was analysed for arbitrary polarisations of counterpropagating waves. These studies show that, in the case of counterpropagating waves with noncollinear polarisations, the coupling coefficients of these waves change and the amplitudes $N_{\pm}$ of population-inversion gratings decrease.

The results obtained for counterpropagating waves with collinear polarisations can be easily generalised to the case of arbitrary polarisations through the introduction of effective coupling coefficients and the inclusion of the change in the depth of the spatial modulation of population inversion. The latter factor can be taken into account with a formal replacement of $N_{\pm}$ by $N_{\pm}(\boldsymbol{e}_1^{\mathrm{p}}\boldsymbol{e}_2^{\mathrm{p}})^2$, where $\boldsymbol{e}_1^{\mathrm{p}}$ and $\boldsymbol{e}_2^{\mathrm{p}}$ are the unit polarisation vectors of the counterpropagating waves [64].

4.2 Stationary regimes of lasing in SSRLs

The influence of the frequency nonreciprocity Ω on the regions of existence and stability of cross mode locking in counterpropagating waves was investigated in Refs. [2, 3, 5]. In the case of strong coupling satisfying the inequality

$$m\left|\sin\frac{1}{2}(\vartheta_1 - \vartheta_2)\right| > \frac{1}{3}\frac{\omega}{Q}\eta \qquad m = m_1 = m_2\ , \qquad (9)$$

the standing-wave regime arises in an SSRL with $\Omega = 0$, $\Delta = 0$, and $\delta \ll 1$. In this regime, lasing usually occurs in several longitudinal modes [3–5]. Frequency nonreciprocity changes intensities and the phase difference of counterpropagating waves. As $|\Omega|$ increases, one of these waves is gradually suppressed. The dependences of $I_{1,2}$ on $|\Omega|$ for $\Delta = 0$ have a characteristic X-like form.

Analysis of such systems shows that frequency nonreciprocity may also change the boundaries of the stability region of the regime considered. This regime is stable when

$$m\left|\sin\frac{1}{2}(\vartheta_1 - \vartheta_2)\right| > \frac{\omega}{Q}\eta\frac{m(m^2+\Omega^2)^{1/2}}{3m^2+2\Omega^2} . \quad (10)$$

As follows from Eqn (10), the mode-locking regime is stable for any Ω once condition (9) is satisfied. This implies that an SSRL may feature frequency locking that does not give rise to a beat regime with increasing in $|\Omega|$: as $|\Omega|$ increases, one of the counterpropagating waves is suppressed before the laser leaves the frequency-locking regime. In other words, an infinite band of frequency locking may exist in an SSRL [2, 5].

Generally, the frequency-locking region of the regime of cross mode locking may have a finite width [67]. In this case, the self-modulation regime of the first type arises in an SSRL beginning with some Ω_1 for $|\Omega| > \Omega_1$. The increase in $|\Omega|$ then leads to a monotonic increase in the amplitude of one of the counterpropagating waves, accompanied by suppression of the second wave. For sufficiently large $|\Omega|$, lasing in such an SSRL becomes nearly unidirectional.

Amplitude nonreciprocity ($\Delta \neq 0$) expands the regions of existence and stability of the running-wave regime. For $\Delta \neq 0$, the intensities $I_{1,2}$ become nonmonotonic functions of Ω [53].

In the case of sufficiently weak coupling, when the inequality

$$m < \frac{1}{2}\left(\frac{\eta\omega}{QT_1}\right)^{1/2}\left|\cos\frac{\vartheta_1-\vartheta_2}{2}\right| \quad (11)$$

is satisfied, the running-wave regime arises and remains stable for $\Omega = 0$ and $\Delta = 0$ if the normalised detuning δ of the lasing frequency is less than some critical value δ_{cr}:

$$|\delta| < \delta_{cr} = \left(\frac{Q}{\omega T_1 \eta}\right)^{1/2} . \quad (12)$$

For small $|\Omega|$, there exists a bistability region, where two running-wave regimes may occur. These regimes are characterised by different directions of propagation of the wave with a higher intensity. Amplitude characteristics as functions of Ω have the form of a hysteresis loop in this region.

4.3 Self-modulation regimes of lasing

Along with stationary lasing regimes with constant intensities of counterpropagating waves, several self-modulation lasing regimes may occur in an independent SSRL.

The self-modulation regime of the first type is of special interest from the viewpoint of laser gyroscopy. This regime is characterised by a nearly harmonic antiphase modulation of intensities of counterpropagating waves. For $\Omega = 0$, the self-modulation regime of the first type arises when

$$\left(\frac{\eta\omega}{QT_1}\right)^{1/2}\left(2\left|\cos\frac{\vartheta_1-\vartheta_2}{2}\right|\right)^{-1} < m < \frac{\eta\omega}{3Q}\left(\left|\sin\frac{\vartheta_1-\vartheta_2}{2}\right|\right)^{-1} . \quad (13)$$

In accordance with these inequalities, the range of absolute values of the coupling coefficients corresponding to self-modulation oscillations has a maximum width when the coupling coefficients are close to complex-conjugate quantities ($|\vartheta_1 - \vartheta_2| \ll 1$). As the phase difference of the coupling coefficients increases, this range narrows, and self-modulation oscillations become impossible for $|\vartheta_1 - \vartheta_2| \to \pi$.

The analysis performed in Refs [2, 5] has shown that, if a self-modulation regime occurs in an SSRL with $\Omega = 0$, then this regime should also be observed with growth in $|\Omega|$ in the range $|\Omega| \leqslant \Omega_1$. The boundary frequency Ω_1 in this case can be found from the expression

$$m\left|\sin\frac{1}{2}(\vartheta_1 - \vartheta_2)\right| = \frac{\omega}{Q}\eta\frac{m(m^2+\Omega_1^2)^{1/2}}{3m^2+2\Omega_1^2} . \quad (14)$$

For $|\Omega| \geqslant \Omega_1$, the laser is switched from the self-modulation regime to the frequency-locking regime.

The frequency ω_m, the intensities of counterpropagating waves, and the depth of intensity self-modulation, are the main characteristics of self-modulation regimes. In the self-modulation regime of the first type, the frequency of self-modulation oscillations increases with increasing $|\Omega|$. The mean intensities of counterpropagating waves differ from each other under these conditions [5]. In the case when $\omega_m \gg \omega_{r0}$ and $\Delta = 0$, the dependence of the frequency of self-modulation oscillations on the nonreciprocity Ω of an optical cavity is given by [68]

$$\omega_m = (\omega_{m0}^2 + \Omega^2)^{1/2} , \quad (15)$$

where

$$\omega_{m0}^2 = m_1 m_2 \cos(\vartheta_1 - \vartheta_2) + \frac{(1+\delta^2)m_1^2 m_2^2 \sin^2(\vartheta_1-\vartheta_2)}{m_1^2 + m_2^2 + 2m_1 m_2\cos(\vartheta_1-\vartheta_2) - \delta m_1 m_2 \sin(\vartheta_1-\vartheta_2)} . \quad (16)$$

In the case of symmetric coupling, Eqn (16) is reduced to

$$\omega_{m0} = m\left|\cos\frac{\vartheta_1-\vartheta_2}{2} - \delta\sin\frac{\vartheta_1-\vartheta_2}{2}\right| . \quad (17)$$

Expressions (13)–(15) ignore the influence of relaxation processes on self-modulation oscillations. By modifying the formula for the self-modulation frequency to include relaxation processes, we arrive at [69]

$$\omega_m = (\omega_{m0}^2 + \Omega)^{1/2} + \frac{1}{4}(\omega_{m0}^2 + \Omega^2)^{-1/2}\frac{\omega\eta}{QT_1} . \quad (18)$$

The above formulas expressions the dependence of ω_m on the frequency nonreciprocity of the cavity provide a reasonable agreement with the experimental data.

We should note that the frequency-locking area, which is characteristic of the beat regime, is not observed in the regime of lasing considered [2, 3, 5]. The quantity ω_m ceases to be a weak function of Ω for small Ω in the self-modulation regime of the first type in the case when the working point is biased. Such a bias occurs in the case of a nonzero constant or sign-alternating pedestal [69, 70]. The frequency response in this case can be represented as

$$\frac{\omega_m}{2\pi} = \left(1 + \frac{M\Omega_p\omega_{rot}}{\omega_{m0}^2 + \Omega_p^2}\right)(\omega_{m0}^2 + \Omega_p^2)^{1/2} , \quad (19)$$

where Ω_p is the amplitude of the frequency bias, and ω_{rot} is the angular velocity to be measured.

Optical nonreciprocity can be detected not only by measuring the frequency of self-modulation oscillations in an SSRL, but also by determining the modulation coefficients and the ratio of radiation intensities for counterpropagating waves. The latter method of measurements makes use of the fact that, when an SSRL operates in the self-modulation regime, a variation in the amplitude or frequency nonreciprocity of the laser cavity changes not only the self-modulation frequency, but also the intensities of counterpropagating waves and their modulation depths at the frequency ω_m.

In the case of the self-modulation regime of the first type, the fields in the counterpropagating waves can be written as [71]

$$E_1 = \mathrm{Re}\{A_1 \exp[\mathrm{i}(\omega_1 + \omega_m)t] + B_1 \exp(\mathrm{i}\omega_1 t)\} ,$$
$$E_2 = \mathrm{Re}\{A_2 \exp[\mathrm{i}(\omega_1 + \omega_m)t] + B_2 \exp(\mathrm{i}\omega_1 t)\} , \quad (20)$$

where $\omega_1 = \omega_c - \delta(\omega/Q_1 + \omega/Q_2)/2 - \omega_m/2$ is the optical frequency of radiation with allowance for frequency pulling toward the centre of the gain line, and ω_c is the eigenfrequency of the laser cavity. The complex amplitudes A_i and B_i ($i = 1, 2$) of spectral radiation components can be found from the equations for the spatial harmonics N_0 and $N_\pm$ of population inversion [68].

The dimensionless intensities of the counterpropagating waves can be represented as

$$a|E_1|^2 = I_{01} + I_1 \sin(\omega_m t + \varphi_1) ,$$
$$a|E_2|^2 = I_{02} + I_2 \sin(\omega_m t + \varphi_2) , \quad (21)$$

where I_{0i} and I_i ($i = 1, 2$) are the constant components and modulation amplitudes of the intensities of the counterpropagating waves.

As shown in Ref. [71], the dependence of Ω on the parameters characterising the self-modulation regime in the case of small frequency nonreciprocities ($|\Omega| \ll \omega_{m0}$) can be written as

$$|\Omega| = \left(\Delta^2 + \omega_m^2\right) \frac{|1 - I_{01}I_2/I_{02}I_1|}{2\omega_m[1 - (I_2/I_{02})^2]^{1/2}} , \quad (22)$$

In the absence of amplitude nonreciprocity ($\Delta = 0$), this dependence is given by

$$|\Omega| = R\omega_m , \quad (23)$$

where

$$R = \frac{|\alpha_0 - \beta_0|}{I_{01}/I_1 + I_{02}/I_2} ;$$

$$\alpha_0 = \left[\left(\frac{I_{02}}{I_2}\right)^2 - 1\right]^{1/2} ; \quad \beta_0 = \left[\left(\frac{I_{01}}{I_1}\right)^2 - 1\right]^{1/2} .$$

Eqn (23) gives the frequency nonreciprocity in terms of experimentally measurable parameters of self-modulation oscillations. By measuring these parameters, one can find Ω.

Another possibility of measuring the frequency nonreciprocity is associated with the dependence on Ω of the constant components and modulation amplitudes of the intensities of counterpropagating waves. This dependence implies that frequency nonreciprocity can be measured through the detection of a signal resulting from the mixing of two counterpropagating waves in a ring laser operating in the self-modulation regime [71].

Consider the signal arising due to the mixing of two counterpropagating waves. Generally this signal can be represented as

$$\tilde{E}_{mix} = \tilde{r}_1 \tilde{E}_1 + \tilde{r}_2 \tilde{E}_2 , \quad (24)$$

where $\tilde{r}_{1,2} = r_{1,2} \exp(\mathrm{i}kl_{1,2})$ are the complex coefficients. The absolute values of these coefficients are determined by the reflection coefficients of the mirrors reflecting radiation to a photomixer. The phases of these coefficients are proportional to the optical paths covered by each of the waves.

The intensity of the signal measured by a photodetector, similar to the intensities of counterpropagating waves, has a constant component I_{0mix} and an alternating component with a modulation amplitude I_{mix} at the frequency ω_m. Let us represent the photomixing signal and the signal of one of the waves as

$$a|\tilde{E}_{mix}|^2 = I_{0mix} + I_{mix} \sin(\omega_m t + \varphi_2 + \psi) ,$$
$$a|\tilde{E}_2|^2 = I_{02} + I_2 \sin(\omega_m t + \varphi_2) . \quad (25)$$

Then, as shown in Ref. [71], the following relations can be used in the case when $\Delta = 0$:

$$I_{mix} \cos\psi = \frac{I_2 r_1^2 \left(|\alpha|^2 - \omega_{m0}^2 - 2\Omega \mathrm{Im}\, s\right)}{|d|^2} ,$$
$$I_{mix} \sin\psi = \frac{I_2 r_1^2 2\omega_m \mathrm{Re}\, s}{|d|^2} . \quad (26)$$

The quantity $s = \tilde{r}_2 d/\tilde{r}_1$ can be expressed in terms of measurable parameters:

$$|s|^2 = \frac{\omega_{m0}^2 r_2^2 I_2}{r_1^2 I_1} . \quad (27)$$

If parameters of an experimental setup are chosen such that $r_2^2 I_2 = r_2^2 I_1$ and the phase difference $k(l_2 - l_1)$ ensures the minimum value of I_{mix}, then we can apply the following formula:

$$|\Omega| = \frac{\omega_{m0}\left|\left(r_1^2 I_1 - r_2^2 I_2\right) + I_{mix}\cos\psi\right|}{2r_1 r_2 (I_1/I_2)^{1/2}} . \quad (28)$$

Thus, in the self-modulation regime of the first type, the frequency nonreciprocity of the laser cavity determines not only the frequency of self-modulation oscillations, but also constant components $\bar{I}_{1,2}$ and modulation amplitudes of the intensities of counterpropagating waves.

The relationships presented above for the self-modulation regime of the first type are applicable when the interaction between self-modulation and relaxation oscillations is sufficiently weak. The situation becomes more complicated in the regions of parametric resonances, when this interaction becomes significant.

Analysis of the interaction between self-modulation and relaxation oscillations performed by Zolotoverkh et al. [20] has shown that the frequencies ω_m and ω_r are related to each other in this case. For $\Omega = 0$, the relation between these frequencies is given by the expressions:

$$\omega_m = \omega_{m0}\left(1 + \frac{\alpha}{4} - \frac{3}{64}\alpha^2\right) , \quad (29)$$

$$\omega_r^2 = \tfrac{1}{2}\left\{\omega_m^2 + \omega_{r0}^2 - [(\omega_m^2 + \omega_{r0}^2)^2 + (\omega_{r0}^4 - 4\omega_{r0}^2\omega_m^2)]^{1/2}\right\}, \quad (30)$$

where $\alpha = \omega_{r0}^2/m_1 m_2$ and $\omega_{r0}^2 = \omega\eta/QT_1$ [see Eqn (1)].

In the case of sufficiently weak coupling, when condition (11) is satisfied, an SSRL with $\Omega = 0$ operates either in the running-wave regime or in the self-modulation regime of the second type. The stability of the regime of lasing depends on the detuning of the lasing frequency from the centre of the gain line ω_0. The structure of the gain line affects considerably the stability condition [4, 72, 73].

In an elementary two-level model, which ignores the asymmetry of the gain line, the self-modulation regime of the second type arises in SSRLs when the frequency detuning $\delta = |\omega - \omega_0|/\Delta\omega_g$ satisfies the condition

$$|\delta| > \delta_{cr} = \left(\frac{Q}{\omega T_1 \eta}\right)^{1/2} , \quad (31)$$

and the SSRL operates in the self-modulation regime of the second type. As $|\Omega|$ increases, the mean intensities of counterpropagating waves are no longer equal to each other, and the SSRL is switched to the stationary running-wave regime for large Ω.

It is of interest also to consider the possibility of measuring Ω by detecting the changes in the frequencies of counterpropagating waves in the case of low-frequency switching of the lasing direction in the self-modulation regime of the second type. Owing to the phase nonreciprocity of the cavity, the changes in the frequencies of counterpropagating waves in this case are proportional to Ω [72, 73].

4.4 Dependence of relaxation processes on the frequency nonreciprocity

Another possibility of detecting the frequency nonreciprocity is associated with the dependence on Ω of the frequencies of relaxation oscillations and the relevant decay rates. As mentioned above, the regime of cross mode locking in counterpropagating waves with substantially different amplitudes (the running-wave regime) is characterised by three relaxation frequencies: the main frequency ω_{r0} (1) and the frequencies

$$\omega_{r1,r2} = \left(\frac{\omega_{r0}^2}{2} + \frac{\Omega^2}{4}\right)^{1/2} \mp \frac{\Omega}{2} . \quad (32)$$

The decay rates of relaxation oscillations with frequencies $\omega_{r1,r2}$ are given by [4]

$$\gamma_{r1,r2} = -\frac{1+\eta}{2T_1}\left[1 \mp \frac{\Omega}{2}\left(\frac{\omega_{r0}^2}{2} + \frac{\Omega^2}{4}\right)^{-1/2}\right] . \quad (33)$$

The self-modulation regime of the first type, as shown above, is characterised by two relaxation frequencies. One of these frequencies is described by Eqn (1), and the other depends on Ω in accordance with the expression [74]

$$\omega_{ra} = \frac{1}{\sqrt{2}}\left[\omega_{r0}^2 + \omega_m^2 - (\omega_m^4 + 2\omega_{r0}^2\Omega^2)^{1/2}\right]^{1/2} . \quad (34)$$

The decay rates of these oscillations are described by the expressions:

$$\gamma_{r0} = -\frac{1+\eta}{2T_1} , \quad (35)$$

$$\gamma_{ra} = -\frac{\omega_{m0}^2}{\omega_{m0}^2 + \Omega^2}\,\frac{1+\eta}{2T_1} . \quad (36)$$

4.5 The beat regime in SSRLs

The beat regime is of special interest in the context of laser gyroscopy applications. Note that this regime is employed in gas laser gyroscopes [46]. The beat regime has much in common with the self-modulation regime of the first type. In both regimes, the intensities and the phase difference of counterpropagating waves are modulated periodically either with the self-modulation frequency ω_m or with the beat frequency ω_b. To implement the beat regime in an SSRL, one has to reduce the competition between counterpropagating waves. This can be done in different ways.

As shown by Dotsenko et al. [58], the beat regime in an SSRL can be stabilised with the use of a feedback loop, introducing the difference in the magnitudes of losses for counterpropagating waves inside the laser cavity proportional to the difference in the intensities of these waves:

$$\Delta = \frac{2(Q_1 - Q_2)}{Q_1 + Q_2} = qa\left(E_1^2 - E_2^2\right) . \quad (37)$$

In an SSRL with a feedback loop, the beat regime with equal mean intensities of counterpropagating waves arises when

$$q \gg \frac{m^2|\sin(\vartheta_1 - \vartheta_2)|Q}{\omega|\Omega|\eta} . \quad (38)$$

The beat regime can also be stabilised with the use of an intracavity nonlinear element (a saturable absorber [75] or a nonlinear crystal [57] for second-harmonic generation). In lasers with intracavity second-harmonic generation, a stable beat regime with equal intensities of counterpropagating waves arises when

$$|\Omega| \gg m^2|\sin(\vartheta_1 - \vartheta_2)|TK^{-1} , \quad (39)$$

where K is the magnitude of losses of fundamental radiation in each of the waves (per single pass) owing to the conversion of radiation into the second harmonic.

The method of self-illumination waves was proposed for the stabilisation of the beat regime in Refs. [43, 60]. This method can be described briefly in the following way. By returning a fraction of output radiation to the cavity at a small angle with respect to the cavity axis, one can induce additional gratings of population inversion in the active medium. Cross diffraction of the counterpropagating waves by these gratings may ensure a high gain for the weak wave, thus stabilising the beat regime. The condition of stabilisation of the beat regime in the case of weakly coupled counterpropagating waves in a laser with self-illumination waves is written as [6]

$$\frac{\varkappa l_0}{l} > \frac{TQ}{\omega T_1 \eta} , \quad (40)$$

where $\varkappa$ is the ratio of the intensity of the self-illumination wave to the intensity of the main wave, and l_0 is the size of the area in the active medium where the main wave overlaps the self-illumination wave. Finally, the beat regime can be stabilised by means of phase conjugation [59] and polarisation – frequency decoupling of counterpropagating waves in a nonplanar ring cavity [75].

The potential advantage of the beat regime in SSRLs is the absence of the frequency-locking region in the case of weak coupling: $|m_{1,2}| \ll \omega_{r0}$. The beat regime gives rise to an additional frequency difference of counterpropagating waves because of the self-diffraction of these waves from the gratings induced in the active medium. In the beat regime, these gratings move because of the difference in the frequencies of the counterpropagating waves. Therefore diffraction from these gratings changes the frequencies of the counterpropagating waves. Consequently, the dependence of the beat frequency ω_b on Ω is described by a nonlinear function even when the counterpropagating waves are not coupled with each other. For $\Omega \to 0$, we have $\omega_b \to \omega_{r0}$. Therefore, the frequency-locking region, in fact, vanishes if $|m_{1,2}| \ll \omega_{r0}$.

An important advantage of this regime is that it can be implemented within a broad range of SSRL parameters, e.g., in the case of weak coupling and, which is of special importance, for large $|\Omega|$.

5. Conclusions

Thus the output characteristics of SSRLs (the frequency difference of counterpropagating waves, the intensities of these waves, the frequencies and the phases of self-modulation oscillations, and decrements of these oscillations) are highly sensitive to the frequency (phase) nonreciprocity of a ring cavity. This circumstance makes SSRLs very attractive for studying the nonlinear dynamics of interaction of counterpropagating waves in the active media of lasers.

The properties of SSRLs, especially the sensitivity of a ring laser to the angular velocity of its rotation, are very important for applications in laser gyroscopy. We should note that impressive progress has been made in laser gyroscopy over more than 30 years of its successful development. Modern laser gyroscopy, which is based on the use of gas (He – Ne) ring lasers, is capable of solving most of the navigation-related problems.

Gyroscopic applications of solid-state lasers, which became realistic only after the advent of diode pumping, are not very broad at the moment. Nevertheless it is obvious that, potentially, solid-state active media have several important advantages over gas active media. Among these advantages we should mention the absence of vacuum elements and high voltages, the high-gain characteristic of solid-state active media, the high reliability and technological efficiency of laser cavities, the possibility of creating miniature devices with low energy consumption, etc. Self-modulation regimes of lasing and relaxation oscillations typical of SSRLs hold much promise for measuring angular velocities.

Analysis performed in this paper shows that the self-modulation regime of the first type in SSRLs is especially attractive (and well understood) for applications in laser gyroscopy. The absence of the frequency-locking region, which is characteristic of gas laser gyroscopes, a high stability attainable with slab ring lasers, and a virtually infinite dynamic range of angular velocity measurements give us grounds to believe that competitive miniature SSRL-based gyroscopes may be created in the near future.

A combination of the beat regime with self-illumination waves in SSRLs also seems to offer much promise for various applications. It would be of special interest to use this regime in SSRLs with forced longitudinal mode locking, when the efficiency of the induced gratings of population inversion reaches its maximum. However, we should note that the stability of this regime is still to be properly studied.

We hope that this review will promote further progress in SSRL applications for laser gyroscopy.

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