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Method for determination of the polarisation nonreciprocity in a fibre ring interferometer

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Abstract. A method is proposed for observation of the polarisation nonreciprocity of fibre ring interferometers (FRIs) by placing a rotating polariser at the output of an interferometer ahead of a photodetector. It is demonstrated theoretically and experimentally that the absence of a signal for any position of the transmission axis of the polariser at the FRI output is a criterion of the absence of the polarisation nonreciprocity. It is suggested that the coaxial alignment of the anisotropic FRI components be monitored during assembly to ensure the polarisation nonreciprocity on the basis of the absence of a signal at the output of a rotating polariser. It is also shown that, when the conditions for the polarisation nonreciprocity are fulfilled, the signal from the output of a beam splitter located flush against the fibre loop output carries information about the phase characteristics of the beam splitter.

The use of the Sagnac effect in fibre ring interferometers (FRIs) for measurement of the rate of rotation is nowadays well known. The phase shift Φ_s at the interferometer output is linked to the angular rate of rotation Ω_s by the relationship [1]

$$\Phi_s = \frac{4\pi^2 N D^2}{\lambda_0 c_0}\Omega_s\,, \qquad (1)$$

where D is the coil diameter; N is the number of turns; λ_0 and c_0 are the wavelength and velocity of light, respectively. One of the main causes of the appearance of an additional (parasitic) phase shift, unrelated to rotation, in the signal arising from the interference of counterpropagating waves at the ring interferometer output is the polarisation nonreciprocity. It arises as a consequence of the deviation from the coaxial alignment of the anisotropy axes of the ring interferometer components. There have been several reports of the calculation and experimental investigation of the polarisation nonreciprocity for different FRI systems [2 – 4]. Ways of reducing the influence of these effects on the output signal have been considered, including introduction into the system of a radiation polariser or depolariser (Fig. 1) whose quality then governs the degree of suppression of the undesirable effects.

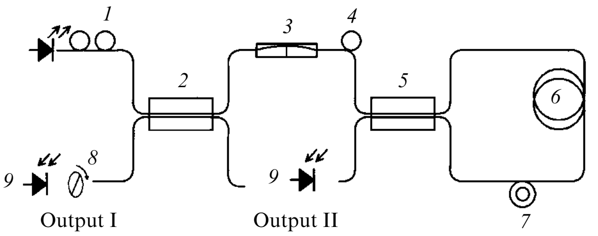

Figure 1. Experimental setup of the FRI: (1, 4) components for polarisation control; (2, 5) fibre beam splitters; (3) depolariser; (7) phase modulator; (6) fibre loop; (8) rotating polariser; (9) photodetectors.

The expression for the nonreciprocal polarisation phase in terms of elements of the Jones matrix of an FRI have been put forward in a most general form by Andronova et al. [4]. An analysis of the results of this study shows that the polarisation nonreciprocity effects are absent in two cases. In the first case, the interferometer system satisfies the condition for the polarisation reciprocity, i.e. the off-diagonal elements of the Jones matrix of the FRI are equal and any change in the polarisation of the radiation at the interferometer input does not lead to the appearance of a parasitic signal. In the second case, the Jones matrix of the FRI does not satisfy the condition for the polarisation reciprocity and the off-diagonal matrix elements are not equal ($M_{12} \neq M_{21}$), but the polarisation of the radiation at the interferometer input is such ($A_x = A_y$ and $\langle A_x A_y \rangle = 0$) that the nonreciprocal effects are not observed. However, any change in the polarisation at the input, arising from heating or mechanical action, may give rise to a parasitic signal. In the course of assembly of the system, it is therefore necessary to align the components in such a way that the system satisfies the polarisation reciprocity condition ($M_{12} = M_{21}$), which corresponds to the condition of coaxial alignment of all the anisotropic FRI components. Since the absence of a parasitic signal at the output is not always a criterion of the reciprocity, it is of interest to examine the methods for monitoring the polarisation reciprocity of FRIs during assembly and for testing the complete systems.

The aim of the present study is examination and experimental demonstration of a method for determination of a deviation from conditions for the polarisation reciprocity of an FRI system regardless of the magnitude of the parasitic signal at its output and also examination of the possibility of measuring the phase characteristics of beam splitters based on FRIs.

The method considered is based on the influence of a polariser placed ahead of a photodetector recording the FRI output signal. The influence of the polariser on the FRI output signal was first observed by Alekseev et al. [5]. The output characteristics of an FRI assembled in accordance with the scheme shown in Fig. 1 were investigated in the present study.

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The polarisation of the radiation at the FRI fibre-loop input and the mutual tilt of the anisotropy axes of the FRI components were varied during our experiment. This made it possible to investigate more fully, compared with the study of Alekseev et al. [5], the influence on the FRI output signal of the position of an output polariser and of its rotation for various polarisations of the counterpropagating waves and mutual alignments of the FRI components.

The investigated interferometer system (Fig. 1) differed from that described in the literature [6, 7] by replacement of a polariser with a depolariser. Two beam splitters (2 and 5), an anisotropic fibre depolariser (3), a phase modulator (7), and two polarisation components (1 and 4), used to vary the polarisation of the radiation at the input to the fibre loop, were placed in the interferometer system. The component (4) is a standard Lefevre polarisation component [8]. In a simplified form, it consisted of a small fibre ring (an analogue of a quarter-wave plate), the tilt of whose axis altered the polarisation. Compression and twisting of the fibre at the inner loop output ahead of the beam splitter made it possible to alter the relative positions of the anisotropy axes of the loop and the beam splitter. The FRI loop, which was 200 m long and made of a single-mode isotropic fibre, was wound on a coil with a radius of 2 cm, which gave rise to a loop anisotropy so that the radiation from a superluminescent source, propagating along mutually orthogonal anisotropy axes, became virtually incoherent at the output.

During assembly of the interferometer, measures were taken to reduce the initial misalignment of the anisotropy axes of the loop and the beam splitter. The interference signal of the counterpropagating waves at the FRI output was detected on the basis of the first-harmonic amplitude at the phase-modulation frequency $f_{\rm m}$ with the aid of the phase modulator (7). After photodetection and amplification, the modulation signal was observed visually on the screen of an oscilloscope. The output signal was related mainly to the polarisation nonreciprocity effects (the contribution of the remaining nonreciprocal effects was negligible).

The experiments were carried out as follows. Positioning of the polarisation components and deformation of the ends of the fibre loop made it possible to suppress the parasitic signal at the FRI output (Fig. 2a), which corresponds to fulfillment of the relevant conditions [5] applicable to the components of the field at the FRI output ($A_x = A_y$ and $\langle A_x A_y \rangle = 0$). The placing of a polariser at the FRI output, ahead of the photodetector, led to the appearance of a signal at the frequency $f_{\rm m}$. The amplitude of this signal depended on the position of the polariser axis in such a way that the rotation of the axis at a frequency Ω induced modulation of the output-signal amplitude at a frequency 2Ω (Fig. 2b). In this case, when a signal at the phase-modulation frequency was observed in the absence of the polariser at the FRI output (Fig. 3a), rotation of the polariser axis changed the amplitude of this signal also at the frequency equal to that of the polariser rotation frequency $\Omega_{\rm s}$ (Fig. 3b). Despite control of the polarisation components outside the loop, introduction of the polariser induced a phase-modulation signal. In order to reduce this signal to zero, it was necessary to change the position of the anisotropy axis of one of the fibre ends within the fibre loop ahead of the beam splitter or to distort it. This procedure made it possible to reduce to zero the signal both without the polariser and with the rotating polariser placed at the photodetector input.

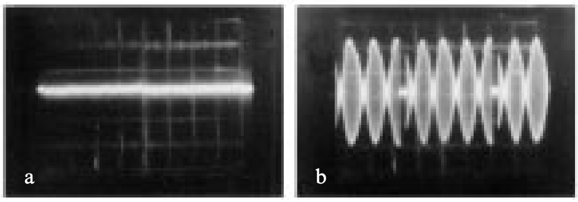

Figure 2. Oscillograms of an FRI signal at the phase-modulation frequency (zero signal) without a polariser (a) and after placing a rotating polariser at the FRI output (b).

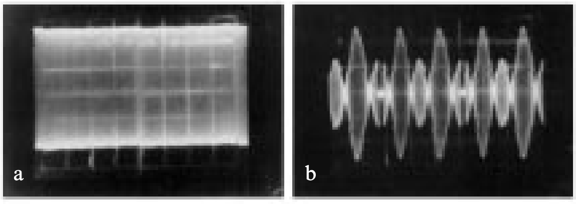

Figure 3. Oscillograms of the FRI signal at the phase-modulation frequency (nonzero signal) without a polariser (a) and after placing a rotating polariser at the FRI output (b)

Analysis of the experimental results makes it possible to put forward a qualitative explanation of this effect, which is that in the first case a change in the positions of the components controlling the polarisation altered the polarisation of the radiation at the input, ensuring conditions favouring 'vanishing' of the output signal. In the second case, twisting or compression of the fibre at one of the ends of the loop ahead of the beam splitter made it possible to alter the relative positions of the anisotropy axes of the loop and the beam splitter, and to fulfill the conditions for the polarisation reciprocity of the interferometer, i.e. to ensure equality of the off-diagonal coefficients. Thus, the absence of the FRI output signal at the phase-modulation frequency for any position of the external polariser ahead of the photodetector is a criterion of the polarisation reciprocity of an FRI.

In order to confirm this qualitative explanation of the experimental results, we calculated the FRI interference signal from output I of the beam splitter after its passage through the polariser ahead of the photodetector. The calculation was performed by the method of Jones matrices [4]. In general, the matrix equation for the field components at the FRI output (beyond the polariser at the photodetector input) in the specified coordinate system assumes the form

$$\boldsymbol{E}^{\pm} = \hat{M}_{\rm p} \cdot \hat{M}_{\rm k}^{\pm} \boldsymbol{A} \,, \tag{2}$$

where $\hat{M}_{\rm k}^{\pm}$ are FRI matrices for counterpropgating waves ($\hat{M}_{\rm k}^{-} = \hat{M}_{\rm k}^{+{\rm T}}$); $\hat{M}_{\rm p}$ is the external-polariser matrix in the specified coordinate system; $\boldsymbol{A} = \{A_x, A_y\}$ and $\boldsymbol{E} = \{E_x, E_y\}$ are the field vectors at the input and output, respectively. The matrices are of the form

$$\hat{M}_{\rm k}^{+} = \begin{vmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{vmatrix}, \quad \hat{M}_{\rm p} = \begin{vmatrix} \cos^2\theta & \dfrac{\sin 2\theta}{2} \\ \dfrac{\sin 2\theta}{2} & \sin^2\theta \end{vmatrix},$$

where $M_{ij} = M'_{ij} + {\rm i}M''_{ij}$ are FRI matrix elements; θ is the angle between the transmission axis of the output polariser and the ordinate axis of the laboratory coordinate system; $M_{12} = M_0 + \Delta M$ and $M_{21} = M_0 - \Delta M$. It follows from the study of Listvin et al. [2] that the photodetector output signal at the first harmonic of the phase-modulation frequency, related to the nonreciprocal effects in the FRI, is determined by the quantity $\mathrm{Re}\,U_{\mathrm{I}}$, whereas the phase shift at the FRI output for $\Omega_{\mathrm{s}} = 0$ is $\Phi = \arctan(\mathrm{Im}\,U_{\mathrm{I}}/\mathrm{Re}\,U_{\mathrm{I}})$, where $U_{\mathrm{I}} = E_x^+ E_x^{-*} + E_y^+ E_y^{-*}$ is the interference signal. The expression for the FRI output signal in the absence of rotation beyond the output polariser at the first harmonic of the modulation frequency, obtained from the solution of Eqn (2) in terms of the M^+ matrix elements, has the form

$$\begin{aligned}
\mathrm{Im}\,U_{\mathrm{I}} &= (A_x^2 - A_y^2)(M_0''\Delta M' - M_0'\Delta M'') \\
&\quad + A_x A_y \gamma \big\{ \cos\psi \big[\Delta M''(M_{11}' - M_{22}') + \Delta M'(M_{22}'' - M_{11}'')\big] \\
&\quad - \sin\psi \big[\Delta M'(M_{11}' + M_{22}') + \Delta M''(M_{22}'' + M_{11}'')\big] \big\} \\
&\quad + \cos 2\theta \big\{ (A_x^2 + A_y^2)(M_0'\Delta M'' - M_0''\Delta M') \\
&\quad + A_x A_y \gamma \cos\psi \big[\Delta M''(M_{11}' + M_{22}') - \Delta M'(M_{11}'' + M_{22}'')\big] \\
&\quad + A_x A_y \gamma \sin\psi \big[\Delta M''(M_{22}' - M_{11}') + \Delta M'(M_{22}'' - M_{11}'')\big] \big\} \\
&\quad + \sin 2\theta \big\{ A_x^2 (M_{11}''\Delta M' - M_{11}'\Delta M'') + A_y^2 (M_{22}''\Delta M'' \\
&\quad - M_{22}''\Delta M') - 2A_x A_y \gamma \sin\psi (\Delta M' M_0' + \Delta M'' M_0'') \big\}. \qquad (4)
\end{aligned}$$

where ψ is the phase difference between the field components A_x and A_y at the input.

Expression (4) was obtained for one of the spectral components. In order to demonstrate the influence of the broadband nature of the radiation-source spectrum on the output signal, the coefficient γ characterising the degree of coherence of the components A_x, A_y was introduced formally into expression (4). The coefficient γ vanishes when the fields are incoherent. An analysis of this expression shows that the FRI output signal amplitude at the modulation frequency vanishes only when the ring interferometer matrix satisfies the condition for polarisation reciprocity $M_{12} = M_{21}$, i.e. when $\Delta M' = \Delta M'' = 0$.

If $M_{12} \neq M_{21}$, the FRI output signal consists of two parts, each of which depends on the difference between the off-diagonal coefficients of the FRI matrix. The expression for $\mathrm{Im}\,U_{\mathrm{I}}$ contains two parts: U_1 and U_2. The first part (U_1) is independent of the input polariser position (i.e. it is independent of θ) and is the polarisation nonreciprocity signal in the absence of the external polariser, as given by expression (13) in Ref. [4]. The second part (U_2) depends on the polariser position and contains terms proportional to $\cos 2\theta$ and $\sin 2\theta$. If $U_1 = 0$, the output signal amplitude varies, when the polariser rotates ($\theta = \omega t$), at the frequency 2Ω equal to twice the rotation frequency. If $U_1 \neq 0$, the addition of U_1 and U_2 yields the variation of the signal amplitude also at the first-harmonic frequency of the polariser rotation. For comparison with experiment, the output signal was calculated at the output of a rotating polariser for an FRI with an anisotropic fibre loop and with an anisotropic beam splitter, examined by Andronova et al. [4]. The phase characteristics of the beam splitter are determined by the parameters $\Delta\varphi = \varphi_{\mathrm{s}} - \varphi_{\mathrm{p}}$, $\Delta\psi = \psi_{\mathrm{s}} - \psi_{\mathrm{p}}$ characterising the phase anisotropy of the beam splitter in reflection and transmission. The anisotropy axes of the fibre at the loop output and input meet at the angles α and β ($\alpha \neq \beta$) with the anisotropy axes of the beam splitter. Following the procedure in Ref. [4], the matrix elements of an anisotropic fibre FRI with linear birefringence can be written down as follows when the beam-splitter characteristics are taken into account in a coordinate system linked to the anisotropy axes of the beam splitter:

$$\begin{aligned}
M_{11}' &= \cos\frac{kL}{2}\cos(\alpha+\beta)\cos(\Delta\varphi+\Delta\psi) \\
&\qquad - \sin\frac{kL}{2}\cos(\alpha-\beta)\sin(\Delta\varphi+\Delta\psi)\ , \\
M_{11}'' &= \sin\frac{kL}{2}\cos(\alpha-\beta)\cos(\Delta\varphi+\Delta\psi) \\
&\qquad + \cos\frac{kL}{2}\cos(\alpha+\beta)\sin(\Delta\varphi+\Delta\psi)\ , \qquad (5) \\
M_0' &= \cos\frac{kL}{2}\sin(\alpha+\beta)\cos(\Delta\varphi+\Delta\psi) \\
&\qquad - \sin\frac{kL}{2}\sin(\alpha-\beta)\sin(\Delta\varphi-\Delta\psi)\ , \\
\Delta M'' &= \sin\frac{kL}{2}\sin(\alpha-\beta)\cos(\Delta\varphi-\Delta\psi) \\
&\qquad + \cos\frac{kL}{2}\sin(\alpha+\beta)\sin(\Delta\varphi-\Delta\psi)\ ,
\end{aligned}$$

where L is the fibre length; $k = (k_x + k_y)/2 = (\pi n_x + \pi n_y)/\lambda_j$; n_x and n_y are the refractive indices along the 'fast' and 'slow' fibre axes; $M_{22} = -M_{11}^*$. Substitution of expression (5) in expression (4) gives the signal envelope at output I at the phase-modulation frequency, which is determined by $\mathrm{Im}\,U_{\mathrm{I}}$ [4]. Subject to the condition $\langle E_x E_y \rangle = 0$, the expression for the signal envelope can be written as follows:

$$\mathrm{Im} U_{\mathrm{I}} = (A_x^2 - A_y^2)S + (A_x^2 + A_y^2)(S\cos 2\theta + R\sin 2\theta)\ , \qquad (6)$$

where

$$\begin{aligned}
S &= (1/2)\sin 2\alpha \sin 2\beta \sin 2(\Delta\varphi - \Delta\psi)\ , \\
R &= \sin 2\beta \cos 2\alpha \sin 2\Delta\varphi - \sin 2\alpha \cos 2\beta \sin 2\Delta\psi\ .
\end{aligned} \qquad (7)$$

It is seen from the set of expressions (7) that both coefficients, S and R, vanish when $\alpha = \beta = 0$, i.e. for coaxial alignment of the anisotropy axes of the beam splitter and the fibre loop. Without this alignment ($\alpha \neq \beta \neq 0$), an additional condition for the 'vanishing' of the coefficient S is the absence of losses in the beam splitter ($\Delta\varphi - \Delta\psi = 0$). This condition is insufficient for 'vanishing' of the coefficient R and the absence of the phase anisotropy of the beam splitter ($\Delta\varphi = \Delta\psi = 0$) is also necessary [4].

Expression (6) was tabulated numerically with the aid of a computer as a function of the angle θ governing the polariser position. The parameters of a discrete beam splitter with 2% losses, for which the deviations of the phase shifts $\varphi_{\mathrm{s}} - \psi_{\mathrm{s}}$ and $\varphi_{\mathrm{p}} - \psi_{\mathrm{p}}$ from $\pi/2$ were 0.57 and 0.4 angular degrees for 1% losses, were used in the estimates. The remaining parameters adopted in the calculation are listed in the captions of Fig 4. Comparison of the calculated (Fig. 4) and experimental (Figs 2 and 3) relationships demonstrates a satisfactory qualitative agreement. Quantitative comparisons of experiment and calculation are extremely difficult. Our calculation thus confirms that the absence of a phase-modulation signal for any positions of the axes of a polariser located ahead of the photodetector may serve as a criterion of the polarisation reciprocity of an FRI. In this connection, it is possible to put forward a method for monitoring the

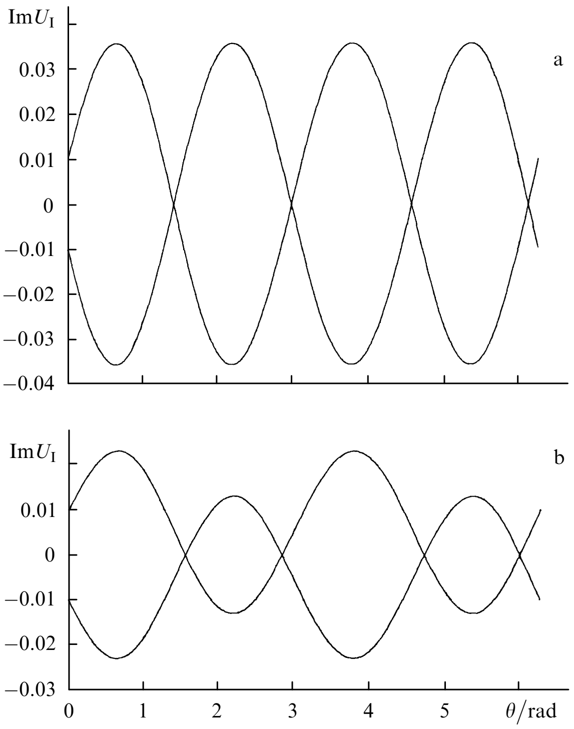

Figure 4. Envelopes of the phase-modulation signal Im $U_{\rm I}$ at the FRI output beyond a rotating polariser, calculated by means of formula (6) for $\alpha = 20°$, $\beta = 45°$, $\Delta\varphi = 1.28°$, $\Delta\psi = 0.83°$ when $E_x = E_y$, $R = 0.017$, $S = 0.005$ (without a polariser, zero signal) (a) and when $E_x = 1$, $E_y = 0.1$, $R = 0.017$, $S = 0.005$ (without a polariser, nonzero signal) (b).

polarisation nonreciprocity with a rotating polariser at the FRI input ahead of the photodetector. It should be pointed out that expression (4) was derived for the general case and, therefore, the proposed method is applicable to systems with or without a polariser. Correct alignment of the FRI components must be estimated not from the absence of a signal at the output for $\Omega = 0$, but from the absence of a signal on rotation of the polariser.

At this stage we are considering output I of the beam splitter (2 in Fig. 1), for which the influence of the beam splitter (5) in the case of each counterpropagating wave is the same because each of the waves is 'reflected' once from the beam splitter and 'passes' once through it.

We shall now discuss the signal at output II of the beam splitter (5). In this case a characteristic feature of the output signal of the beam splitter (5) is that, together with the signal related to the nonreciprocal FRI phase, it also contains a signal related to the beam splitter, because one wave is reflected twice from it and acquires the doubled reflection phase $2\Delta\varphi$, whereas the counterpropagating wave passes twice through the beam splitter and acquires the doubled transmission phase $2\Delta\psi$. The difference between these two phases in fact yields an additional contribution to the measured signal at output II:

$$\mathrm{Im}U_{\rm II} = A_x^2 \sin 2(\varphi_{\rm s} - \psi_{\rm s}) + A_y^2 \sin 2(\varphi_{\rm p} - \psi_{\rm p})\ . \tag{8}$$

It has been shown [4] that, in the absence of losses, the difference is $\varphi_{\rm s} - \psi_{\rm s} = \varphi_{\rm p} - \psi_{\rm p} = \pi/2$ and it does not contribute to the measured signal Im $U_{\rm II}$. In this case, when the polarisation nonreciprocity signal is reduced to zero at output I (Im $U_{\rm II} = 0$), the output II signal is defined by expression (8), i.e. it is determined by the losses in the beam splitter. In separate measurements of the losses, when the beam splitter axes are aligned along the x and y axes and there is a polariser at output II, the absence of the phase-modulation signal at output I (Im $U_{\rm I} = 0$) is insufficient and it is also necessary to fulfill the FRI reciprocity condition ($M_{12} = M_{21}$), i.e. the absence of a signal at output I beyond the rotating polariser.

Thus, in determination of the FRI polarisation reciprocity in the case when the output characteristics are monitored with the aid of a rotating polariser at output I, the signal from output II may be used for comparative estimation of the quality and phase anisotropy of the beam splitters. The calibration phase shift may be determined from expression (1), provided that the fibre length and the FRI loop perimeter are known.

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