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# Mechanism of the appearance of a diffractive nonreciprocity in a ring gas laser

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**Abstract. It is shown theoretically that diffraction leads to a difference between the losses and phase velocities of counterpropagating waves in ring gas lasers. This is the cause of the appearance of the amplitude and frequency nonreciprocity of these waves.**

## 1. Introduction

The diffractive frequency instability of a ring gas laser is one of the causes of the drift of the zero point (i.e. of the existence of an initial difference between the frequencies of counterpropagating waves in a laser at rest in the absence of nonreciprocal devices) in a laser gyroscope. Although the splitting of the counterpropagating wave frequencies on stopping down of the laser beam by a razor's edge or a needle was first observed by Cheo and Cooper [1], the study of this phenomenon and of its causes has continued up to the present time [2 – 7].

It has been suggested [7] that the appearance of the amplitude and frequency nonreciprocity is associated with the difference between the losses of the counterpropagating waves, which in the author's view arises owing to the cavity misalignment when the aperture is shifted at right angles to the beam. The hypothesis of the inequality of the frequency-dependent losses of counterpropagating waves was put forward by Garside [8], who made an experimental attempt to achieve unidirectional lasing in a ring laser. The possibility of implementing a unidirectional regime in a laser with an unstable cavity was pointed out also in a theoretical study [9], where it was demonstrated that the spatial configurations of counterpropagating waves are significantly different in such cavities.

The frequency and amplitude nonreciprocities were studied experimentally in a ring He – Ne laser operating at the wavelength $\lambda = 3.39$ μm, both on the basis of the pure $^{20}$Ne isotope and on the basis of a mixture of isotopes [4, 7]. Lasers with three-mirror cavities having approximately equal values of the parameter g and generating a single longitudinal-mode were used in these investigations. A shift of the aperture placed in the cavity perpendicularly to the beam was accompanied by the appearance of a counterpropagating-wave frequency splitting: $2\Delta\omega = \omega_r - \omega_l$. The authors [4, 7] studied the dependence of the intensity $I_j$ ($j = r, l$) and of the frequency nonreciprocity $\Delta\omega$ on the detuning of the cavity frequency from the centre $\omega_{ab}$ of the atomic-transition line. It was established that the frequency and amplitude nonreciprocities are maximal near the line centre and vanish at the limits of the lasing range, where the behaviour of the intensities of these studies proved to be somewhat different.

It was demonstrated in Ref. [4] that, for a symmetrical disposition of the intensity curves relative to $\omega_{ab}$, the counterpropagating wave frequencies were equal and the appearance of asymmetry was accompanied by the appearance of beats of $\Delta\omega$. Under these conditions, a change in the sign of $\omega - \omega_{ab}$ was always accompanied by a change in the sign of $2\Delta I = I_r - I_l$. In Ref. [7], the dependences of the counterpropagating wave intensities on the detuning were asymmetric for $\Delta\omega = 0$, but they had the same profile. After introduction of an aperture, the dependences acquired a different asymmetry (without a change in the sign of $\Delta l$) and the wave with the greater asymmetry had the higher frequency.

It is striking that the curves presented in Ref. [7] are in many respects similar to those obtained in Ref. [10], where a device giving rise to nonidentical losses (amplitude nonreciprocity) was used. The dependences presented in Ref. [4] repeat the form of the curves obtained in Ref. [11] for nonidentical gains (phase nonreciprocity) of the counterpropagating waves.

The results of studies reported in Refs [10, 11] can in the main be described quite satisfactorily in terms of the plane-wave model [12, 13], where the dependences of the intensities $I_j$ and of the frequencies $\omega_j$ of the counterpropagating waves on the frequency detuning relative to the central transition frequency $\omega_{ab}$ are described by the equations

$$I_r = \frac{\eta_r \beta_l'' - \eta_l \theta_l''}{\beta_r'' \beta_l'' - \theta_r'' \theta_l''}\,, \quad \eta_j = \frac{\alpha_j - \varepsilon_j}{KH}\,, \tag{1}$$

$$\omega_r = \Omega + \frac{c}{L} KH \left[ Z'(\zeta_r) - I_r \beta_r' - I_l \theta_l' \right]\,. \tag{2}$$

The equations for the $l$th wave are obtained by replacing the subscript r by l. Here, $\Omega$ is the cavity frequency; L is the cavity length; H is the length of the cell with the active medium; $\alpha_j = KHZ''(\zeta_j)$ is the linear gain; $\varepsilon_j$ represents the linear losses of the $j$th wave. The function $Z = Z' + iZ''$, the gains K, and the self-saturation ($\beta = \beta' + i\beta''$) and cross-saturation ($\theta = \theta' + i\theta''$) coefficients are listed in Appendix 1. Fig. 1 illustrates such functions for the following parameters occurring in these functions: $\lambda = 3.39$ μm, $\gamma_a = 16$ MHz, $\gamma_b = 24$ MHz, $\gamma_{ab} = 100$ MHz, and Doppler broadening $ku = 300$ MHz. These values will also be used in all subsequent calculations.

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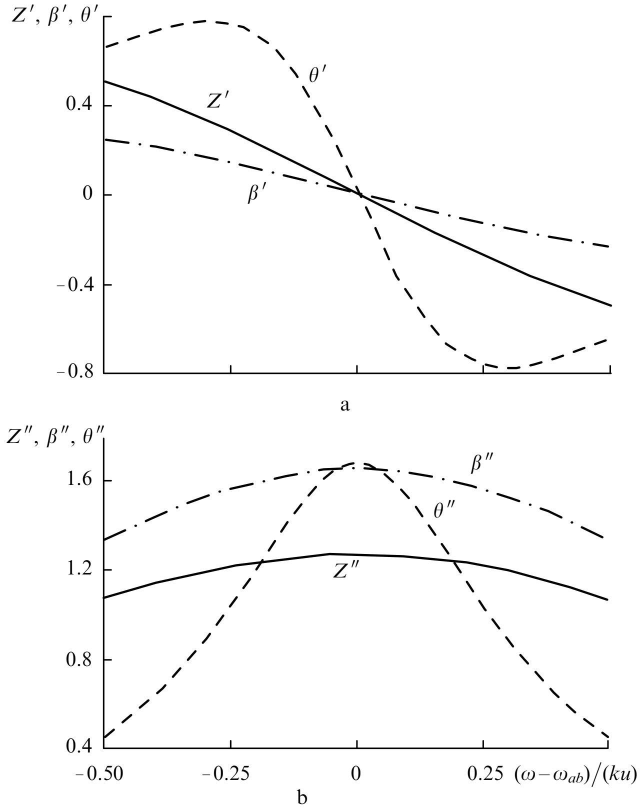


**Figure 1.** Dependences of the real (a) and imaginary (b) parts of the coefficients Z, $\beta$, and $\theta$ on the detuning.

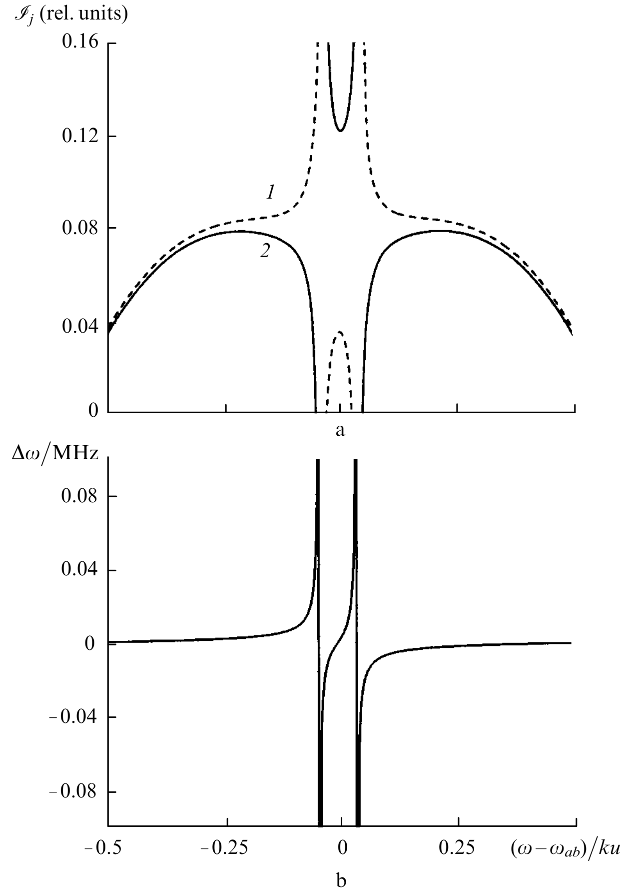


**Figure 2.** Dependences of the intensities of the counterpropagating waves (1, 2) (a) and of the frequency nonreciprocity (b) on the detuning, obtained in the plane-wave model for a cavity containing a device generating different counterpropagating wave losses; $\varepsilon_r = 1$, $\varepsilon_l = 1.002$, $KH = 1$, $L = 1$ n.

In the absence of the nonreciprocity, we have $I_r = I_l = \eta/(\beta'' + \theta'')$. In the presence of the amplitude and/or phase nonreciprocity in the cavity, an instability region is observed near the lasing line centre. This can easily be shown, for example, in the case where the counterpropagating wave losses are different.

After introducing the notation $\eta = (\eta_r + \eta_l)/2$, $\Delta\eta = (\eta_r - \eta_l)/2$ and putting $\beta_r = \beta_l$ and $\theta_r = \theta_l$, we obtain the following equation from Eqn (2):

$$I_{r,l} = \eta\frac{1}{\beta'' + \theta''} \pm \Delta\eta\frac{1}{\beta'' - \theta''} = I \pm \Delta I \ , \qquad (3)$$

showing that the functions $I_j(\omega - \omega_{ab})$ have a discontinuity at the frequencies characterised by $\beta'' = \theta''$ (Fig. 1), but the sum of the intensities remains a smooth function. This has been confirmed experimentally in many studies. Depending on the losses, either both waves or (by virtue of the strong cross-saturation) a wave with the greater losses may be generated within this region.

As a result of elimination of the nonreciprocity, both waves exist in the entire lasing region [10, 11], i.e. the presence of a region of instability indicates the amplitude or phase nonreciprocity. Figs 2 and 3 give the results of a numerical calculation of the normalised intensities $\mathscr{I}$ and of the differences $\Delta\omega$ between the frequencies of the counterpropagating waves [the systems of Eqns (1) and (2)] (Fig. 2 represents the experiment reported in Ref. [10] and Fig. 3 that described in Ref. [11]).

Similarity of the dependences of $\mathscr{I}$ and $\Delta\omega$ on the detuning from the resonance frequency, reported in Refs [4] and [11] as well as in Refs [7] and [10], may indicate operation of the same mechanism of the appearance of nonreciprocities, but in the absence of nonreciprocal elements it cannot be accounted for in terms of the language of the Lamb equations. In Ref. [7], it is simply postulated that the difference between the losses arises on misalignment of the cavity and manifests itself solely through the real part of the cross-saturation. The asymmetry of the Lamb dip, observed in the dependence of $2I = I_r + I_l$ on $\omega - \omega_{ab}$, also cannot be described by the plane-wave model.

We shall show that, in a laser cavity containing at least one spherical mirror and an aperture, the losses and the phase velocities of the counterpropagating waves are indeed different. We shall use an approach, different from that adopted in Refs [14]–[16], in order to find the natural oscillations of a cavity containing a transversely inhomogeneous nonlinear active medium and an aperture. This approach is based on the standard asymptotic expansion procedure [17, 18]. However, in contrast to Ref. [14], we shall take into account the multiplicative effect of the diffraction and saturation on the natural oscillations of the cavity.

We shall show that the radial distribution of the saturated gain forms an extended amplitude–phase corrector, the optical power and the stopping down properties of which are proportional to the induced transverse inhomogeneity parameter $W_j = 2L/kw_j^2$, where $k = \omega/c$; $w_j$ are the halfwidths of the transverse wave distributions. Since they are different in an inhomogeneous cavity, the effective optical

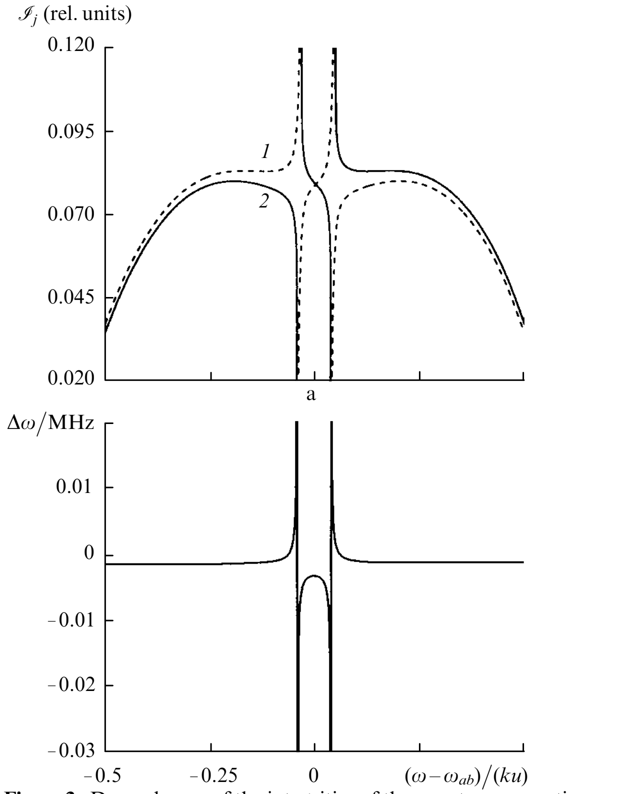


**Figure 3.** Dependences of the intensities of the counterpropagating waves (1, 2) (a) and of the additional frequency nonreciprocity (b) on the detuning, obtained in the plane-wave model for a cavity containing a device ensuring different frequencies of the counterpropagating waves; $\delta\omega = 0.5$ MHz, $L = 1$ m, $KH = 1$.

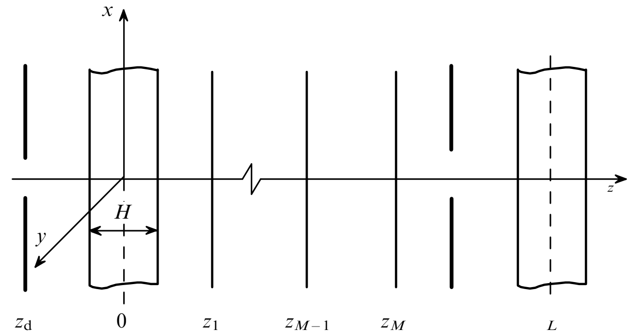


**Figure 4.** Schematic diagram of a ring cavity in the form of an equivalent periodic waveguide [$z_d$ is the position of the aperture, $z_m$ is the position of the mirrors ($m = 1, 2, \ldots, M$), H is the length of the cell with the active medium, and L is the perimeter of the cavity].

power and the size of the induced-corrector aperture are also different for the opposite directions. This gives rise to a nonreciprocal frequency-dependent change in the parameter g for counterpropagating waves, so that one can in fact explain the frequency-dependent inequality of their losses and phase velocities, and hence the inequality of the frequencies and intensities. The inequality of the counterpropagating wave losses leads to the possibility of existence of a region of unidirectional lasing. We shall also demonstrate that the losses may decrease abruptly at the lasing threshold.

An allowance for the multiplicative effect of the active medium and of the aperture explains the asymmetric nature of the losses and hence also of the intensities, relative to the central transition frequency.

## 2. Principal equations. Numerical calculations

We shall follow the formulation of the problem in Ref. [14] and consider a stable ring optical cavity with a perimeter L formed by an arbitrary number of mirrors (Fig. 4). The cavity contains an aperture with a Gaussian transmission coefficient. There is no retroreflection from the cavity components.

If a monochromatic wave propagates in each direction, we have a field of counterpropagating waves in the quasi-optical approximation:

$$E_j(x, y, z) = E_{0j}\psi_j(x, y, z) \times \exp\left[\mathrm{i}k_j\int_0^z n_{zj}(\tilde{z})\,\mathrm{d}\tilde{z}\right] + \text{c.c.}\,, \tag{4}$$

where $k_j = \omega_j/c$; $E_{0j}$ are the constant amplitudes of the counterpropagating waves; $\psi_j(x, y, z)$ are the functions which depend slightly on the coordinates. Following the standard asymptotic expansion procedure [14], we obtain for each wave a system of equations for nonlinear refractive indices of the active medium

$$n_{zr}(z) = 1 + \frac{1}{k}K[-Z + \beta_r\mathscr{I}_r(z) + \theta_l\mathscr{I}_l(z)] \tag{5}$$

and for slow amplitudes of the counterpropagating waves

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \pm 2\mathrm{i}n_{zj}(z)\frac{\partial}{\partial z} + n_{xj}(z)x^2 + n_{yj}(z)y^2\right]\psi_j(r) = 0 \tag{6}$$

with the coefficients $n_{zj}$ [Eqn (5)] and

$$n_{pr}(z) = -2KH[\beta_r W_{pr}(z)\mathscr{I}_r(z) + \theta_l W_{pl}(z)\mathscr{I}_l(z)] \tag{7}$$

($p = x, y$). The dimensionless intensities along the cavity axis to be determined

$$\mathscr{I}_j = I_j f_j(z)\,, \qquad f_j(z) = |m_{xj}m_{yj}|^{-1}\exp\left[-2k_j\int_0^z n''_{zj}(t)\,\mathrm{d}t\right] \tag{8}$$

are slow functions of the longitudinal coordinate z. The expression for $I_j$ is given by formula (A1.1).

Eqn (4) is supplemented by the conditions for the transformation of the field of a counterpropagting wave when the latter passes through the corresponding cavity components and also by the conditions for the reproducibility of the fields after a round trip through the cavity.

The distribution of the fields of the zeroth-mode counterpropagating waves is sought in the following form:

$$\psi_j(x, y, z) = \psi_{xj}(x, z)\psi_{yj}(y, z)\,,$$

$$\psi_{xj}(x, z) = \exp\left(\pm P_{xj} + \frac{\mathrm{i}x^2}{2q_{xj}}\right) = \frac{1}{\sqrt{m_{xj}}}\exp\frac{\mathrm{i}x^2}{2q_{xj}}\,. \tag{9}$$

A similar formula can be written also in the coordinate plane yz. The parameters $q^{-1}$ and $m_{pj}$ are transformed in accordance with the law

$$q_{pj}^{-1}(z) = \frac{c_{pj}(z) + d_{pj}(z)q_{pj}^{-1}(0)}{a_{pj}(z) + b_{pj}(z)q_{pj}^{-1}(0)}\,,$$
$$m_{pj} = a_{pj} + b_{pj}q_{pj}^{-1}(0) \qquad (p = x, y)\,. \tag{10}$$

Here, $q_{pj}(0)$ is the parameter $q_{pj}$ in the $z = 0$ section.

The matrix for the transformation of the beam parameters on passage through a longitudinally and transversely inhomogeneous medium has been published [16]. In view of its complexity, the 'short tube' approximation is used to elucidate the physical cause of the nonreciprocity. Physically, this approximation means that in the interval $h = H/L$ the changes in $n_{zj}(z)$ and $n_{pj}(z)$ along the z axis are negligible and these functions may be replaced by their values at the point $z_0$ when the familiar matrix [19, 20] for a transversely inhomogeneous medium may be used (see Appendix 2). The question of the influence of the extent of the active medium will be discussed in a separate communication.

Having determined the elements $A_{pj}$, $B_{pj}$, $C_{pj}$, $D_{pj}$ of the matrix representing a round trip of the counterpropagating waves through the cavity (here, the ABCD matrix represents the product of the abcd matrices describing the individual components in the cavity), we find $G_{pj} = \frac{1}{2}(A_{pj} + D_{pj})$ and then, by using the periodicity conditions, we find the parameters

$$q_{pr,l}^{-1} = \pm\frac{D_{pj} - A_{pj}}{2B_{pj}} + \mathrm{i}\frac{(1 - G_{pj})^{1/2}}{B_{pj}} = S_{pr,l} + iW_{pr,l} \ , \quad (11)$$

characterising the curvature $s_{pj}$ ($S_{pj} = s_{pj}L$) of the wavefronts and their half-width $w_{pj}$ ($W_{pj} = 2L/kw_{pj}^2$). Having calculated the complex magnification of the beam $m_{pj}$ [expression (10)] along the cavity length, taking into account expression (11), we can determine the propagation constants for the counterpropagating waves:

$$\Gamma_j = \sum_{p=x,y}\Gamma_{pj} = \Gamma_j' + \mathrm{i}\Gamma_j'' \, , \quad \Gamma_{pj} = \mathrm{i}\ln[G_{pj} \pm (G_{pj}^2 - 1)^{1/2}] \, . \quad (12)$$

The field periodicity conditions [Eqn (4)], which we shall formulate as [14]

$$\Gamma_j + k_jL\int_0^1 n_{zj}^{(3)}\,\mathrm{d}z = 0 \, , \quad n_{zj} = 1 + (\delta n_{zj}' + i\delta n_{zj}')/k_j \, , \quad (13)$$

lead to the phase and amplitude balance equations

$$\Gamma_j' + k_jL(1 + \delta n_{zj}'h) = 0 \, , \quad \Gamma_j'' + \delta n_{zj}''h = 0 \ , \quad (14)$$

which define the lasing intensities and frequencies; here, $\Gamma_j'$ are the phase shifts additional to the geometrical-optical shift and $\Gamma_j''$ are the counterpropagating wave losses in a cavity containing a nonlinear active medium.

Fig. 5 presents the results of a numerical calculation of the lasing intensities. Evidently, the total dimensionless intensity $2\mathscr{I} = \mathscr{I}_r + \mathscr{I}_l$ has a dip near the line centre, typical of the intensity of radiation from a linear laser. The asymmetry of the curve is clearly seen and $\mathscr{I}$ is greater in the low-frequency range (for $\omega < \omega_{ab}$), which agrees well with the results of measurements reported in Ref. [7]. The dependence of each intensity $\mathscr{I}_{r,l}$ on the detuning is also asymmetric. This asymmetry is different for a counterpropagating wave, which also agrees with Ref. [7]. The appearance of the amplitude nonreciprocity is accompanied by the frequency nonreciprocity.

An asymmetry of the region of instability is also observed. The mode with the lower intensity is preferred within this region. On increase in the losses, unidirectional generation of a weak mode may occur, as is in fact noted in Ref. [21].

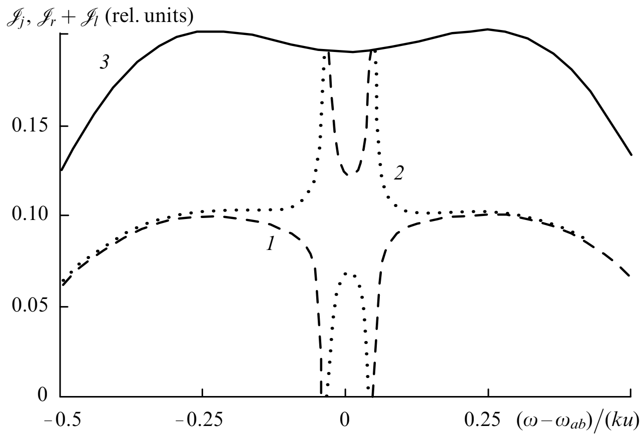


**Figure 5.** Dependences of the intensities $\mathscr{I}_r$ (1) and $\mathscr{I}_l$ (2) of the counterpropagating waves and of their sum (3) on the detuning, obtained in the Gaussian-beam model for a three-mirror cavity with one spherical mirror having the radius $R = 1.2$ m and a Gaussian aperture with the half-width $a_x = a_y = 0.4$ mm; $z_{\mathrm{d}} = 0.5$ m, $L = 1$ m, and $KH = 1.5$.

## 3. Weak perturbation approximation

We shall demonstrate analytically that the difference between the frequencies and intensities of the counterpropagating waves results from the difference between their losses and phase velocities. The calculations will be made for a ring laser, the cavity of which has an arbitrary number of mirrors and is illustrated schematically in Fig. 4. The centre of the active medium layer is chosen as the initial section, which makes it possible to select an explicit form of the matrix $A_{0p}B_{0p}C_{0p}D_{0p}$ corresponding to the empty cavity (without the medium and aperture). In the weak cavity-perturbation approximation ($n_{pj} < N_p < 1$), we shall describe $\Gamma_j$ [expression (13)] by

$$\Gamma_{pj} = \Gamma_{p0} + \delta\Gamma_{pj}, \quad \Gamma_{p0} = \mathrm{i}\ln\left[g_\mathrm{p} + (g_\mathrm{p}^2 - 1)^{1/2}\right] - (\arccos g_\mathrm{p} + 2\pi q), \quad \delta\Gamma_{pj} = \delta\Gamma'_{pj} + \mathrm{i}\Gamma''_{pj},$$

where $g_\mathrm{p}$ is the parameter g of the unperturbed cavity; q is the longitudinal mode index; $\delta\Gamma'_{pj}$ and $\Gamma''_{pj}$ are defined in Appendix 2.

Having substituted $n_{pj}$ [expression (8)] in expression (A2.5), we easily find that in the case of a weak cavity perturbation the counterpropagating wave losses are given by the following expression which takes into account the terms due to the transverse inhomogeneity of a nonlinear medium:

$$\varepsilon_r = \Gamma''_r = \varepsilon_\mathrm{d} - KH\left[\left(\beta''_r \mathscr{I}_r W_r + \theta''_l \mathscr{I}_l W_l\right) W_{0p}^{-1}\mu_{1p} + \left(\beta'_r \mathscr{I}_r W_r + \theta'_l \mathscr{I}_l W_l\right) W_{0p}^{-1} N_p \mu_{2p}\right], \quad (15)$$

where

$$\varepsilon_\mathrm{d} = \sum_{p=x,y}\left(\frac{w_{0p}}{a_p}\right)^2 \mu_{3p}$$

are the cavity losses introduced by a Gaussian aperture; $w_{0p}$ is the beam half-width in the reference section, deduced ignoring the perturbation. The parameters $\mu_{ip}$ ($i = 1, 2, 3$) are determined by the cavity geometry [see expression (A2.6)].

$$\eta_r = \mathscr{I}_r(\beta''_r \upsilon_{1r} - \beta'_r \upsilon_{2r}) + \mathscr{I}_l(\theta''_l \upsilon_{1l} - \theta'_l \upsilon_{2l}), \quad (16)$$

$$\omega_r = \Omega - \frac{c}{L} KH\left[\mathscr{I}_r(\beta'_r \upsilon_{1r} + \beta''_r \upsilon_{2r}) + \mathscr{I}_l(\theta'_l \upsilon_{1l} + \theta''_l \upsilon_{2l})\right]. \quad (17)$$

Here, $\eta_j$ is the relative excess of the unsaturated gain over the linear losses defined by Eqn (1);

$$\Omega = \frac{c}{2L}\sum_{p=x,y}\Gamma_{p0}$$

is the frequency for the unperturbed cavity;

$$\upsilon_{1r,l} = \upsilon_1 \mp \sum_{p=x,y}\frac{\mu_{1p}\Delta W_p}{W_{0p}}; \quad \upsilon_1 = 1 - \sum_{p=x,y}\mu_{1p}\left(1 + \frac{\delta W_p}{W_{0p}}\right); \quad (18)$$

$$\upsilon_{2r,l} = \upsilon_2 \pm \sum_{p=x,y}\frac{\mu_{2p}N_p\Delta W_p}{W_{0p}}; \quad \upsilon_2 = \sum_{p=x,y}\mu_{2p}N_p\left(1 + \frac{\delta W_p}{W_{0p}}\right). \quad (19)$$

We expressed the parameters $W_{pj}$ [expression (11)] in the form $W_{pr,l} = W0p + \delta W_\mathrm{p} \pm \Delta W_\mathrm{p}$, where $\delta W$ is responsible for the reciprocal and $\Delta W$ for the nonreciprocal deformation of the fields of the counterpropagating waves by the transverse inhomogeneity of the medium and by the aperture. In the case of a weak perturbation of the cavity, $\delta W$ is determined mainly by the real part of the refractive index, i.e. it exhibits an odd dependence on the detuning. On the other hand, $\Delta W$ is determined by the position and dimensions of the aperture and also by the imaginary part of the refractive index, and its frequency-dependent component exhibits an even dependence on the detuning.

The intensities of the counterpropagating waves can be found from system of equations (16) and they can be represented in the following form, taking into account relationships (18) and (19):

$$\mathscr{I}_r = \mathscr{I} + \Delta\mathscr{I}, \quad \mathscr{I}_l = \mathscr{I} - \Delta\mathscr{I},$$

where

$$\mathscr{I} = \eta\frac{1}{(\beta''+\theta'')\upsilon_1 - (\beta'+\theta')\upsilon_2}; \quad (20)$$

$$\Delta\mathscr{I} = \eta\sum_{p=x,y}\frac{\Delta W_p}{W_{0p}}\frac{(\beta''-\theta'')\mu_{1p} + (\beta'-\theta')\mu_{2p}N_p}{(\beta''\upsilon_1 - \beta'\upsilon_2)^2 - (\theta''\upsilon_1 - \theta'\upsilon_2)^2}. \quad (21)$$

After substitution of these relationships in formula (15), we find that the counterpropagating wave losses in a cavity with a nonlinear medium are different:

$$\varepsilon_{r,l} = \varepsilon \pm \Delta\varepsilon,$$

where

$$\Delta\varepsilon = -KH\mathscr{I}\sum_{p=x,y}\frac{\Delta W_p}{W_{0p}}\left(1 + \frac{W_p\Delta\mathscr{I}}{\mathscr{I}\Delta W_p}\right) \times\left[(\beta''-\theta'')\mu_{1p} + (\beta'-\theta')\mu_{2p}N_p\right], \quad (22)$$

$$\frac{W_p\Delta\mathscr{I}}{\mathscr{I}\Delta W_p} = \frac{W_p}{W_{0p}}\frac{(\beta''-\theta'')\mu_{1p} + (\beta'-\theta')\mu_{2p}N_p}{(\beta''-\theta'')\upsilon_{1p} - (\beta'-\theta')\upsilon_{2p}}. \quad (23)$$

It follows from these relationships that $\Delta\varepsilon$ is proportional to the lasing intensity and is determined by the nonreciprocal deformation of the distributions of the counterpropagating-wave fields. The reciprocal part of the losses is given by

$$\varepsilon = \varepsilon_\mathrm{d} - KH\mathscr{I}\sum_{p=x,y}\frac{W_p}{W_{0p}}\left[(\beta''+\theta'')\mu_{1p} + (\beta'+\theta')\mu_{2p}N_p\right]. \quad (24)$$

Taking into account expressions (20) and (21), it is easy to deduce from Eqn (17) that

$$\omega_r - \omega_l = -(\alpha - \varepsilon)\frac{c}{2L} \times\sum_{p=x,y}\frac{\Delta W_p}{W_{0p}}N_p\mu_{2p}\upsilon_{1p}\frac{(\beta'-\theta')^2 + (\beta''-\theta'')^2}{(\beta''\upsilon_1 - \beta'\upsilon_2)^2 - (\theta''\upsilon_1 - \theta'\upsilon_l)^2}. \quad (25)$$

We have thus established that an inequality of the transverse counterpropagating-wave field distributions, resulting from diffraction by the aperture and by the active medium, leads to an inequality of the saturation of the transverse components of the complex refractive indices [expression (7)] for the counterpropagating waves. These indices are responsible for the lens-like and stopping-down properties of the active medium. In a cavity with spherical mirrors, this leads to an inequality of the nonlinear losses and phase velocities of the counterpropagating waves, which induces the frequency [expression (25)] and amplitude [expression (21)] nonreciprocities. These nonreciprocities are complex functions of the parameters which determine the cavity geometry and the properties of the active medium. The amplitude nonreciprocity [expression (21)] is an even (although asymmetric) function of the detuning, whereas the frequency nonreciprocity [expression (25)] may exhibit an even or an odd dependence on $\omega - \omega_{ab}$. The nature of the dependence is determined by the cavity geometry and by the parameter $v_1$ [set of expressions (18)].

The difference between the experimental curves presented in Refs [4] and [7] is associated, in our view, with the difference between the diameters of the discharge tubes (3 mm in Ref. [4] and 6 mm in Ref. [7]). Indeed, the linear effect of the gas lens, arising owing to the transversely inhomogeneous distribution of the perturbation density (the effect of this mechanism has been investigated in Ref. [16]) and because of the influence of the misalignment of the cavity, manifests itself more strongly in tubes of small diameter. Three-mirror cavities were used in the investigations described in Refs [4] and [7].

A cavity with an odd number of mirrors is known to be immune to misalignment [19]. A shift of the aperture at right angles to the plane of a cavity with a wide tube is equivalent to the use of a smaller symmetrical aperture. However, for a small diameter of the discharge tube, even slight misalignments may necessitate allowance for the Langmuir flow [22], the existence of which creates an additional phase nonreciprocity. The predominance of the phase mechanism of the nonreciprocity has in fact been observed earlier [4], although it is the difference between the counterpropagating wave losses that plays the main role, as reported in Ref. [7].

Expression (20) describes the Lamb dip profile and shows that the depth of the dip of the total-intensity curve $2\mathscr{I}$ (Fig. 5), observed near the line centre, depends on the degree of saturation of the losses. A 'saturation aperture' diminishes the losses defined by expression (24) and thereby influences the depth of the dip in the intensity curve [expression (20)] (via $v_{1p}$). The term proportional to $v_{2p}$ [set of expressions (19)] describes the dip asymmetry associated with the asymmetric nature of the total losses [expression (24)]: the 'saturation lens' alters the transverse distributions of the counterpropagating wave fields in accordance with the dispersion law, by virtue of which the losses on the aperture acquire an additional component which is odd in terms of the detuning. The influence of the nonlinear lens on the losses manifests itself solely when account is taken of the multiplicative effect of the aperture and of the transverse inhomogeneity of the medium on the natural cavity oscillations.

## Appendix 1

Calculations of the polarisation of a medium consisting of two-level atoms, made adopting the standard approximations [12] of the third-order perturbation theory in terms of the small parameter

$$I_j = \frac{(\gamma_a + \gamma_b) d^2 |E_{0j}|^2}{\gamma_a \gamma_b \gamma_{ab} \hbar^2}\,, \tag{A1.1}$$

are reported above. Here, d is the dipole moment; $\gamma_{ab}$ is the half-width of the homogeneous line of the $a \leftrightarrows b$ transition; $\gamma_a$ and $\gamma_b$ are the half-widths of the levels a and b; $j = r$, l are the counterpropagating wave indices; $P_r(r) = (\varkappa + \chi)E_r(r)$ is the polarisation.

The linear part of the polarisability $\varkappa$ of the medium is defined by the following relationships:

$$2\pi\varkappa = -KZ(\zeta)k^{-1}\,, \qquad K = 2\pi d^2 N(\hbar u)^{-1}\,, \tag{A1.2}$$

where

$$Z(\zeta) = 2\mathrm{i}\int_0^\infty \exp(-\rho^2 + 2\mathrm{i}\rho\zeta)\,\mathrm{d}\rho$$

is the plasma function;

$$\zeta = \frac{\omega - \omega_{ab}}{ku} + \frac{\mathrm{i}\gamma_{ab}}{ku}\,;$$

ku is the half-width of the Doppler profile, k is the wave number.

The nonlinear part of the wave polarisability r is given by

$$\begin{aligned}
2\pi\chi^{(3)} &= K\frac{2\mathrm{i}d^2}{k\hbar^2\gamma_a}\frac{4}{(ku)^2} \\
&\times \int_0^\infty \mathrm{d}\rho_1 \int_0^\infty \mathrm{d}\rho_2 \int_0^\infty \mathrm{d}\rho_3 \exp\left[-\frac{2(\rho_1+\rho_3)]\gamma_{ab}}{ku} - \frac{2\rho_2\gamma_a}{ku}\right] \\
&\times \Bigg(|E_{0r}|^2|\psi_r|^2 \exp\left(-2k\int n''_{zr}\,\mathrm{d}z\right)\Big\{\exp[-(\rho_1-\rho_3)^2 \\
&+ 2\mathrm{i}\xi_r(\rho_1-\rho_3)] + \exp[-(\rho_1+\rho_3)^2 + 2\mathrm{i}\xi_r(\rho_1+\rho_3)]\Big\} \\
&+ |E_{0l}|^2|\psi_l|^2 \exp\left(-2k\int n''_{zl}\,\mathrm{d}z\right)\Big\{\exp[-(\rho_1-\rho_3)^2 \\
&+ 2\mathrm{i}\xi_r\rho_1 + 2\mathrm{i}\xi_l\rho_3] + \exp[-(\rho_1+\rho_3)^2 + 2\mathrm{i}\xi_r\rho_1 - 2\mathrm{i}\xi_l\rho_3] \\
&+ \exp[-(\rho_1+2\rho_2+\rho_3)^2 + 2\mathrm{i}\xi_r\rho_1 + 2\mathrm{i}\xi_{rl}\rho_2 - 2\mathrm{i}\xi_l\rho_3] \\
&+ \exp[-(\rho_1+2\rho_2+\rho_3)^2 + 2\mathrm{i}\xi_r(\rho_1+\rho_3) + 2\mathrm{i}\xi_{rl}\rho_2]\Big\}\Bigg) \\
&+ M(a\to b) = \frac{1}{k}K\left[\beta_r I_r|\psi_r|^2 \exp\left(-2k\int n''_{zr}\,\mathrm{d}z\right)\right. \\
&\left. + \theta_l I_l|\psi_l|^2 \exp\left(-2k\int n''_{zl}\mathrm{d}z\right)\right]\,.
\end{aligned} \tag{A1.3}$$

Here $M(a \to b)$ is the previous expression in which $\gamma_a$ has been replaced by $\gamma_b$;

$$\beta_r = \mathrm{i}Z''(\zeta_r) + 2\mathrm{i}\frac{\gamma_{ab}}{ku}[1 + \zeta_r Z(\zeta_r)]\,; \tag{A1.4}$$

$$\theta_r = \sum_{t=1}^{4}\theta_{rt}\,; \qquad \theta_{r1} = \frac{1+\mathrm{i}\varDelta}{2(1+\varDelta^2)}[Z(\zeta_r) + Z(\zeta_l)]\,;$$

$$\theta_{r2} = -\frac{\mathrm{i}}{2\varDelta}[Z(\zeta_r) + Z^*(\zeta_l)]\,, \tag{A1.5}$$

$$\begin{aligned}
\theta_{r3} &= \frac{\gamma}{\gamma_{ab}}\frac{p}{4}\sum_{n=a,b}|v_n|^{-2}\Big\{Z(\zeta_{rl}^{(n)}) \\
&-\frac{1}{2}\left[Z(\zeta_r) - Z^*(\zeta_l)\right] - \alpha_n\theta_{r2}\Big\}\,;
\end{aligned} \tag{A1.6}$$

$$\begin{aligned}
\theta_{r4} &= \frac{\gamma}{\gamma_{ab}}\frac{p}{4}\sum_{n=a,b}|v_n|^{-2}\Big\{Z(\zeta_r)\left[1 - \frac{2\gamma_{ab}^2}{(ku)^2}v_n v\right] \\
&- Z(\zeta_{rl}^{(n)}) + \frac{2v_n\gamma_{ab}}{ku}\Big\}\,;
\end{aligned} \tag{..1.7}$$

$$\zeta_j = \xi_j + \frac{\mathrm{i}\gamma_{ab}}{ku}\ (j = r,\, l)\,; \quad \zeta_{rl}^{(a,b)} = \xi_{rl} + \frac{\mathrm{i}\gamma_{a,b}}{2ku}\,; \quad \text{(A1.8)}$$

$$\xi_j = \frac{\omega_j - \omega_{ab}}{ku}\,; \quad \xi_{rl} = \frac{\xi_r - \xi_l}{2}\,; \quad \xi = \frac{\xi_r + \xi_l}{2} = \frac{\Delta\gamma_{ab}}{ku}\,;$$

$$v = \Delta + \mathrm{i}\,; \quad v_{a,b} = \Delta + i\alpha_{a,b}\,; \quad \alpha_{a,b} = 1 - \frac{\gamma_{a,b}}{2\gamma_{ab}}\,; \quad \text{(A1.9)}$$

$$p = \frac{\gamma_a\gamma_b}{\gamma^2}\,; \quad \gamma = \frac{\gamma_a + \gamma_b}{2}\ .$$

## Appendix 2

The matrix for a cavity with a Gaussian aperture and a plane-parallel layer of a quadratically inhomogeneous active medium of length $h = H/L$ is given by

$$\begin{pmatrix} A_{pr} & B_{pr} \\ C_{pr} & D_{pr} \end{pmatrix} = \begin{pmatrix} \cosh\sqrt{n_{pj}}\dfrac{h}{2} & \dfrac{1}{\sqrt{n_{pj}}}\sinh\sqrt{n_{pj}}\dfrac{h}{2} \\ \sqrt{n_{pj}}\sinh\sqrt{n_{pj}}\dfrac{h}{2} & \cosh\sqrt{n_{pj}}\dfrac{h}{2} \end{pmatrix}$$

$$\times\begin{pmatrix} 1 & z_{\mathrm{d}} - \frac{h}{2} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2\mathrm{i}N_p & 1 \end{pmatrix}\begin{pmatrix} 1 & -z_d \\ 0 & 1 \end{pmatrix}\begin{pmatrix} A_{0p} & B_{0p} \\ C_{0p} & D_{0p} \end{pmatrix} \text{(A2.1)}$$

$$\times\begin{pmatrix} 1 & -\frac{h}{2} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \cosh\sqrt{n_{pj}}\dfrac{h}{2} & \dfrac{1}{\sqrt{n_{pj}}}\sinh\sqrt{n_{pj}}\dfrac{h}{2} \\ \sqrt{n_{pj}}\sinh\sqrt{n_{pj}}\dfrac{h}{2} & \cosh\sqrt{n_{pj}}\dfrac{h}{2} \end{pmatrix}\ .$$

The first matrix corresponds to a layer of the active medium [19] of length $h/2$, the second to a free gap, the third to a Gaussian aperture [$N_p = L/(ka_p^2)$] ($a_p$ are the half-widths of the aperture in the directions of the corresponding axes), etc. The coefficients $n_{pj}$ are defined by formula (7).

If the cavity-perturbing parameters are small ($n_{pj} < N_p < 1$), we can determine the parameter g of a cavity by calculating the elements of the matrix (A2.1) to within terms linear in $n_{pj}h$ and quadratic in $N_p$, taking into account terms of the order $n_{pj}hN_p$:

$$G_{pj} = \frac{A_{pj} + D_{pj}}{2} = g_p + \delta g_p\,, \quad g_p = \frac{A_{0p} + B_{0p}}{2}\,, \quad \text{(A2.2)}$$
$$\delta g_p = \mathrm{i}N_p f_5 + n_p h f_3 + \mathrm{i}N_p n_p h f_4\ ,$$

$$f_1 = A_{0p} - C_{0p}z_{\mathrm{d}}\ ,$$
$$f_2 = B_{0p} - D_{0p}z_{\mathrm{d}}\ ,$$
$$f_3 = 0.5(B_{0p} - \tfrac{1}{12}C_{0p}h^2)\ , \quad \text{(A2.3)}$$
$$f_4 = f_2 z_d - \tfrac{1}{12}f_1 h^2\ ,$$
$$f_5 = B_{0p}(1 + 2z_t S_{0p} - z_{\mathrm{d}}^2 C_{0p}/B_{0p})\ ,$$
$$S_{0p} = \frac{1}{2B_{0p}}(D_0 - A_0)\ .$$

Assuming that $\delta g_p g_p/(1 - g_p^2)$ is small, we find the parameter $\delta\Gamma_p = \delta\Gamma_p' + \mathrm{i}\Gamma_p''$ which occurs in expression (22):

$$\delta\Gamma_{pj}' = (2W_{0p})^{-1}\left[n_{pj}'h\mu_{1p} - n_p''hN_p\mu_{2p} - \frac{|g_p|}{(1 - g_p^2)^{1/2}}N_p^2 W_{0p}^{-1}\tilde{f}_{5p}\right]\,, \quad \text{(A2.4)}$$

$$\Gamma_{pj}'' = (2W_{0p})^{-1}(N_p\mu_{3p} + n_{pj}''h\mu_{1p} + n_{pj}'hN_p\mu_{2p})\ , \quad \text{(A2.5)}$$

where

$$\mu_{1p} = \tilde{f}_{3p}(1 + 2W_{0p}^{-1}\tilde{f}_{5p})\,; \quad \mu_{2p} = \tilde{f}_{4p} + 2W_{0p}^{-1}|g_p|\tilde{f}_{3p}\tilde{f}_{5p}\ ;$$
$$\mu_{3p} = \tilde{f}_{5p}(1 + N_p W_{0p}^{-1}\tilde{f}_{5p})\,; \quad W_{0p} = (g_p^2 - 1)^{1/2}/B_{0p}\ ;$$
$$\tilde{f}_i = f_i/B_{0p} \quad (i = 1, \ldots, 5)\ . \quad \text{(A2.6)}$$

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