Quantum Electronics 30 (2) 147 – 151 (2000) 

PACS numbers: 85.70.Sq; 78.20.Ls
DOI: 10.1070/QE2000v030n02ABEH001676

# Characteristic features of the operation of different designs of the Faraday isolator for a high average laser-radiation power

E A Khazanov

**Abstract. The influence of the thermal self-interaction of the laser radiation in the first pass through a Faraday isolator on the power losses in the main spatiopolarisation mode is analysed. Analytical expressions for these losses were obtained for different designs of the isolators. It is shown that the design with two 22.5° Faraday rotators and with a 67.5° reciprocal rotator between the former devices is best from the standpoint of all the isolator parameters. The results obtained for a Gaussian beam were extended to the case of super-Gaussian and Π-shaped (step-like) beams. It was established that the Π-shaped beam exhibits a minimal self-interaction.**

## 1. Introduction

The average radiation powers of laser systems, which have significantly increased in recent years, make ever more topical the study and search for ways of suppressing the thermal effects induced by the absorption of laser radiation in the bulk of optical components. The Faraday isolator is one of the components most strongly subjected to thermal self-interaction, since the absorption in magnetically active media is relatively high — of the order of $10^{-3}$ cm$^{-1}$ [1]. A number of studies have been devoted to the self-induced thermal effects in magnetically active media [2 – 9].

The absorption of radiation in optical Faraday components induces a temperature distribution inhomogeneous over the cross-section. This is the reason for the presence of an isotropic thermal lens and of the inhomogeneous distribution of the angle of rotation of the polarisation plane induced by the temperature dependence of the Verdet constant and the appearance of a linear birefringence, caused by the photoelastic effect, together with the circular birefringence (the Faraday effect). The latter two mechanisms alter the polarisation state of the radiation transmitted through the magnetically active medium, which impairs the quality of the decoupling.

It has been shown [4] that the photoelastic effect makes the greatest contribution to the impairment of the quality of the decoupling. New designs for the Faraday isolator, which make it possible to avoid this impairment, have been proposed and theoretically justified [5]. Experiments carried out in Ref. [6] confirmed the high efficiency of the new designs compared with the traditional design.

Decoupling is as a rule the main but by no means the only parameter of the Faraday isolator. Other important characteristics of this device are aberrations of the beam and power losses in the first pass. We may note that in certain applications these characteristics may be no less important than decoupling. As an example, one may quote a laser interferometer for the detection of gravitational waves [10], in which the distortions of the laser beam play a significant role. The influence of the absorption of laser radiation in the isolator on the distortion of the first path has not been investigated hitherto for either the traditional or the new designs. We may note that all the previous studies were limited to the Gaussian beam profile. On the other hand, the magnitude of the self-induced effects may depend significantly on the beam profile.

Analytical expressions were obtained in the present study for the power losses and the degree of aberration of the beam in the first pass for both the traditional design and for two new designs [5, 6] of the Faraday isolator. A combined comparison of all three designs from the standpoint of operation at a high average power of the laser radiation was carried out on the basis of the results obtained as well as the results from Refs [5, 9]. The results obtained for a Gaussian beam in the present and previous studies [4 – 6, 9] were extended to super-Gaussian and Π-shaped beams.

## 2. Distortions of a linearly polarised Gaussian beam

We shall examine three designs of the Faraday isolator: the traditional design (Fig. 1a) and two new designs [5] (Figs 1b and 1c). Instead of the 45° Faraday rotator (3), in the new designs use is made of two 22.5° rotators, between which there is a half-wave plate (6) with the optical axis positioned at an angle of 22.5° relative to the polariser plane (Fig. 1b), or a reciprocal rotator (8) of polarisation by an angle of 67.5° (Fig. 1c). In the absence of thermal effects, after the first pass through the isolator (from left to right) the beam retains the horizontal (in the plane of the figure) polarisation in all the designs and passes through the polariser (4), whereas in the reverse pass the polarisation changes to vertical (at right angles to the plane of the figure) and the beam is reflected by the polariser (1). The linear birefringence in Faraday rotators induced by the photoelastic effects in the reverse pass leads to the appearance of the nonisolation $\gamma$ defined by the relationship

**E A Khazanov** Institute of Applied Physics, Russian Academy of Sciences, ul. Ul'yanova 46, 603600 Nizhny Novgorod, Russia

Received 31 August 1999
Kvantovaya Elektronika **30** (2) 147 – 151 (2000)
Translated by A K Grzybowski

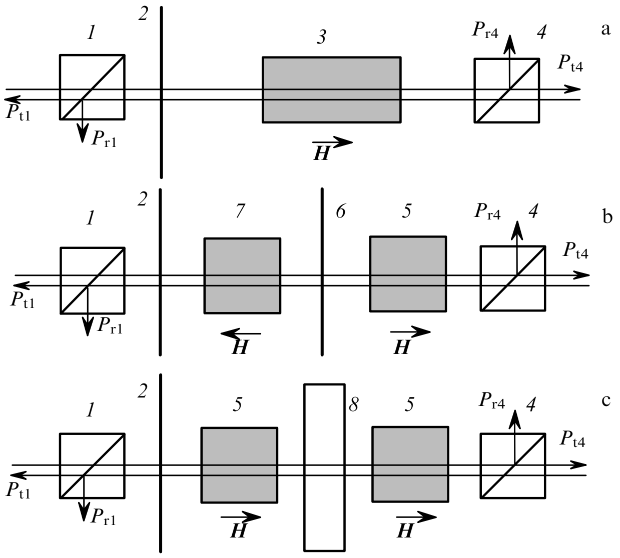


**Figure 1.** Traditional design for a Faraday isolator (a) and its new designs [5] with a $\lambda/2$ plate (b) and a reciprocal rotator (c): (1, 4) polarisers; (2, 6) $\lambda/2$ plates; (3) 45° Faraday rotator; (5, 7) 22.5° Faraday rotators rotating the polarisation plane anticlockwise and clockwise, respectively; (8) 67.5° reciprocal polarisation rotator.

$$\gamma = \frac{P_{t1}}{P_{t1} + P_{r1}} ,$$

where $P_{t1}$ and $P_{r1}$ are the powers of the radiation transmitted through the polariser (1) and of the radiation reflected from it. When the distortions were small, the following expressions for $\gamma$ were obtained [5] in the case of a Gaussian beam:

$$\gamma_0 = p^2 \frac{A_1}{\pi^2} , \quad \gamma_L = p^4 \frac{8A_2}{\pi^4} \xi_a^2 (2a^2 + b^2) ,$$

$$\gamma_R = p^4 \frac{6a^2 A_2}{\pi^4} \left( 1 + \frac{2}{3} \xi_a^2 + \xi_a^4 \right) , \tag{1}$$

where

$$a = \frac{\pi}{8} - \frac{1}{\sqrt{8}} ; \quad b = \frac{1}{2} - \frac{1}{\sqrt{8}} ; \quad \xi_a = \frac{2p_{44}}{p_{11} - p_{12}} ;$$

$$p = \frac{L}{\lambda} \frac{\alpha Q}{\varkappa} P_0 ; \quad Q = \frac{1}{L} \frac{dL}{dT} \frac{n_0^3}{4} \frac{1+\nu}{1-\nu} (p_{11} - p_{12}) ; \tag{2}$$

$$A_1 = \int_0^\infty \left[ \frac{1}{y} - \frac{\exp(-y)}{y} - 1 \right]^2 \frac{dy}{\exp y} \approx 0.137 ;$$

$$A_2 = \int_0^\infty \left[ \frac{1}{y} - \frac{\exp(-y)}{y} - 1 \right]^4 \frac{dy}{\exp y} \approx 0.042 ;$$

L, T, $n_0$, $\alpha$, $\varkappa$, $\nu$, and $p_{ij}$ are the total length, temperature, refractive index, absorption coefficient, thermal conductivity, the Poisson ratio, and the photoelasticity coefficients of the magnetically active medium, respectively; $P_0$ is the total power of the heating laser radiation; $\lambda$ is the wavelength. The subscripts '0', 'L', and 'R' refer here and henceforth to the traditional design (Fig. 1a), the design with a $\lambda/2$ plate (Fig. 1b), and the design with the polarisation rotator (Fig. 1c), respectively.

We shall now consider the propagation of a beam in the first pass and shall find expressions for the power losses and beam distortions. We suppose that, after passing through the polariser (1), the beam has a horizontal polarisation, a Gaussian intensity distribution with a radius $r_0$, and a plane wavefront, i.e. the complex field amplitude can be expressed in the form

$$\boldsymbol{E}_1 = E_0 \boldsymbol{x}_0 \exp\left( -\frac{r^2}{2r_0^2} \right) , \tag{3}$$

where $\boldsymbol{x}_0$ is a unit vector directed along the x axis and r is the polar radius.

Owing to polarisation distortions during the first pass, part of the radiation is reflected by the polariser (1). The polarisation power losses $\gamma_p$ are defined by the expression

$$\gamma_p = \frac{P_{r4}}{P_{t4} + P_{r4}} ,$$

where $P_{t4}$ and $P_{r4}$ are the powers of the radiation transmitted through the polariser (4) and reflected from it. In order to calculate $\gamma_p$, it is convenient to use the relationship

$$\gamma_{p0,pL,pR} = \frac{\int_0^{2\pi} d\varphi \int_0^\infty |\boldsymbol{E}_{40,4L,4R} \boldsymbol{y}_0|^2 r \, dr}{\int_0^{2\pi} d\varphi \int_0^\infty |\boldsymbol{E}_{40,4L,4R}|^2 r \, dr} , \tag{4}$$

where $\boldsymbol{y}_0$ is a unit vector directed along the y axis; $\boldsymbol{E}_4$ is the complex field amplitude before the polariser (4). Here and henceforth, we assume that the optical diameter of the Faraday isolator is such that the aperture losses may be neglected.

Apart from polarisation distortions, the beam undergoes also amplitude and phase distortions arising during propagation through an inhomogeneously heated magnetically active medium. For a quantitative description of these distortions, it is convenient to introduce the quantities

$$\gamma_{t0,tL,tR} = 1 - \frac{\left| \int_0^{2\pi} d\varphi \int_0^\infty \boldsymbol{E}_{404L,4R} \boldsymbol{E}_{ref}^* r \, dr \right|^2}{\int_0^{2\pi} d\varphi \int_0^\infty |\boldsymbol{E}_{40,4L,4R}|^2 r \, dr \int_0^{2\pi} d\varphi \int_0^\infty |\boldsymbol{E}_{ref}|^2 r \, dr} , \tag{5}$$

characterising the difference from unity of the projection of the laser field $\boldsymbol{E}_4$ onto an ideal field, i.e. onto a field in the absence of thermal effects $\boldsymbol{E}_{ref} = E_0 \boldsymbol{y}_0 \exp(-r^2/2r_0^2)$. The quantity $\gamma_t$ determines all the distortions and represents the losses in the initial spatiopolarisation mode, whereas $\gamma_p$ describes only the polarisation losses and represents the power losses of the radiation with linear polarisation. The difference $\gamma_t - \gamma_p$ characterises the amplitude–phase distortions. We may note that the isotropic thermal lens and the depolarisation of the radiation contribute to $\gamma_t - \gamma_p$. The contribution of the latter factor is due to the fact that the depolarised radiation is inhomogeneous over the cross-section and that amplitude and phase distortions appear in the beam after reflection of the depolarised radiation by the polariser (4).

The formal treatment involving Jones polarisation matrices can be used conveniently to determine the field $\boldsymbol{E}_4$. The matrices of the rotator of the polarisation plane by an angle $\beta_R[R(\beta_R)]$, of the $\lambda/2$ plate, the optical axis of which is directed at an angle $\beta_L[L(\beta_L)]$, and of the Faraday rotator

$[F(\Phi,\ \delta_1,\ \Psi)]$ can be found, for example, in Refs [4, 5, 11]. Here, $\Phi$ is the angle of rotation of the polarisation plane, $\delta_1$ and $\Psi$ are the phase difference and the angle of inclination of the eigenpolarisation of the linear birefringence induced in the rotator.

In order to take into account the isotropic thermal lens in Faraday rotators, it is necessary to multiply the field by the scalar factor

$$\exp(\mathrm{i}kLn)\ ,$$

where $k = 2\pi/\lambda$ is the wave vector; $n = n(r)$ is the refractive index. Here and henceforth we assume that the transverse distribution of temperature and hence of the refractive index is homogeneous along the direction of propagation of the beam. Furthermore, we shall assume that the diffraction length of the beam is much greater than the length of the Faraday isolator, even when account is taken of the induced distortions.

The distribution of temperature in an optical component and hence the laser-beam phase, taking into account the aberrations, is frequently close to parabolic. For this reason, most of the phase distortions can be compensated with the aid of an ordinary lens or a telescope (not shown in the figure), which introduces an additional curvature (radius of curvature $R_0$) into the wavefront. This is equivalent to the multiplication of the field by the phase factor

$$\exp\left[\pm \mathrm{i}kL\left(\frac{r}{R_0}\right)^2\right]\ .$$

The 'plus' sign is necessary to compensate for the negative thermal lens and the 'minus' sign is necessary to compensate for the positive thermal lens. By varying $R_0$, it is possible to minimise the phase distortions for $R_0 = R_{0\,\mathrm{opt}}$. Thus we may find the field $\boldsymbol{E}_1$ for all three designs:

$$\begin{aligned}
\boldsymbol{E}_{40} &= \exp\left[\pm \mathrm{i}kL\left(\frac{r}{R_0}\right)^2\right]\exp(\mathrm{i}kLn) \\
&\quad\times F\left(\Phi = \frac{\pi}{4},\ \delta_1\right)L\left(-\frac{\pi}{8}\right)\boldsymbol{E}_1\ , \\
\boldsymbol{E}_{4\mathrm{L}} &= \exp\left[\pm \mathrm{i}kL\left(\frac{r}{R_0}\right)^2\right]\exp(\mathrm{i}kLn) \\
&\quad\times F\left(\Phi = \frac{\pi}{8},\ \frac{\delta_1}{2}\right)L\left(\frac{\pi}{8}\right)F\left(\Phi = -\frac{\pi}{8},\ \frac{\delta_1}{2}\right)L\left(\frac{\pi}{4}\right)\boldsymbol{E}_1\ , \\
\boldsymbol{E}_{4\mathrm{R}} &= \exp\left[\pm \mathrm{i}kL\left(\frac{r}{R_0}\right)^2\right]\exp(\mathrm{i}kLn) \\
&\quad\times F\left(\Phi = \frac{\pi}{8},\ \frac{\delta_1}{2}\right)R\left(\frac{3\pi}{8}\right)F\left(\Phi = \frac{\pi}{8},\ \frac{\delta_1}{2}\right)L\left(\frac{\pi}{16}\right)\boldsymbol{E}_1\ ;
\end{aligned} \tag{6}$$

We assume that the polarisation distortions in the two Faraday rotators in the new designs are identical. The phase difference for the purely linear birefringence in each rotator is in this case $\delta_1/2$, i.e. $\delta_1$ is the total phase shift for all the designs in the presence of purely linear birefringence throughout the entire length of the magnetically active medium.

In order to calculate $\gamma_\mathrm{t}$ and $\gamma_\mathrm{p}$, it is necessary to determine only $\delta_1$, $\Psi$, and $n(r)$. When a cylindrical crystal with the [001] orientation and the angle $\theta$ between the crystallographic axis and the x axis is used, these quantities can be determined in terms of the temperature distribution $T(r)$ [12]:

$$\begin{aligned}
&\delta_1(r,\ \varphi) = 4\pi\frac{L}{\lambda}Q\left[\frac{1+\xi_\mathrm{a}^2\tan^2(2\varphi-2\theta)}{1+\tan^2(2\varphi-2\theta)}\right]^{1/2}\frac{1}{r^2}\int_0^r r^2\frac{\mathrm{d}T}{\mathrm{d}r}\mathrm{d}r\ , \\
&\tan(2\Psi-2\theta) = \xi_\mathrm{a}\tan(2\varphi-2\theta)\ , \\
&n(r) = n(0) + [T(r)-T(0)]P, \\
&P = \frac{\mathrm{d}n}{\mathrm{d}T} - \frac{1}{L}\frac{\mathrm{d}L}{\mathrm{d}T}\frac{n_0^3}{4}\frac{1+v}{1-v}(p_{11}+p_{12}).
\end{aligned} \tag{7}$$

The temperature distribution $T(r)$ can be easily found from the thermal conductivity equation (see, for example, Ref. [5]).

We shall consider the case of a weak linear birefringence, i.e. when

$$\delta_1 \ll 1\ . \tag{8}$$

Expressions (7) are valid also for glass magneto-optical components, for which $\xi_\mathrm{a} = 1$ and there is no dependence on $\theta$. When a crystal is used, $\delta_1$ and $\psi$ and hence also the distortions depend on the angle $\theta$. We shall consider the optimum angle $\theta_{0,\mathrm{L,R}}^{\mathrm{opt}}$ for which the nonisolation $\gamma$ is minimal [5]:

$$\begin{aligned}
&\theta_0^{\mathrm{opt}} = -\frac{\pi}{8}\ , \quad \theta_\mathrm{L}^{\mathrm{opt}} = \frac{\pi}{16} + \frac{1}{4}\arcsin\left(\frac{\pi\sqrt{2}/4-1}{\sqrt{2}-1}\frac{\xi_\mathrm{a}^2+1}{\xi_\mathrm{a}^2-1}\right)\ , \\
&\text{arbitrary } \theta_\mathrm{R}^{\mathrm{opt}}\ .
\end{aligned} \tag{9}$$

If the expression in brackets is greater than unity, i.e. $\xi_\mathrm{a} < 1.315$, then $\theta_\mathrm{L}^{\mathrm{opt}} = 3\pi/16$. An important advantage of the design in Fig. 1c is that $\gamma_\mathrm{R}$ does not depend on the angle $\theta$ at all; this makes it possible to choose the optimum $\theta$ from the standpoint of the minimisation of $\gamma_\mathrm{pR}$ and $\gamma_\mathrm{tR}$. Using expressions (3)–(11), it can be shown that $\gamma_\mathrm{tR}$ does not depend on $\theta$ at all, whereas $\gamma_\mathrm{pR}$ has a minimum at

$$\theta_\mathrm{R}^{\mathrm{opt}} = \frac{5\pi}{16}\ , \tag{10}$$

On substituting formula (7) in formula (6) and the result in formulas (4) and (5), we obtain expressions for the losses in all three designs taking into account formulas (3) and (8)–(10):

$$\gamma_{\mathrm{t0,tL,tR}} = \gamma_{\mathrm{pO,pL,pR}} + \gamma_{\mathrm{aO,aL,aR}} + \gamma_\mathrm{i}\ , \tag{11}$$

$$\begin{aligned}
&\gamma_\mathrm{p0} = p^2\frac{A_1}{\pi^2}\xi_\mathrm{a}^2\ , \\
&\gamma_\mathrm{pL} = p^2\frac{A_1}{\pi^2}\begin{cases}(2-\pi/2)(\xi_\mathrm{a}^2+1)\ , & \xi_\mathrm{a} > 1.315\ , \\ 2(2-\sqrt{2})\ , & \xi_\mathrm{a} < 1.315\ ,\end{cases} \\
&\gamma_\mathrm{pR} = p^2\frac{A_1}{\pi^2}(2-\sqrt{2})\ ,
\end{aligned} \tag{12}$$

$$\gamma_\mathrm{a0} = p^2\frac{A_1}{\pi^2}\ , \quad \gamma_\mathrm{aL} = 0\ , \quad \gamma_\mathrm{aR} = p^2\frac{A_1}{\pi^2}(2-\sqrt{2})\xi_\mathrm{a}^2\ , \tag{13}$$

$$\gamma_\mathrm{i}(R_0 = \infty) = \frac{A_3}{4}p_\mathrm{i}^2\ , \quad \gamma_\mathrm{i}(R_0 = R_\mathrm{opt}) = \frac{A_4}{4}p_\mathrm{i}^2\ , \tag{14}$$

where

$$A_3 = \int_0^\infty f^2(y)\exp(-y)\mathrm{d}y - \left[\int_0^\infty f(y)\exp(-y)\mathrm{d}y\right]^2 \approx 0.268\,;$$

$$A_4 = \int_0^\infty [f(y) - 0.5y]^2 \exp(-y)\mathrm{d}y - \left\{\int_0^\infty [f(y) - 0.5y]\exp(-y)\mathrm{d}y\right\}^2 \approx 0.0177\,; \qquad (15)$$

$$f(y) = \int_0^\infty \frac{1-\exp(-y)}{y}\mathrm{d}y\,; \qquad p_\mathrm{i} = \frac{L}{\lambda}\frac{\alpha P}{\varkappa}P_0\,.$$

We shall now discuss the results obtained. In the first place, we note that all the losses for all three designs are independent of the beam radius $r_0$ and are proportional to the square of the heating radiation power $P_0$. The nonisolation of the Faraday isolator [formula (1)] is likewise independent of $r_0$ and is proportional to $P_0^2$ for the traditional design and to $P_0^4$ for the new designs.

The distortion $\gamma_\mathrm{t}$ [formula (11)] consists of the sum of three terms. The first two terms are related to the depolarisation of the radiation induced by the photoelastic effect, whereas the last term reflects the purely phase distortions induced by the isotropic thermal lens. We may note that both the temperature dependence of the refractive index and the 'isotropic' part of the photoelastic effect (see the two corresponding terms in the expression for P) contribute to this lens. The isotropic distortions $\gamma_\mathrm{i}$ are determined solely by the parameter $p_\mathrm{i}$. As was to be expected, $\gamma_\mathrm{i}$ is the same for all three designs and depends on whether the parabolic aberrations are compensated ($R_0 = R_0^\mathrm{opt}$) or not ($R_0 \to \infty$). It is seen from formulas (14) and (15) that the distortions induced by the thermal lens decrease by the factor $A_3/A_4 = 15$ when the compensation is optimal.

The parameter $p$ characterises the distortions introduced by the anistropy induced by the photoelastic effect. These distortions [the first two terms in formula (11)] can be divided into polarisation ($\gamma_\mathrm{p}$) and amplitude – phase ($\gamma_\mathrm{a}$) distortions. It follows from formulas (12) and (13) that, from the standpoint of the minimisation of $\gamma_\mathrm{p} + \gamma_\mathrm{a}$, both new designs are a little better than the traditional design. It was indicated above that $\gamma_\mathrm{p}$ determines the power losses in the first pass through the Faraday isolator and that for a number of applications it is a more important characteristic than the total spatiopolarisation distortions $\gamma_\mathrm{t}$.

We shall now compare the different designs from the standpoint of the minimum in $\gamma_\mathrm{p}$. The quantity $\gamma_\mathrm{p}$ depends strongly on which optical component is used (glass or crystalline). For glass, $\xi_\mathrm{a} = 1$ and, as can be seen from formula (12), the design in Fig. 1c is somewhat better than the other two. For the terbium-gallium garnet (TGG) (the crystal most widely used in Faraday isolators) $\xi_\mathrm{a} = 3.6$ [4]. When account is taken of this fact, the designs in Figs 1b and 1c have power losses smaller by factors of 2 and 20 than the traditional design (Fig. 1a) when a TGG crystal is used. Having adopted $Q = 4\times10^{-6}$ K$^{-1}$, $P = 2\times10^{-5}$ K$^{-1}$ for the purpose of estimates, we find from formulas (12) – (14) for the TGG that the losses induced by depolarisation ($\gamma_\mathrm{a} + \gamma_\mathrm{p}$) are comparable with the losses associated with the isotropic lens [$\gamma_\mathrm{i}(R_0^\mathrm{opt})$].

The formulas for $\gamma_\mathrm{p}$, $\gamma_\mathrm{a}$, and $\gamma_\mathrm{i}$ obtained in this section are valid when condition (8) holds. In the general case, when this condition does not hold, the results of the numerical integration show that expressions (12) – (14) are valid when $\gamma_\mathrm{p}$, $\gamma_\mathrm{a}$, $\gamma_\mathrm{i} < 0.1$.

## 3. The influence of beam shape

In the previous section we considered the self-induced thermal distortions of a Gaussian beam during the first pass through a Faraday isolator. The nonisolation was investigated for the same profile as in Refs [3 – 9]. Since a laser beam records the distortions (being a source of heat) and counts them simultaneously, the self-interaction may depend strongly on the transverse intensity distribution, in particular on the rate of its decrease from the centre to the periphery. In this section, we shall extend the results obtained to the case of a super-Gaussian beam:

$$\boldsymbol{E}_1 = E_0\boldsymbol{x}_0\exp\left(-\frac{r^{2m}}{2r_0^{2m}}\right)\,. \qquad (16)$$

The parameter m characterises the rate of decrease in the intensity. For $m = 1$ , the intensity falls fairly slowly (a Gaussian beam). With increase in m, the rate of decrease in intensity rises and, when $m \to \infty$, the beam is converted into a Π-shaped (step-like) one.

Repeating for the laser beam defined by formula (16) the procedure described in the previous section, it can be shown that the relationships for the power losses in the first pass [formulas (11) – (14)] remain valid for any value of m provided that the expressions for $A_1$, $A_3$, and $A_4$ are replaced by

$$A_1(m) = \frac{1}{\sigma_0^3}\int_0^\infty \left[\int_0^y \mathrm{d}z\int_0^z \exp(-y^m)\,\mathrm{d}y\right]^2 \frac{\mathrm{d}y}{y^2\exp y^m}\,,$$

$$A_3(m) = \frac{1}{\sigma_0^3}\int_0^\infty g^2(y)\exp(-y^m)\,\mathrm{d}y - \frac{1}{\sigma_0^4}\left[\int_0^\infty g(y)\exp(-y^m)\,\mathrm{d}y\right]^2\,, \qquad (17)$$

$$A_4(m) = A_3(m) - \frac{\sigma_1}{\sigma_0^4}\frac{1}{\sigma_2\sigma_0 - \sigma_1^2} \times\left[\int_0^\infty g(y)\left(1 - \frac{y\sigma_0}{\sigma_1}\right)\exp(-y^m)\,\mathrm{d}y\right]^2\,,$$

where

$$\sigma_k = \int_0^\infty y^k\exp(-y^m)\,\mathrm{d}y\,; \qquad k = 0,\,1,\,2\,;$$

$$g(y) = \int_0^y\left[\int_0^z \exp(-y^m)\,\mathrm{d}y\right]\frac{\mathrm{d}z}{z}\,.$$

By carrying out similar [5] analytical calculations for the nonisolation $\gamma$, it can be shown that relationships (1) also remain in force for any value of m provided hat the following expression is used for $A_2$:

$$A_2(m) = \frac{1}{\sigma_0^5}\int_0^\infty \left[\int_0^y \mathrm{d}z\int_0^z \exp(-y^m)\,\mathrm{d}y\right]^4 \frac{\mathrm{d}y}{y^4\exp y^m}\,, \qquad (18)$$

and expression (17) is used for $A_1$. The values of $A_{1-4}$ for certain values of m are presented in Table 1. Thus the expressions for all types of losses in all three designs remain qualitatively unchanged whatever the value of m. Only the numerical multiplier $A_{1-4}$ depends on the beam profile. On the other hand, these changes may be extremely large (see Table 1). We shall now discuss the most important of these.

**Table 1.**

| $m$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|---|---|---|---|---|
| 1 | 0.137 | 0.042 | 0.268 | 0.0177 |
| 2 | 0.111 | 0.0265 | 0.158 | 0.00205 |
| 4 | 0.095 | 0.0181 | 0.111 | $1.7 \times 10^{-4}$ |
| 8 | 0.087 | 0.0145 | 0.092 | $1.0 \times 10^{-5}$ |
| 16 | 0.085 | 0.0131 | 0.086 | $4.4 \times 10^{-7}$ |
| $\infty$ | 1/12 | 1/80 | 1/12 | 0 |

In the first place we note that the parameters $A_{1-4}$ decrease with increase in m. This means that all the distortions diminish on passing from a Gaussian to a $\Pi$-shaped beam. In particular, the isotropic thermal lens induced by this beam can be fully compensated, i.e. $A_4^{(\infty)} = 0$. This has a simple physical explanation: for a homogeneous heat source in a cylindrical medium, a strictly parabolic temperature distribution is established.

For all three designs, the distortions $\gamma_a$ and $\gamma_p$ induced by depolarisation are proportional to $A_1$, whereas the isotropic distortions $\gamma_i$ are proportional to $A_{3,4}$. For this reason, the comparison of the three designs in the previous section for a Gaussian beam is valid for any beam defined by formula (16). The nonisolation $\gamma$ is proportional to $A_1$ for the traditional design and to $A_2$ for the new designs [see formula (1) and Figs 1b and 1c]. The ratio $A_2(m)/A_1(m)$ shows how the gain in terms of $\gamma$ in the new designs, compared with the traditional design depends on m. It is seen from Table 1 that, on passing from a Gaussian to a $\Pi$-shaped beam, this gain increases by a factor of 2. The ratio of the nonisolation induced by the temperature dependence of the Verdet constant to the nonisolation induced by the photoelastic effect is proportional to the ratio $A_3(m)/A_1(m)$ [4]. Since it decreases with increase in m, the conclusion, reached in Ref. [4] for a Gaussian beam, that the temperature dependence of the Verdet constant makes a negligible contribution, remains in force for any beam defined by formula (16).

Thus one may reiterate that a $\Pi$-shaped beam is optimal from the standpoint of all thermal effects in all the Faraday-isolator designs considered, whereas a Gaussian beam exhibits the strongest self-interaction.

## 4. Conclusions

In conclusion, we shall list the most important results. The thermal self-interaction of laser radiation in the first pass through a Faraday isolator leads to power losses in the initial spatiopolarisation laser-radiation mode. These losses consist of the losses induced by the isotropic thermal lens ($\gamma_i$) and depolarisation. The latter represents in its turn the sum of the purely polarisation losses $\gamma_p$ (the power losses in linear polarisation) and the losses $\gamma_a$ associated with the amplitude – phase distortions due to depolarisation. For all three designs of the Faraday isolator (see Fig. 1), analytical expressions were obtained for all types of power losses in terms of the small-distortions approximation. The design with a reciprocal polarisation rotator (Fig. 1c) is best from the standpoint of both purely polarisation losses $\gamma_p$ and the losses associated with depolarisation ($\gamma_p + \gamma_a$). We may note that it ensures minimal nonisolations defined by formula (1). From the standpoint of the isotropic thermal lens, all the designs are identical and for a Gaussian beam the compensating lens makes it possible to diminish by a factor of 15 the losses $\gamma_1$ induced by the thermal lens. All the self-induced distortions in all three designs decrease on passing from a Gaussian to a super-Gaussian beam and further to a $\Pi$-shaped beam.

**Acknowledgements.** This study was supported by a grant from the National Foundation of the USA (NSF) PHY-9900786 and a grant from the Russian Foundation for Basic Research No. 99-02-17257.

## References

1. Zarubina T V, Petrovskii G T Opt. Zh. **59** (11) 48 (1992)
2. Zarubina T V, Mal'shakov A N, Pasmanik G A, Potemkin A K Opt. Zh. **64** 11, 67 (1997)
3. Khazanov E A, Kulagin O V, Yoshida S, Reitze D Abstracts of Papers presented at the International Conference on Lasers and Electrooptics, San Francisco, 1998, paper CWF34
4. Khazanov E A, Kulagin O V, Yoshida S, Tanner D B, Reitze D H IEEE J. Quantum Electron. **35** 1116 (1999)
5. Khazanov E A Kvantovaya Elektron. (Moscow) **26** 59 (1999) [Quantum Electron. **29** 59 (1999)]
6. Khazanov E, Andreev N, Babin A, Kiselev A, Palashov O, Reitze D J. Opt. Soc. Am. B **17** 99 (2000)
7. Andreev N F, Babin A A, Kiselev A M, Palashov O V, Khazanov E A, Zarubina T V, Shchavelev O S Opt. Zh. (in press)
8. Eichler H J, Mehl O, Eichler J Proc. SPIE Int. Soc. Opt. Eng. **3613** 166 (1999)
9. Khazanov E A Proc. SPIE Int. Soc. Opt. Eng. **3609** 18 (1999)
10. Strain K A, Danzmann K, Muzino J, Nelson P G, Rudiger A, Schilling R, Winkler W Phys. Lett. A **194** 124 (1994)
11. Jaecklin A A, Lietz M Appl. Opt. **11** 617 (1972)
12. Mezenov A V, Soms L N, Stepanov A I Termooptika Tverdotel'nykh Lazerov (Thermo-Optics of Solid-State Lasers) (Leningrad: Mashinostroenie, 1986)