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Dynamics of generation of subpicosecond pulses in semiconductor injection lasers

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Abstract. A theory of the generation of subpicosecond pulses in semiconductor injection lasers taking into account multimode generation and a band structure of energy levels in doped semiconductors has been developed. Spatial-temporal dynamics of mode locking in a triple-section laser with a saturable absorber has been investigated. The analysis carried out has allowed us to determine the optimum conditions for the generation of ultimately short pulses. It has also permitted a study of the dependencies of parameters of the generated pulses on the pumping current, the reverse voltage, and a ratio of the lengths of the amplifying and absorbing sections.

1. Introduction

There has in recent years been considerable interest in investigations into the possibile generation of subpicosecond pulses in semiconductor lasers. For instance, subpicosecond pulses have been generated in a triple-section AlGaAs/GaAs heterolaser with a Fabry – Perot cavity operating in the superradiant regime [1, 2]. The laser consisted of two amplifying sections of length 30 μm and a saturable absorber 10 μm long located in the middle of the cavity. The total cavity length was 100 μm. The amplifying section of the laser was pumped by current pulses of amplitude 200 – 450 mA and duration a few nanoseconds, with a repetition rate of 1 – 10 MHz. A reverse bias down to −7 V was applied to the absorber section.

Theoretical investigations of the dynamics of the generation of semiconductor lasers carried out in Refs [1 – 3] have been based on the shortened Maxwell – Bloch equations, i.e. on a model of a single-mode emission field interacting with a system of two-level atoms. The model allows for a qualitative study of parameters of the generated pulses on parameters of the medium and the pumping current. However, a detailed description of the dynamics of the process which permits us to carry out a quantitative comparison of experimental data with theoretical calculations requires the use of more complicated models [4, 5].

In the present paper, a multimode theory of the dynamics of generation of semiconductor injection lasers taking into account the band structure of energy levels of the medium has been developed. The derived theory has allowed for a description of the spatial-temporal dynamics of self-mode-locking and the dynamics of evolution of the distribution of electrons between energy levels in the bands in lasers with a saturable absorber. A dependence of the temporal profile and spectrum of the generated pulses on the pumping current, reverse bias, and a ratio of the lengths of the amplifying and absorbing sections and their spatial location has been studied on the basis of the this theory.

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2. Basic equations

The equations for the generation of a semiconductor injection laser with the interband recombination in the general case are [4 – 6]:

$$\Delta A - \frac{\varepsilon}{c^2}\frac{\partial^2 A}{\partial t^2} = -\frac{4\pi}{c}\int j(E_2, E_1) g_2(E_2) g_1(E_1) \mathrm{d}E_1\,\mathrm{d}E_2 \ ,$$

$$\frac{\partial j}{\partial t} + \left(\Gamma - \mathrm{i}\frac{E_2 - E_1}{\hbar}\right) j = \frac{\mathrm{i}}{\hbar c}|m(E_2, E_1)|^2 A[\rho_2(E_2) - \rho_1(E_1)] \ ,$$

$$\frac{\partial \rho_\alpha}{\partial t} + \frac{\rho_\alpha - \rho_\alpha^{(e)}}{\tau} + \frac{\rho_\alpha - \rho_\alpha^{(0)}}{T_1} = (-1)^\alpha \frac{\mathrm{i}}{\hbar c}\left(jA - Aj^*\right) + d_\alpha \Delta\rho_\alpha + P_\alpha(E_\alpha, t) \ , \quad (1)$$

where $\alpha = 1, 2$; A is the vector potential of the field; $j(E_2, E_1)$ is the current density of the polarisation of the transition between a level in the conduction band with energy E_2 and a level in the valence band with energy E_1; $m(E_2, E_1)$ is the matrix element of the current density of the transition; $\rho_2(E_2)$ is the electron density of a donor level in the conduction band with energy E_2; $\rho_1(E_1)$ is the hole density in the valence band; $\rho_\alpha^{(e)}(E_\alpha)$ are their quasi-equilibrium values; τ is the intraband relaxation time; $\rho_\alpha^{(0)}(E_\alpha)$ is the steady-state value of the population of the levels; T_1 is the interband relaxation time; $g_\alpha(E_\alpha)$ is the density of states with an energy E_α; Γ is the transverse relaxation rate of the transition $E_2 \leftrightarrow E_1$; P_α is the injection pumping rate of the levels; d_α is the diffusion coefficient.

Expanding $A(\boldsymbol{r}, t)$ on the cavity modes $u_n(\boldsymbol{r})$,

$$A(\boldsymbol{r}, t) = \sum_n A_n(z, t) u_n(\boldsymbol{r}, t) \exp(\mathrm{i}\omega_n t) \ ,$$

it is easy to obtain the following equations for the slowly varying amplitudes:

$$\gamma_n \frac{\partial A_n}{\partial z} + \frac{1}{v}\frac{\partial A_n}{\partial t} = -\frac{\mathrm{i}2\pi n_0(z)}{\omega_n c\sqrt{\varepsilon}} \times \int q_n(E_2, E_1) g_2(E_2) g_1(E_1)\,\mathrm{d}E_1\,\mathrm{d}E_2 \,,$$

$$\frac{\partial q_n}{\partial t} + [\Gamma + \mathrm{i}(\omega_n - \omega_{21})]q_n = \frac{\mathrm{i}|m|^2}{\hbar c}\sum_m R_m A_{n-m} \,, \qquad (2)$$

$$\frac{\partial R_m}{\partial t} + \frac{R_m - R_m^{(e)}}{\tau} = \frac{2\mathrm{i}}{\hbar c}\sum_n \left(q_n A^*_{n-m} - q^*_n A_{n+m}\right) + P_m \,,$$

where $\gamma_n = (\boldsymbol{k}_n \boldsymbol{e}_z)/k_n$; $\boldsymbol{e}_z$ is the unity vector along the z axis which is perpendicular to the output facet of the cavity; $v = c/\sqrt{\varepsilon}$; $\omega_{21} = (E_2 - E_1)/\hbar$. Taking into account that $T_1 \gg \tau$, we omitted the interband relaxation rate, which is proportional to $1/T_1$, in the equations for the populations. The current density of the polarisation and the population difference of the levels, $\rho(E_2, E_1, z) = \rho_2(E_2, z) - \rho_1(E_1, z)$ are normalised to the total volume density of electrons n_0 on the levels of the resonant amplifying region of the active medium. The quasi-equilibrium population difference $\rho^{(e)}(E_2, E_1, z) = f_2(E_2, z) - f_1(E_1, z)$ is defined by the distribution function of electrons between the energy levels

$$f_\alpha(E_\alpha, z) = \left[1 + \exp\frac{E_\alpha - F_\alpha(z)}{kT}\right]^{-1} ,$$

where $F_\alpha(z)$ represents the electron (for $\alpha = 2$) and hole (for $\alpha = 1$) Fermi quasi-level in the resonant amplifying sections ($0 \leqslant z < l/2$ and $l/2 + d \leqslant z \leqslant l + d$) and absorbing section ($l/2 \leqslant z < l/2 + d$) of the laser; d and $l/2$ are the lengths of the absorbing section and of one of the two amplifying sections.

3. Quasi-stationary generation

Let us define the main parameters of pulses generated in the quasi-stationary regime. In this regime, the system of equations (2) is transformed into

$$\frac{\mathrm{d}a_n}{\mathrm{d}z} = \frac{\mu_0}{2\sqrt{\varepsilon}\gamma_n}\sum_m \int \frac{\Gamma[\Gamma - \mathrm{i}(\omega_n - \omega_{21})]}{\Gamma^2 + (\omega_n - \omega_{21})^2} \times R_m\, g_2(E_2) g_1(E_1)\,\mathrm{d}E_1\,\mathrm{d}E_2 a_{n-m} \,,$$

$$R_m = R_m^{(e)} - \mu_0 \Gamma \tau c \sum_{n,m'} R_{m'}\left[\frac{\Gamma\left(a_{n-m'}a^*_{n-m} + a_{n+m'}a^*_{n+m}\right)}{\Gamma^2 + (\omega_n - \omega_{21})^2} - \frac{\mathrm{i}(\omega_n - \omega_{21})\left(a_{n-m'}a^*_{n-m} - a_{n+m'}a^*_{n+m}\right)}{\Gamma^2 + (\omega_n - \omega_{21})^2}\right], \qquad (3)$$

where the steady-state value of the resonant gain coefficient μ_0 is connected to the time T_1 by the well-known relation

$$\mu_0 = \frac{4\pi|m|^2 n_0}{\hbar\omega_n \Gamma c} = \frac{3\lambda^2}{4\pi}\frac{n_0}{\Gamma T_1} \,.$$

A dimensionless vector potential

$$A_n(z, t) = \left(\frac{2\pi\hbar c^2 n_0}{\omega_n}\right)^{1/2} a_n(z, t) \qquad (4)$$

has been introduced into system (3). With Eqn (4) normalised, $|a_n(z, t)|^2$ represents the density of the number of quanta in the mode n normalised on the electron density n_0.

The second item on the right-hand side of the second equation of the system of equations (3) describes the saturation of the population difference and induction of a population grating. Omitting this item (i.e., not considering the mode-locking dynamics of the laser for the time being), it is easy to obtain the following expression for the field amplitude at the output of the laser cavity:

$$a_n(z = l + d, t) = c_n \exp\left\{\frac{1}{2\sqrt{\varepsilon}\gamma_n n_0}[\mu_1(\omega_n, U_1) n_0 l - \mu_2(\omega_m, U_2) n_1 d] - \mathrm{i}\varkappa_n (l + d)\right\} , \qquad (5)$$

where n_1 is the quasi-equilibrium electron density of the acceptor levels within the absorbing section of the laser; U_1 is the voltage on the $p - n$ junction in the amplifying sections of the laser. Assuming that the homogeneous bandwidth of the laser transition Γ is much smaller than the inhomogeneous one, it is easy to obtain for the resonant gain coefficient

$$\mu_1(\omega_n, U_1) = \mu_0 \pi \int g_2(E_1 + \hbar\omega_n) g_1(E_1) \times\left[\tanh\left(\frac{F_p + U_1 - E_1 - \hbar\omega_n}{2kT}\right) - \tanh\left(\frac{F_p - E_1}{2kT}\right)\right]\mathrm{d}E_1 \,, \qquad (6)$$

where F_p is the Fermi quasi-level for holes. The absorption coefficient $\mu_2(\omega_n, U_2)$ expresses in a similar by structure manner.

The integrated gain coefficient

$$G(\omega_n, U_1, U_2) L = \mu_1(\omega_n, U_1) n_0 l - \mu_2(\omega_n, U_2) n_1 d \,, \qquad (7)$$

where $L = l + d$, is maximum at a frequency of ω_0 which is defined by

$$\frac{\partial G(\omega_0)}{\partial \omega} = 0 \,. \qquad (8)$$

Expanding Eqn (7) into a row with an accuracy to the second order on $\omega_n - \omega_0$, one gets

$$G(\omega_n) = G(\omega_0) - \frac{1}{2}\left|\frac{\partial^2 G}{\partial \omega^2}(\omega_0)\right|(\omega_n - \omega_0)^2 \,.$$

In the mode-locking regime, $c_n = c_0$ in Eqn (5) (dynamics of the evolution and build-up of mode locking will be investigated by numerical simulation). In this case, by using the expression for the field intensity at the output of the laser cavity with a coordinate $z = l + d$,

$$I(t) = \left|\sum_n a_n(z = l + d, t)\exp(\mathrm{i}\omega_n t)\right|^2 , \qquad (9)$$

it is easy to obtain the following expression for the duration of a single pulse in a train

$$\tau_\mathrm{p} = \left[|\mu''_1(\omega_0)| n_0 l - \mu''_2(\omega_0) n_1 d\right]^{1/2} , \qquad (10)$$

where $\mu'' = \partial^2\mu/\partial\omega^2$. To carry out numerical estimations, it is possible to approximate Eqn (6) with a Gaussian function for which $\mu''(\omega_0) = \mu(\omega_0)/\Delta\omega^2$. Therefore the gain bandwidth $\Delta\omega$ must be greater than 10^{-3} eV in order to obtain the generation of subpicosecond pulses.

Thus an analysis of the regime of quasi-stationary generation has shown that, for the optimum selection of the absorbing section, the effective gain bandwidth in a triple-section laser should be larger than that in the amplifying section, i.e., $\Delta\omega_1 = (|\mu_1''(\omega_0)|n_0 l)^{-1/2}$. As a result, the duration of generated pulses decreases. Eqn (10) shows that the pulse duration is substantially dependent on the injection current, the reverse bias, and the ratio of the lengths of the amplifying and absorbing sections. The formulas allow for an optimum selection of these parameters in order to achieve subpicosecond pulse generation.

4. Dynamics of the multimode generation

Dynamics of multimode generation has been studied by a numerical solution of the system of equations given above with the parameters which were similar to the experimental results [1, 2]. A difference scheme of running calculations of the first accuracy order with respect to time, and the second order with respect to time and spatial coordinate, respectively, has been chosen. To provide stability of the scheme, a small diffusion on the mesh was artificially introduced.

According to the assumptions made, the stability of the numerical scheme is provided by two conditions. First, the step length of the spatial mesh must be larger than the wavelength. Second, the time step must be small.

The temporal profile and the intensity spectrum of the emission at the laser output [see Eqn (9)] without an absorber section are shown in Fig. 1. It is seen that the duration of the first pulse is 0.7 ps. However, the duration of all subsequent pulses is 3 ps and is connected to the cavity round-trip time for a cavity of length 100 μm. Thus a stable generation of subpicosecond pulses has not been observed without the absorber.

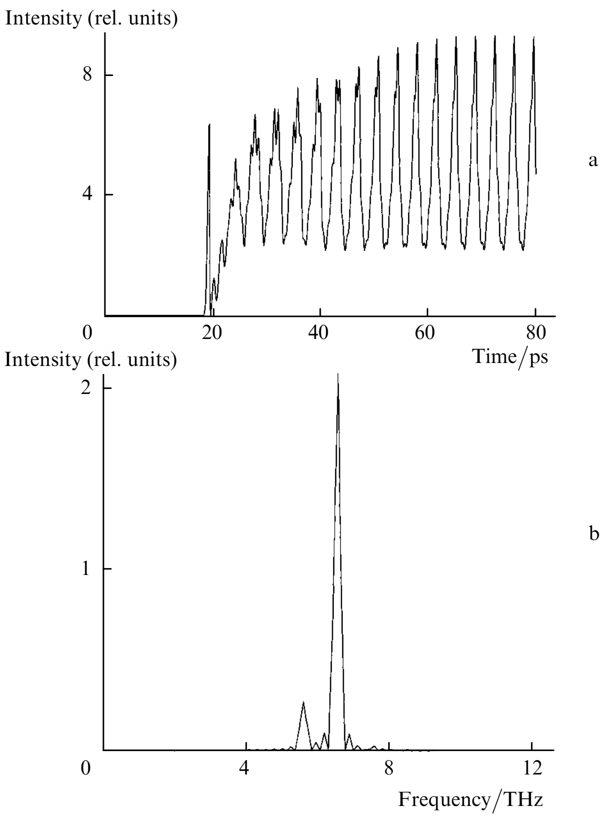

Figure 1. Temporal profile (a) and intensity spectrum (b) of the emission at the output of a laser without an absorbing section.

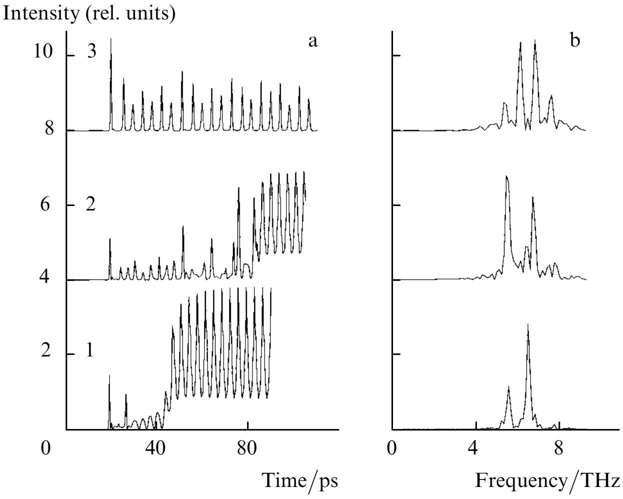

Figure 2. Temporal profile (a) and intensity spectrum (b) of the emission at the output of the cavity for a fixed reverse bias and with the lengths of the absorber section of a laser located in the middle of the cavity $d = 10$ (1), 20 (2), and 40 μm (3).

The temporal profile and the intensity spectrum of the emission at the laser output for a fixed reverse bias and for different lengths of the absorber section located in the middle of the laser cavity are shown in Fig. 2 (10 μm, curves 1; 20 μm, curves 2; 40 μm, curves 3). In the first case, the duration of the first pulse was 0.5 ps, and the durations of the subsequent pulses varied from 2 to 2.5 ps. Here, a fairly deep intensity modulation occured.

An increase in the length of the absorber section up to 20 μm resulted in an increase in the low-frequency peak in the intensity spectrum (curve 2 in Fig. 2b). Almost full-intensity modulation was achieved for $d = 40$ μm. The duration of the first pulse in the train was varied from 0.7 ps to 1.1 ps. Steady-state generation was observed at a time above 70 – 80 ps. The emission spectrum is transformed from single-mode (the curve 1 in Fig. 2b) to a symmetric four-peak type with two strongly pronounced lines (curve 3 in Fig. 2b). This corresponds to the experimental results [1, 2].

A similar transformation of the spectrum occured when the reverse bias applied to the absorbing section was varied. In this case, however, a shift of the central frequency of the spectrum occured in addition to broadening because a variation of the reverse bias U_2 resulted in a change of the frequency ω_0 which is determined by Eqn (8).

Let us investigate a dependence of the pulse shape of the generated pulses on the location of the absorbing section within the laser cavity. Temporal intensity profiles of the emission at the laser output for a fixed length ($d = 40$ μm) and different displacements Δh of the absorber section centre with respect to the middle of the cavity equal to 7.5, 15, and 30 μm (the curves 1 – 3, correspondingly) are shown in Fig. 3. Note that the absorber section occupies the edge area of the laser in the latter case. It is seen from Fig. 3 that the generation of the subpicosecond pulse train becomes unstable when the absorbing section is displaced from the middle of the cavity. The intensity of the pulses subsequent to the first one drops dramatically when the absorber section approaches the resonator edge, with the duration of the first pulse decreasing to 0.5 – 0.7 ps.

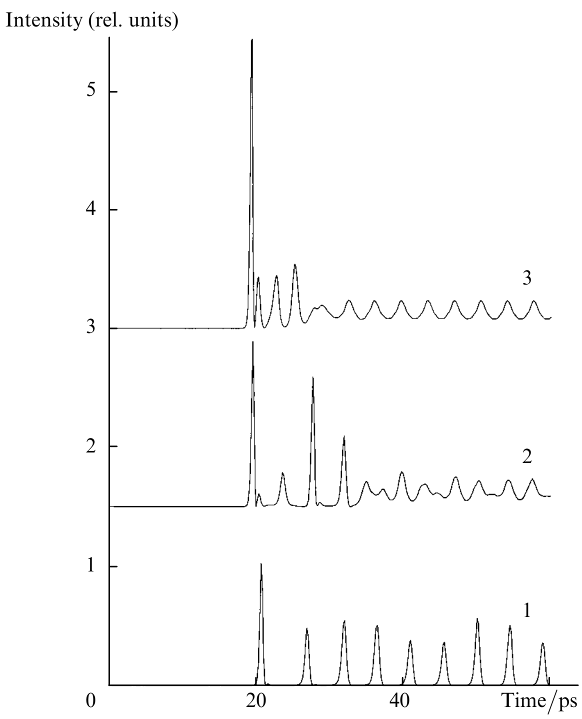

Figure 3. Temporal profile of the emission intensity at the output of the laser cavity for a fixed length ($d = 40$ μm) and for a displacement of the centre of the absorbing section with respect to the middle of the cavity $\Delta h = 7.5$ (1), 15 (2), and 30 μm (3).

5. Conclusion

The results of the theoretical study and mathematical modelling carried out here have shown that the introduction of an absorbing layer into the laser cavity allows for control of parameters of the generated pulses. Optimum selection of the layer thickness and the reverse bias results in an increase in the effective gain bandwidth and, hence, in a decrease in the duration of the generated pulses.

It follows from the results that parameters of the generated pulses depend also on the spatial distribution of the gain coefficient, i.e. on the position of the reverse-biased layer in the laser cavity. The location of the absorber in the middle of the laser active area results in the stable generation of trains of subpicosecond pulses. In order to realise a regime of generation of a single subpicosecond pulse, the absorber layer must be located closer to the edge of the active area of the laser.

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