

PACS numbers: 41.20.Jb; 42.79.Gn
DOI: 10.1070/QE2000v030n02ABEH001684

Relativistic kinematics of the electromagnetic fields of a guided mode

L A Rivlin

Abstract. **It is shown that during the observation of a wave in a waveguide from a comoving reference system travelling at a velocity equal to the group velocity of the wave, the wave propagation is halted and the electromagnetic energy contained in the waveguide proves to be stationary. The nonzero rest mass of the photons in the waveguide is equivalent to this rest energy and is identical with the rest mass measured in dynamic experiments.**

1. Introduction

All the electromagnetic fields which really exist in nature differ, without exception, from the ideal representation of an unlimited plane wave which has never been attained. The fields of a dipole radiation and diffraction by a slit, of a quasi-Gaussian laser beam, a guided mode, etc. may serve as clear examples. In contrast to the ideal plane wave, all these fields have a standing component and longitudinal components of the electrical and magnetic vectors, a group velocity less than the velocity of light c in free space, dispersion, a momentum defect, etc. A mass-like quantity $M > 0$ may be assigned to the photons represented by such real fields. In various thought dynamic experiments with accelerated motion, this quantity exhibits features which are indistinguishable from those with the inertial and gravitational rest mass of the usual massive bodies (see the present author's review [1] and the references to the original studies quoted there).

It has been found that the mass-like behaviour of the photons of really existing fields is manifested also when the standard transformations of the special theory of relativity are applied to them, i.e. in a purely kinematic experiment differing from dynamic problems [1]. Evidently, in order to observe the properties characteristic of a mass at rest, it is simply necessary to halt the test object, i.e. to observe it in a comoving coordinate system where its relative velocity is zero. Such an attempt is not impossible in relation to a photon because, as mentioned above, in real wave fields the group velocity of the transportation of electromagnetic energy, which is identifiable with the velocity of photons, is less than the velocity of light in free space c.

L A Rivlin Moscow State Institute of Radio Engineering, Electronics, and Automation (Technical University), prospekt Vernadskogo 78, 117454 Moscow, Russia, e-mail: rla@superlum.msk.ru

Received 29 July 1999
Kvantovaya Elektronika **30** (1) 185 – 188 (2000)
Translated by A K Grzybowski

2. A two-dimensional infinitely deep potential well for photons (metal waveguide)

The electromagnetic field in an infinitely deep two-dimensional potential well for photons, i.e. the mode of a hollow metal waveguide, may serve as the most convenient model for analysis by virtue of the simplicity of its boundary conditions. The mode field of such a waveguide of arbitrary cross section (Fig. 1) is known [2] to consist of the travelling waves

$$\begin{aligned} \boldsymbol{E}(x, y, z, t) &= \boldsymbol{e}(x, y)\exp[\mathrm{i}(\omega t - kz)] , \\ \boldsymbol{H}(x, y, z, t) &= \boldsymbol{h}(x, y)\exp[\mathrm{i}(\omega t - kz)] \end{aligned} \tag{1}$$

with the transverse eigenfunctions $\boldsymbol{e}(x, y)$ and $\boldsymbol{h}(x, y)$ and the eigenvalues (critical frequencies) ω_{nm} with the integer indices n and m, where ω is the frequency;

$$k = \frac{\omega}{c}\left[1 - \left(\frac{\omega_{nm}}{\omega}\right)^2\right]^{1/2} \tag{2}$$

is the propagation constant; x and y are the transverse coordinates; t is the time. The phase velocity of the wave along the longitudinal z axis is then given by

$$v = \frac{\omega}{k} = c\left[1 - \left(\frac{\omega_{nm}}{\omega}\right)^2\right]^{-1/2} > c , \tag{3}$$

whereas the group velocity of the transportation of electromagnetic energy and of the longitudinal photon migration along the waveguide is

$$u = \frac{\mathrm{d}\omega}{\mathrm{d}k} = c\left[1 - \left(\frac{\omega_{nm}}{\omega}\right)^2\right]^{1/2} < c . \tag{4}$$

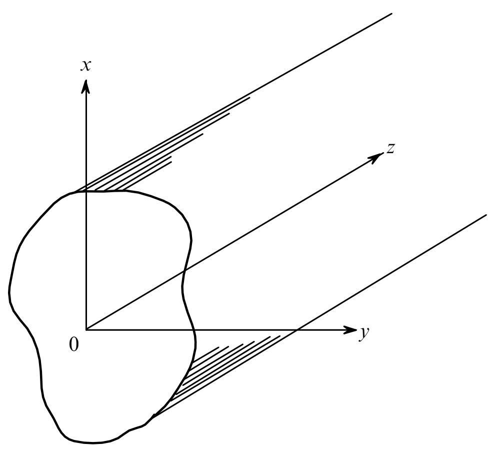

Figure 1.

The eigenvalues and the propagation constants are linked to the wave frequency by the dispersion relationship

$$\omega^2 = \omega_{nm}^2 + (ck)^2 \,, \tag{5}$$

whereas the transverse components of the eigenfunctions e_x, e_y, h_x, and h_y are expressed in terms of the spatial derivatives of the longitudinal components e_z and h_z:

$$\begin{aligned} e_x &= -\mathrm{i}\left(\frac{c}{\omega_{nm}}\right)^2\left(k\frac{\partial e_z}{\partial x} + \mu_0\omega\frac{\partial h_z}{\partial y}\right) , \\ e_y &= -\mathrm{i}\left(\frac{c}{\omega_{nm}}\right)^2\left(k\frac{\partial e_z}{\partial y} - \mu_0\omega\frac{\partial h_z}{\partial x}\right) , \\ h_x &= -\mathrm{i}\left(\frac{c}{\omega_{nm}}\right)^2\left(k\frac{\partial h_z}{\partial x} - \varepsilon_0\omega\frac{\partial e_z}{\partial y}\right) , \\ h_y &= -\mathrm{i}\left(\frac{c}{\omega_{nm}}\right)^2\left(k\frac{\partial h_z}{\partial y} + \varepsilon_0\omega\frac{\partial h_z}{\partial x}\right) , \end{aligned} \tag{6}$$

where μ_0 and ε_0 are the permeability and permittivity of vacuum. The longitudinal components e_z and h_z are in their turn solutions of the equations

$$\begin{aligned} \nabla_\perp e_z + \left(\frac{\omega_{nm}}{c}\right)^2 e_z = 0 \,, \\ \nabla_\perp h_z + \left(\frac{\omega_{nm}}{c}\right)^2 h_z = 0 \end{aligned} \tag{7}$$

subject to the boundary conditions on the metal surface ($\nabla_\perp$ is the Laplace operator in terms of the transverse coordinates x and y).

Depending on the direction of the polarisation vector, there are two types of solutions: solutions with TM-polarisation when $h_z = 0$ and solutions with TE-polarisation when $e_z = 0$, to which corresponds the following simplification of the system of equations (6) with single-term right-hand sides.

The longitudinal Poynting vector along the z axis of the waveguide is

$$P = \left(\frac{c}{\omega_{nm}}\right)^2\left(\frac{\omega}{\omega_{nm}}\right)^2\left\{\left[1 + \left(\frac{u}{c}\right)^2\right]\left(\frac{\partial e_z}{\partial x}\frac{\partial h_z}{\partial y} - \frac{\partial e_z}{\partial y}\frac{\partial h_z}{\partial x}\right) + \left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\frac{u}{c}\left[\left(\frac{\partial e_z}{\partial x}\right)^2 + \left(\frac{\partial e_z}{\partial y}\right)^2\right] + \left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2}\frac{u}{c}\left[\left(\frac{\partial h_z}{\partial x}\right)^2 + \left(\frac{\partial h_z}{\partial y}\right)^2\right]\right\} \tag{8}$$

or for the TM- and TE-polarisations we have, respectively,

$$P_{\mathrm{TM}} = \left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\left(\frac{c}{\omega_{nm}}\right)^2\frac{u/c}{1-(u/c)^2}\left[\left(\frac{\partial e_z}{\partial x}\right)^2 + \left(\frac{\partial e_z}{\partial y}\right)^2\right] , \tag{9}$$

$$P_{\mathrm{TE}} = \left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2}\left(\frac{c}{\omega_{nm}}\right)^2\frac{u/c}{1-(u/c)^2}\left[\left(\frac{\partial h_z}{\partial x}\right)^2 + \left(\frac{\partial h_z}{\partial y}\right)^2\right] . \tag{10}$$

In a metal waveguide, the set of mass-like quantities M is defined unambiguously by the eigenvalues (critical frequencies) ω_{nm} [1, 2]:

$$M = \frac{\hbar\omega_{nm}}{c^2} \,, \tag{11}$$

whereas the total rest mass of all the photons of the mode nm is

$$M(N+1) = \frac{\hbar\omega_{nm}}{c^2}(N+1) \,, \tag{12}$$

where N is the photon occupation number of the mode nm, whereas unity $(1 = 1/2 + 1/2)$ reflects the contribution of the zero-point vacuum oscillations along both directions of propagation.

3. Relativistic transformation of the frequency and propagation constant

We shall consider what happens to the frequency and the propagation constant of the wave, specified in the laboratory reference system, on transition to another inertial system migrating along the z axis with a velocity $c\beta$. The standard relativistic derivation of the formula for the Doppler effect is based on the postulate that the wave phase $\varphi = \omega t - kz =$ const is invariant in different inertial coordinate systems. On substituting in the formula for φ the ratios

$$t = \frac{t' + \beta z'/c}{(1-\beta^2)^{1/2}}, \quad z = \frac{z' + c\beta t'}{(1-\beta^2)^{1/2}} \,, \tag{13}$$

expressed in terms of the Lorentz-transformed primed coordinates of the moving system, it is easy to obtain the relationship for the phase in the primed system:

$$\varphi = \omega\frac{1 - c\beta k/\omega}{(1-\beta^2)^{1/2}}t' - k\frac{1 - \beta\omega/ck}{(1-\beta^2)^{1/2}}z' \,, \tag{14}$$

where the multipliers of t' and z' are the frequency ω' and the propagation constant k' in the moving system, whereas the second terms in the numerators are transformed with the aid of formula (4) and the dispersion relationship (5), so that

$$\omega' = \omega\frac{1 - \beta[1 - (\omega_{nm}/\omega)^2]^{1/2}}{(1-\beta^2)^{1/2}} = \omega\frac{1 - \beta(u/c)}{(1-\beta^2)^{1/2}} \,, \tag{15}$$

$$k' = k\frac{1 - \beta[1 - (\omega_{nm}/\omega)^2]^{-1/2}}{(1-\beta^2)^{1/2}} = k\frac{1 - \beta(c/u)}{(1-\beta^2)^{1/2}} \,. \tag{16}$$

Expression (15) is none other than the formula for the Doppler effect in the waveguide, demonstrating the expected dependence of the frequency on the relative velocity of the source and detector.

Remarkable consequences follow from formulas (15) and (16). If the velocity of the migration of the primed system $c\beta$ is identical with the group velocity of the wave u in the laboratory reference system (which is possible by virtue of the fact that $u < c$), then

$$\omega' = \omega_{nm} = \omega(1-\beta^2)^{1/2} \tag{17}$$

and

$$k' = 0 \,, \quad u' = 0 \,. \tag{18}$$

The latter means that in the primed system the wave stops and the transportation of electromagnetic energy ceases (which was in fact to be expected since $c\beta = u$), the travelling component of the wave vanishes altogether, and only the transverse standing component, with the frequency ω' [formula (17)], corresponding to a transverse second-order Doppler effect, remains. The electromagnetic rest energy $N\hbar\omega_{nm}$ is accumulated in this stopped optical wave or, in other words, the wave contains N stopped photons each with the energy $\hbar\omega' = \hbar\omega_{nm}$ equal to the energy of a quantum of the natural (critical) frequency of the guided mode. In conformity with the principle of equivalence, there is only one step from the photon rest energy $\hbar\omega_{nm}$ to the stopped mass [formula (11)], which is exactly identical with the nonzero photon rest mass observed in a series of dynamic experiments [1].

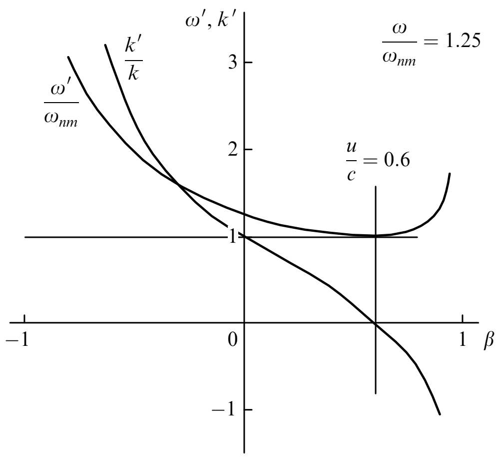

Figure 2.

On further increase in the velocity of the motion of the primed reference system ($c\beta > u$), the direction of propagation of the wave is reversed ($k' < 0$) and its frequency ω' increases (Fig. 2).

4. Relativistic transformations of the field vectors

On passing to the moving primed coordinate system, the field vectors are also transformed. The longitudinal components of the field vectors then remain unchanged ($e_z' = e_z$, $h_z' = h_z$), whereas the transverse components are transformed in accordance with the familiar relativistic rules:

$$\begin{aligned}
e_x' &= (1-\beta^2)^{-1/2}[e_x - (\mu_0/\varepsilon_0)^{1/2}\beta h_y]\,,\\
e_y' &= (1-\beta^2)^{-1/2}[e_y + (\mu_0/\varepsilon_0)^{1/2}\beta h_x]\,,\\
h_x' &= (1-\beta^2)^{-1/2}[h_x + (\varepsilon_0/\mu_0)^{1/2}\beta e_y]\,,\\
h_y' &= (1-\beta^2)^{-1/2}[h_y - (\varepsilon_0/\mu_0)^{1/2}\beta e_x]
\end{aligned} \tag{19}$$

or, taking into account formula (6),

$$\begin{aligned}
e_x' &= -\mathrm{i}\frac{c}{\omega_{nm}}\frac{\omega}{\omega_{nm}}\left(1-\beta^2\right)^{-1/2}\left[\left(\frac{u}{c}-\beta\right)\frac{\partial e_z}{\partial x}\right.\\
&\quad\left.+\left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2}\left(1-\beta\frac{u}{c}\right)\frac{\partial h_z}{\partial y}\right],\\
e_y' &= -\mathrm{i}\frac{c}{\omega_{nm}}\frac{\omega}{\omega_{nm}}\left(1-\beta^2\right)^{-1/2}\left[\left(\frac{u}{c}-\beta\right)\frac{\partial e_z}{\partial y}\right.\\
&\quad\left.-\left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2}\left(1-\beta\frac{u}{c}\right)\frac{\partial h_z}{\partial x}\right],\\
h_x' &= -\mathrm{i}\frac{c}{\omega_{nm}}\frac{\omega}{\omega_{nm}}\left(1-\beta^2\right)^{-1/2}\left[\left(\frac{u}{c}-\beta\right)\frac{\partial h_z}{\partial x}\right.\\
&\quad\left.-\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\left(1-\beta\frac{u}{c}\right)\frac{\partial e_z}{\partial y}\right],\\
h_y' &= -\mathrm{i}\frac{c}{\omega_{nm}}\frac{\omega}{\omega_{nm}}\left(1-\beta^2\right)^{-1/2}\left[\left(\frac{u}{c}-\beta\right)\frac{\partial h_z}{\partial y}\right.\\
&\quad\left.+\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\left(1-\beta\frac{u}{c}\right)\frac{\partial e_z}{\partial x}\right].
\end{aligned} \tag{20}$$

Hence the Poynting vector, characterising the electromagnetic energy flux along the waveguide z' axis in the primed reference system, is given by

$$\begin{aligned}
P' &= \left(\frac{c}{\omega_{nm}}\frac{\omega}{\omega_{nm}}\right)^2\left(1-\beta^2\right)^{-1/2}\left(\left[\left(1-\beta\frac{u}{c}\right)^2\right.\right.\\
&\quad\left.+\left(\frac{u}{c}-\beta\right)^2\right]+\left(\frac{\partial e_z}{\partial x}\frac{\partial h_z}{\partial y}-\frac{\partial e_z}{\partial y}\frac{\partial h_z}{\partial x}\right)\\
&\quad+\left(\frac{u}{c}-\beta\right)\left(1-\beta\frac{u}{c}\right)\left\{\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\left[\left(\frac{\partial e_z}{\partial x}\right)^2\right.\right.\\
&\quad\left.\left.\left.+\left(\frac{\partial e_z}{\partial y}\right)^2\right]+\left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2}\left[\left(\frac{\partial h_z}{\partial x}\right)^2+\left(\frac{\partial h_z}{\partial y}\right)^2\right]\right\}\right),
\end{aligned} \tag{21}$$

or, taking into account formula (10) for the TM- and TE-polarisations, we have, respectively

$$\begin{aligned}
P'_{\mathrm{TM}} &= \left(1-\beta\frac{u}{c}\right)\left(1-\beta\frac{c}{u}\right)\left(1-\beta^2\right)^{-1/2}P_{\mathrm{TM}}\,,\\
P'_{\mathrm{TE}} &= \left(1-\beta\frac{u}{c}\right)\left(1-\beta\frac{c}{u}\right)\left(1-\beta^2\right)^{-1/2}P_{\mathrm{TE}}\,.
\end{aligned} \tag{22}$$

Ultimately, the Poynting vector vanishes from the comoving reference system with $\beta = u/c\,(P'_{\mathrm{TM}} = 0, P'_{\mathrm{TE}} = 0)$, i.e. the transportation of energy ceases and the accumulated electromagnetic energy becomes stationary, whereas the photons in the waveguide are found to be at rest. On further increase in the velocity of the primed system ($\beta > u/c$), the energy flux changes sign and its direction is reversed. This picture repeats precisely the results in Section 3, obtained in the analysis of the behaviour of the propagation constant k.

5. Conclusions

Thus, in addition to the conclusions in Ref. [1] concerning the mass-like behaviour of photons in real electromagnetic fields in diverse dynamic situations, one should note that a nonzero photon rest mass is observed following its purely kinematic arrest as a result of the relativistic transformation of the coordinates.

It is important to emphasise that, despite the apparently absolute experimental indistinguishability of this photon rest mass from the inertial and gravitational masses in the standard interpretation, it is hardly possible to regard the rest mass as an immanent property of the photon, similar to the invariant properties of massive particles, because the photon rest mass depends on the external conditions (the method of excitation of the electromagnetic field, the boundary conditions, etc.; for example, the mass varies as one moves along a waveguide with a variable cross section along its length).

The results obtained are more likely to be of heuristic importance, demonstrating the way in which a rest mass is acquired by particles which initially belong to mass-free fields, without an a priori introduction of the concept of mass.

Finally, we may note incidentally the possibility of carrying out in the laboratory yet another real relativistic experiment, this time within the framework of the general theory of relativity. This concerns a 'tabletop black hole' [1], the experimental observation of which is precluded by the unavoidable increase in the attenuation of the wave in the waveguide as critical phenomena are approached, which is accompanied by the loss of the abrupt threshold character by the latter. This obstacle may be eliminated by compensating for the attenuation by introducing into the waveguide an amplifying medium with a gain bandwidth appreciably exceeding the detuning of the wave frequency from the critical frequency.

References

1. Rivlin L A Usp Fiz. Nauk **167** 309 (1997) [Phys.-Usp. **40** 291 (1997)]
2. de Broglie L Problèmes de Propagations Guidées des Ondes Électro-Magnetiques (Paris: Gauthier-Villars, 1941)

REVIEWS

PACS numbers: 01.30.Vv

'Luminescence of molecules and crystals' by M D Galanin

M V Fok

In M D Galanin's book 'Luminescence of Molecules and Crystals', published by the P N Lebedev Physics Institute of the Russian Academy of Sciences in 1999, an attempt is made to provide information about the physics of luminescence in a compact form. The author traces the history of the development of ideas about luminescence and introduces its fundamental characteristics. The luminescence and excitation spectra, the quantum yield, the duration of glow, and the degree of polarisation are considered. Much attention is devoted to fundamental questions, for example, the difference between luminescence and other kinds of secondary emission and the difference between the average lifetime of the excited state of molecules and the duration of their afterglow under the conditions of quenching by extraneous molecules. The results of experimental studies of the luminescence of molecules, of the impurity centres in crystals, and of the exciton mechanism of luminescence are analysed in detail.

The merits of the book include the fact that it is well written without excessive detail. However, the lack of detail sometimes obliges the reader to ponder profoundly on what he has read in order to understand and assimilate it.

M V Fok P N Lebedev Physics Institute, Russian Academy of Sciences

Received 11 January 2000
Kvantovaya Elektronika **30** (2) 000 – 000 (2000)
Translated by A K Grzybowski