PACS numbers: 42.65.Wi; 42.82.Et
DOI: 10.1070/QE2000v030n04ABEH001725

Theory of light propagation in three-core nonlinear directional couplers

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Abstract. An exact analytic solution is obtained for the problem of light propagation in a three-core nonlinear directional coupler with identical parallel fibres in the geometry of a regular triangle for an arbitrary dependence of the propagation constant on the light intensity.

The theory of stationary propagation of laser radiation in nonlinear directional couplers (NDCs) usually assumes that the propagation constants β depend on the intensity J of the propagating waves. By now, properties of two-core NDCs, whose propagation constants contain the Kerr correction (a term proportional to the light intensity [1 – 3]) have been studied in detail. A system of nonlinear differential equations that describes the propagation of light in coupled parallel fibres has been constructed and analytic solutions of these equations, which are expressed in terms of elliptic functions, have been obtained.

At the same time, attempts were made in papers [3 – 7] to apply numerical methods to the study of properties of three- and multicore NDCs, taking into account the Kerr correction to the propagation constant. It was shown in Ref. [4] that three-core NDCs offer several advantages over two-core NDCs. Numerical methods were used to study the light switching in three-core NDCs in a linear geometry and a regular triangle geometry [3]; and in Ref. [6] these methods were applied to three-, four-, and five-core NDCs. However, so far as we know, general analytic solutions of a system of coupled nonlinear differential equations for the field amplitudes in cores of the multicore NDCs have not yet been obtained, even for Kerr media.

Below we present analytic quadrature solutions of a system of nonlinear equations for coupled waves propagating along a three-core NDC in the geometry of a regular triangle for an arbitrary nonlinear dependence of the propagation constant β on the wave intensity J.

Consider an NDC consisting of three identical parallel fibres, for which the propagation constant is described by the expression

$$\beta = \beta_0 + f(J) , \tag{1}$$

where β_0 is a constant and $f(J)$ is an arbitrary function of the wave intensity. The fibres are arranged in the NDC cross section perpendicular to the direction of light propagation so that their axes are located at the angles of an equilateral triangle. We assume that the coupling constant γ of each of the fibres with two other fibres is independent of the light intensity, which is always satisfied [1, 3]. In this case, the nonlinear differential equations for coupled waves with amplitudes E_1, E_2, E_3 propagating along the x axis of each of the fibres of the NDC have the form [1 – 7]

$$E_1' = -\mathrm{i}[\beta_0 + f(J_1)]E_1 + \mathrm{i}\gamma(E_2 + E_3) , \tag{2}$$

$$E_2' = -\mathrm{i}[\beta_0 + f(J_2)]E_2 + \mathrm{i}\gamma(E_1 + E_3) , \tag{3}$$

$$E_3' = -\mathrm{i}[\beta_0 + f(J_3)]E_3 + \mathrm{i}\gamma(E_1 + E_2) , \tag{4}$$

where the prime in the left-hand side of the equations denotes differentiation with respect to the coordinate x (the x axis of the NDC coincides with the direction of light propagation).

Let us find solutions of this system by assuming that laser radiation with the field amplitude E_0 (the intensity J_0) is coupled into one of the fibres of the NDC (for example, the first one). It is clear from physical and symmetry considerations that because the fibres are identical, the field amplitudes in the second and third fibres of the NDC will be the same: $E_3 = E_2$. Then, the system of Eqns (2) – (4) is reduced to the system of two nonlinear equations of the type

$$E_1' = -\mathrm{i}[\beta_0 + f(J_1)]E_1 + 2\mathrm{i}\gamma E_2 , \tag{5}$$

$$E_2' = -\mathrm{i}[\beta_0 + f(J_2)]E_2 + 2\mathrm{i}\gamma(E_1 + E_2) . \tag{6}$$

Therefore the consideration of a three-core NDC with identical fibres was reduced to the consideration of an equivalent two-core NDC with two different fibres with propagation constants $\beta_1 = \beta_0 + f(J_1)$ and $\beta_2 = \beta_0 + f(J_2) - \gamma$, the coupling constant between the first and second fibres being twice as large as that between the second and third fibres.

Let us introduce the functions

$$J_1 = \frac{c}{8\pi}|E_1|^2, \quad J_2 = \frac{c}{8\pi}|E_2|^2 ,$$

$$Q = \mathrm{i}\frac{c}{8\pi}\left(E_1^* E_2 - E_1 E_2^*\right) , \tag{7}$$

$$R = \frac{c}{8\pi}\left(E_1^* E_2 + E_1 E_2^*\right) .$$

By using Eqns (5) and (6) and their complex conjugate equations we obtain the following system of nonlinear equations for new functions

$$J_1' = -2\gamma Q , \quad J_2' = \gamma Q , \tag{8}$$

$$Q' = [\, f(J_1) - f(J_2) + \gamma]R + 2\gamma(J_1 - 2J_2) , \tag{9}$$

$$R' = -[\, f(J_1) - f(J_2) + \gamma]Q . \tag{10}$$

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Received 4 October 1999
Kvantovaya Elektronika **30** (4) 349 – 350 (2000)
Translated by M N Sapozhnikov

We will find the solution of this system under the boundary conditions

$$J_{1|x=0} = J_0 \, , \quad J_{2|x=0} = 0 \, , \quad Q_{|x=0} = R_{|x=0} = 0 \, . \qquad (11)$$

From expressions (8) we obtain the first integral of motion

$$J_1 + 2J_2 = J_0 \, , \qquad (12)$$

which follows from the law of conservation of energy in the system. The second integral of motion is readily obtained from expressions (8)–(11):

$$Q^2 + R^2 = 4J_1 J_2 \, . \qquad (13)$$

Finally, the third integral of motion can be readily obtained from expressions (8), (10), and (12):

$$R = \frac{1}{2\gamma}\left[F(J_1) - F(J_0) + 2F(J_2)\right] - J_2 \, , \qquad (14)$$

where

$$F(J) = \int_0^J f(x)\,\mathrm{d}x \qquad (15)$$

and $F(0) = 0$.

By using expressions (8), (13), and (14), we obtain the quadrature solution for the spatial distribution of the light intensity J_2 in the second (third) fibre:

$$\int_0^{J_2} \left[-W(y)\right]^{-1/2} \mathrm{d}y = \gamma x \, , \qquad (16)$$

where

$$W(J_2) = \left\{ \frac{1}{2\gamma}\left[F(J_1) - F(0) + 2F(J_2)\right] - J_2 \right\}^2 - 4J_1 J_2 \qquad (17)$$

plays the role of the potential energy of a conservative nonlinear oscillator whose vibrations exist only in the region of J_2 where $W(J_2) \leqslant 0$. The maximum intensity of light $J_{2\,\max}$ in the second (third) fibre can be readily found from the condition $W(J_2) = 0$. If $f(J)$ is a complicated function and integral (16) cannot be represented in terms of the known functions, then at this (last) stage we can use the numerical methods and integrate one equation rather than the systems (8)–(10) or (2)–(4).

Assuming that $f(J) = \alpha J$ (the Kerr correction, which is often used [1–7]), we obtain $F(J) = \frac{1}{2}\alpha J^2$ and

$$W(J_2) = J_2\left[J_2\left(\frac{3}{2}\frac{\alpha}{\gamma}J_2 - \frac{\alpha}{\gamma}J_0 - 1\right)^2 - 4(J_0 - 2J_2)\right] \, . \qquad (18)$$

In this case, the integral in Eqn (16) can be expressed in terms of the elliptical functions. The propagating light transfers periodically from the first fibre to the second and third fibres, and vice versa. The coupling length and the maximum light intensity transferred into neighbouring fibres are determined by the intensity J_0 of the light incident on the front end of the first fibre. It follows from Eqn (18) that $J_{2\,\max} = \frac{4}{9}J_0$ for $\alpha = 0$ and it decreases monotonically with increasing J_0.

If we assume that $f(J) = \alpha(1 + J/J_s)^{-1}$ (the propagation constant saturates with increasing J), where J_s is the saturation intensity and α is a constant, then $F(J) = \alpha J_s \ln(1 + J/J_s)$ and

$$W(J_2) = \left\{ J_2 - \frac{\alpha J_s}{2\gamma}\ln\frac{(1 + J_2/J_s)^2\left[1 + (J_0 - 2J_2)/J_s\right]}{1 + J_0/J_s} \right\}^2 - 4J_2(J_0 - 2J_2) \, . \qquad (19)$$

In this case, the integral in Eqn (16) cannot be represented in terms of the known functions. Nevertheless, one can see that the periodic transfer of light from the first fibre to the second (third) one, and vice versa, still occurs, and that the coupling length and the maximum possible transfer depend on the excitation level J_0.

In conclusion the problem of propagation of laser radiation in a three-core NDC containing identical fibres in the geometry of a regular triangle can be solved in quadratures for an arbitrary nonlinear dependence of the propagation constant on the propagating light intensity.

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