Quantum Electronics **30** (4) 365 – 369 (2000)

PACS numbers: 42.25.Fx; 42.15.Dp
DOI: 10.1070/QE2000v030n04ABEH001726

Focused Bessel beams

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Abstract. The diffraction broadening of a focused beam with a Bessel amplitude distribution is examined. Calculations are reported not only of the traditional differential characteristics (radial distributions of the electric-energy densities and of the axial total electromagnetic energy flux in the beam), but also of integral quantities characterising the degree of transverse localisation of the radiation in a tube of specified radius within the beam. It is shown that in a large-aperture Bessel beam only a very small fraction of the total beam power is concentrated in its central core and that a focal point is also observed on intense focusing of the Bessel beam. This spot is not in the geometric-optical focal plane but is displaced from the latter by a certain distance.

New types of solutions of the wave equation for free space, which differ in principle from the traditional plane waves or classical Gaussian beams, have attracted considerable attention in recent years [1 – 4]. One interesting representative of such solutions is a paraxial beam with a Bessel amplitude distribution [5 – 14]. However, paraxial beams suffer from a significant shortcoming, which is a very weak transverse localisation of the intensity distribution as a whole.

This communication considers, first, the change in the transverse distribution and in the degree of concentration of radiation in Bessel beams as a result of the application of a large-aperture lens in the formation of these beams by the classical annular slit method. Second, it describes also a study of the spatial distribution of the optical field in the focal region of a large-aperture focusing system with a Bessel amplitude – phase filter. (In other words, a study was made of the field arising as a result of the intense focusing of a beam with a Bessel amplitude distribution.)

We shall examine the properties of focused Bessel beams within the framework of the vector diffraction theory [15, 16], since in the case of large apertures the standard scalar approximation can no longer provide a satisfactory description. We shall assume at the same time that the Fresnel number of the optical system $N_{\mathrm{F}} = a^2/\lambda f$ (where a is the radius of the exit pupil, λ is the wavelength, and f is the focal distance) is significantly greater than unity (as a rule, $N_{\mathrm{F}} \gg 1$ in the usual large-aperture focusing systems). The classical Debye approximation is then sufficiently precise [17] and only allowance for the vector character of the field leads now to a vector diffraction integral [16].

We shall employ the relative distributions of the longitudinal electromagnetic-energy flux (the axial component of the Poynting vector) p and of the spatial electric-energy density w (in the paraxial case, $w \approx p$ is the light intensity in relative units) for the description of the focused field. Assuming that the optical system has a rotational symmetry and is aberration-free and aplanatic (satisfies the Abbé sine condition [18]) we have in the case when the source emits a linearly polarised field

$$p(r, z) = |I_0|^2 - |I_2|^2 \,, \tag{1}$$

$$w(r, \varphi, z) = |I_0|^2 + 4|I_1|^2 \cos^2\varphi + |I_2|^2 + 2\cos 2\varphi \,\mathrm{Re}(I_0 I_2^*), \tag{2}$$

$$I_n = \int_{\theta_0}^{\theta_m} t(\theta)(\cos\theta)^{1/2} f_n(\theta) J_n(\lambda^{-1} 2\pi r \sin\theta)\exp(\mathrm{i}\lambda^{-1}2\pi z\cos\theta)\mathrm{d}\theta, \tag{3}$$

$$f_0 = \sin\theta(1+\cos\theta)\,, \quad f_1 = \sin^2\theta\,, \quad f_2 = \sin\theta(1-\cos\theta), \tag{4}$$

where r, φ, and z are cylindrical coordinates (the z axis coincides with the optical axis of the focusing system, the geometrical focus is at the point $r = z = 0$, φ is determined relative to the axis along which the electric vector of the incident field is directed); J_n is a Bessel function of the first kind with an integral index n; $0 \leqslant \theta_0 < \theta_m$; θ_m is half the aperture angle in the image space; $t(\theta)$ defines the relative angular dependence of the amplitude on the exit pupil (exit sphere) formed by the amplitude – phase filter of the system. Instead of the two true spatial coordinates r and z, we shall henceforth use the traditional normalised quantities — 'optical coordinates' $v = 2\pi r\lambda^{-1}\sin\theta$ and $u = 2\pi z\lambda^{-1}\sin 2\theta$.

We may note that the diffraction integral (3) works only in a focal region which is located at a distance from the exit pupil or, in other words, the distance to the geometric-optical focal plane should be significantly shorter than the distance to the exit sphere ($z \ll f$). Since in large-aperture systems we virtually have $a \approx f$ and $N_{\mathrm{F}} \approx a/\lambda$, we find that, for $a = 10$ cm and $\lambda = 0.5$ μm ($N_{\mathrm{F}} = 2\times 10^5$), the maximum value of $|u|$ may be $\sim 10^4$ assuming that the condition $z/f \lesssim 0.01$ is sufficient. Attention should be drawn also to the fact that, as can be readily seen from formulas (1) – (3), in the Debye approximation the diffraction field in the focal region always possesses specular reflection symmetry relative to the geometric-optical focal plane $z \equiv 0$, i.e. $p(-z) = p(z)$ and $w(-z) = w(z)$. The familiar 'focal shift' [19] arises if $N_{\mathrm{F}} \lesssim 1$ (the question of the focusing of Bessel beams in this case has already been examined [20]).

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Received 22 October 1999
Kvantovaya Elektronika **30** (4) 365 – 369 (2000)
Translated by A K Grzybowski

The dependence of the quantities p and w on the coordinates v and u yields a satisfactory idea about the field extrema, but the efficiency of the spatial localisation of the radiation in Bessel beams cannot be inferred from them. This is because precisely the numerous weak side lobes carry the bulk of the total power of such beams. In order to describe the degree of spatial localisation of the light, we introduce the integral characteristics

$$K_p(v,\,u) = \int_0^v p(v,\,u)\mathrm{d}v \Big/ \int_0^\infty p(v,\,u)\mathrm{d}v\ , \tag{5}$$

$$K_w(v,\,u) = \int_0^v\!\!\int_0^{2\pi} w(v,\,\varphi,\,u)\mathrm{d}v\,\mathrm{d}\varphi \Big/ \int_0^\infty\!\!\int_0^{2\pi} w(v,\,\varphi,\,u)\mathrm{d}v\,\mathrm{d}\varphi, \tag{6}$$

which describe precisely (quantitatively) the change in the degree of transverse concentration of the radiation in the beam when the latter propagates in free space. The width of a Bessel beam is understood as the width of the distribution as whole ($K_{p,w} \to 1$), and not only the width of its central core.

We shall now examine the large-aperture Bessel beams generated by the method of a narrow annular slit [6]. It is then necessary to adopt $\theta_0 = \theta_m - \Delta\theta_m$ in expression (3) ($\Delta\theta_m$ is the angular slit width; $\Delta\theta_m \ll \theta_m$) and $t(\theta) \equiv 1$. The results of numerical calculations of the quantities p, w, K_p, and K_w by formulas (1), (2), (5), and (6), respectively, are presented in Figs 1–3. We note immediately that the large-aperture beams considered are vector beams — the electric (magnetic) field vector has not only transverse components but also a perceptible axial component which depends on the transverse coordinates; in other words, the direction of the electric vector (the polarisation of light) at a certain point within the beam depends on the coordinates of this point. A characteristic feature of such beams is that only the quantity p has rotational symmetry, while a dependence on the angle φ (in the radial direction) within the beam arises for w (Fig. 1a). Furthermore, $w = p$ only for $v \equiv 0$, whereas outside the optical axis ($v \neq 0$) the transverse distribution of w differs from the corresponding distribution for p (w may be assumed to be symmetrical relative to rotation for small apertures — in paraxial beams w hardly differs from p). Indeed, our calculations have demonstrated directly (Fig. 1a) that in such beams the transverse distribution w, arising mainly owing to the contribution of the integrals $I_{1,2}$ [see formulas (1) and (2)], differs appreciably from the function $J_0^2(r)$.

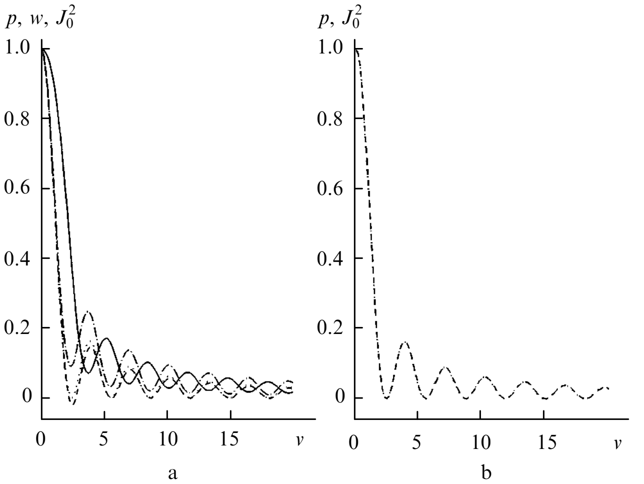

Figure 1. Dependences of p (dashed curves), w for $\varphi = 0$ (continuous curve) and 90° (chain curve), and the function J_0^2 (dotted curve; in Fig. 1b, the dotted curve merges with the dashed curve) on v in the geometric-optical focal plane ($u = 0$) for $\theta_m = 60°$ (a) and 6° (b) and for $\Delta\Theta_m = 0.5°$ (a) and 0.05° (b).

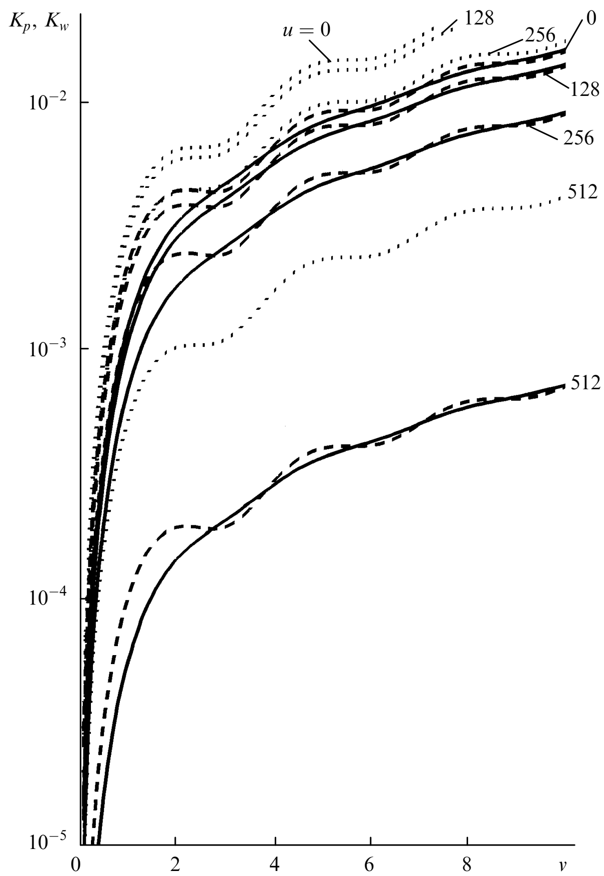

Figure 2. Dependences of K_w (continuous and dotted curves) and K_p (dashed curves) on v for different values of u and $\theta_m = 6°$ (dotted curves) and 60° (remaining curves) and for $\Delta\Theta_m = 0.05°$ (dotted curves) and 0.5° (remaining curves).

The main feature of paraxial Bessel beams, namely that only a small fraction of the total beam power is localised at its central maximum, clearly characterises also their large-aperture (vector) analogues. However, large-aperture beams are on the whole more likely to be concentrated around their axis than paraxial beams. Calculations have shown that, in terms of optical coordinates, the integral beam characteristics $K_{p,w}$ for different aperture angles have the same order of magnitude (specific results for $\theta_m = 6°$ and 60° are presented in Figs 2 and 3). Since $r \sim v/\sin\theta_m$, it follows that, for a fixed value of v, for example for $\theta_m = 6°$ and 60°, the ratio of the true spatial radii, which cover an approximately the same proportion of the total beam power, is $r(\theta_m = 6°)/r(\theta_m = 60°) = \sin 60°/\sin 6° \approx 8.3$, i.e. the radius of the central part (the central maximum) of a large-aperture Bessel beam with $\theta_m = 60°$ is virtually an order of magnitude smaller than for a small-aperture beam with $\theta_m = 6°$.

During the propagation of the beam, the quantities p and w on the optical axis of the system and at certain distances from it decrease in almost the same way. In other words, the consumption of energy in the beam cross section is practically uniform. The latter fact is indicated also by the dependences of

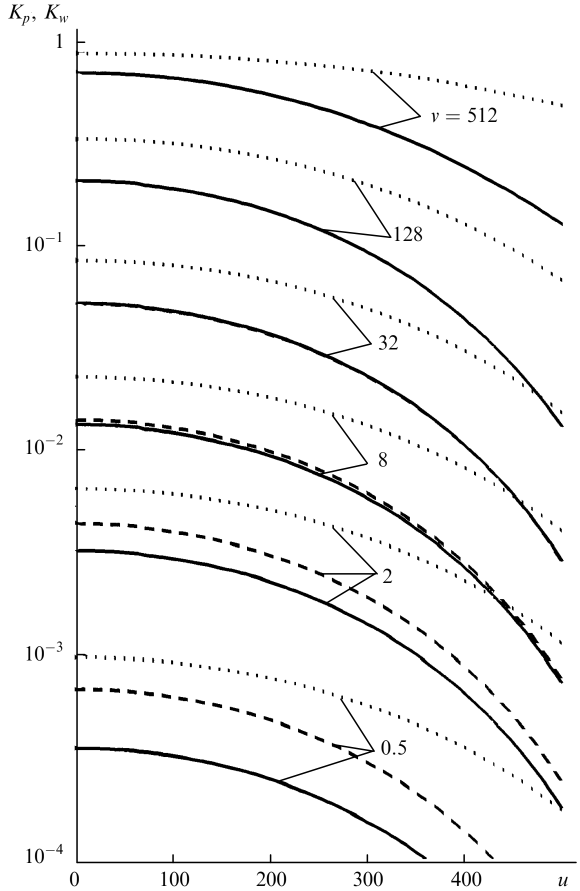

Figure 3. Dependences on the optical coordinate u of K_w (continuous curves) and K_p (dashed curves) for $\Theta_m = 60°$ and $\Delta\Theta_m = 0.5°$ and also of K_w (dotted curves) for $\theta_m = 6°$ and $\Delta\Theta_m = 0.05°$ in the axial direction for fixed values of v.

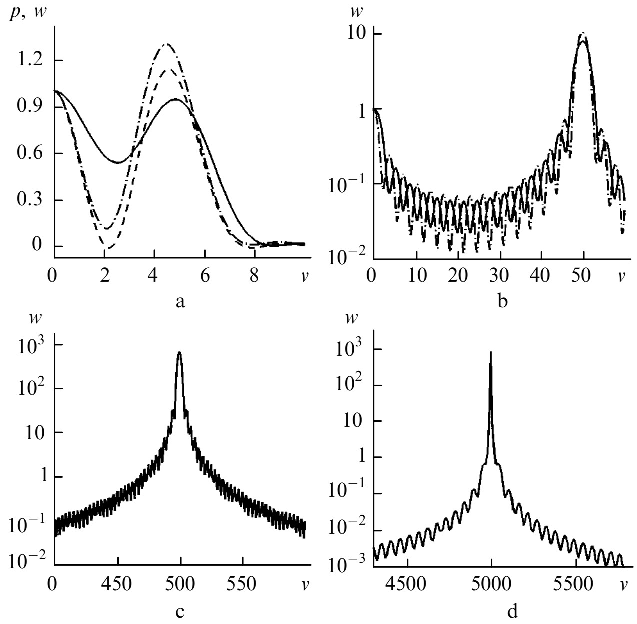

Figure 4. Dependences of p (dashed curve) and w for $\varphi = 0$ (continuous curves) and 90° (chain curve) on v in the geometric-optical focal plane ($u = 0$) for $\theta_m = 60°$ and $C = 5$ (a), 50 (b), 500 (c), and 5000 (d).

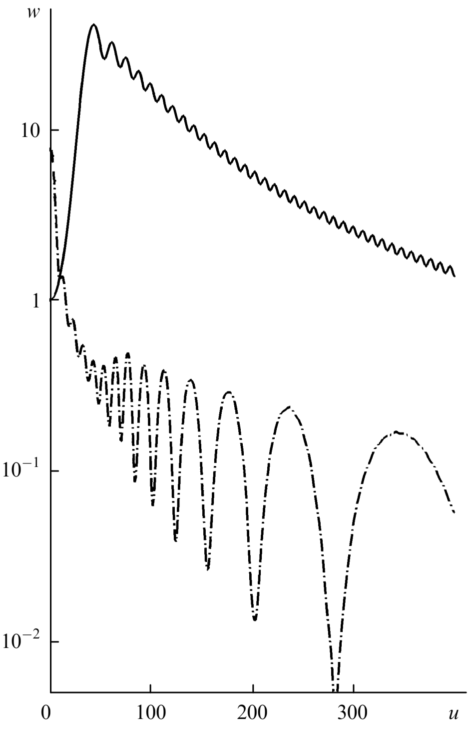

Figure 5. Dependences of w in the axial direction on the optical coordinate u along the beam symmetry axis ($v = 0$) (continuous curve) and at a distance $v = 50$ from the axis (chain curve) for $C = 50$ and $\Theta_m = 60°$.

$K_{p,w}$ on u (Fig. 3) and the difference between the integral parameters K_p and K_w is less notable than the difference between p and w. We may also note that, for a fixed aperture, the degree of transverse localisation of the beams is smaller the smaller the width of the annular slit (to a first approximation, this dependence may be regarded as linear). For a specified slit width, the degree of spatial localisation of the beam in the transverse direction may be increased by increasing the aperture angle of the focusing system (however, this leads to a rapid diffraction broadening along the axis).

We shall next consider the case where $\theta_0 = 0$ in formula (3) and

$$t(\theta) = J_0(C \sin\theta / \sin\theta_m) , \tag{7}$$

where the real constant is $C > 2.405$. In other words, we investigate the optical field in the focal region of the focusing systems with amplitude–phase filters and transmission of type (7). For large values of C, such filters form a convergent spherical wave with a virtually Bessel amplitude distribution at the wave front, whereas for small values of C they represent truncated Bessel filters. The dependence of the diffraction pattern on C along the geometric-optical focal plane is illustrated in Fig. 4. For $C \to \infty$, the diffraction image is transformed into a single light ring, which was demonstrated a long time ago in the scalar theory [7, 21]. In principle, this follows simply from the fact that the Fourier transform of the function $J_0(x)$ is a circle [9]. Consequently, when the zeroth Bessel beam is focused, its bright central maximum is fully suppressed in the geometric-optical focal plane and only a single intense side maximum at a distance $v = C$ is formed there.

However, this is valid only in the geometrical-optical focal plane $u = 0$ and in its immediate vicinity. The dependence of w along the beam axis (for $v = 0$) on u (Fig. 5) shows that, on moving away from this plane, w begins to increase, reaches its maximum at a certain distance $u = C$, and beyond this point (in the direction of increase in u) diminishes extremely slowly for large values of C, retaining for a long time its high value, as happens in the large-aperture Bessel beams examined above directly beyond their geometric-optical focus.

At the same time, sufficiently far from the optic axis the quantity w exhibits the opposite behaviour: on moving away from the geometric-optical focal plane, the energy density decreases (Fig. 5). The appearance of intense field localisation on the beam axis at the point $u = C$ is demonstrated best in Fig. 6, which presents the dependences of the quantities p and w along the transverse plane $u = C$ on the radial optical coordinate v for different values of C. Thus the focus, in the usual (true) sense, i.e. defined as a point on the beam axis where the energy density is a maximum, exists also in the given instance. It is merely displaced from the geometrical focus by a certain distance $u \approx C$. For high values of C, the energy density at the true focus exceeds the energy density at the geometric-optical focus by several orders of magnitude (see Figs 6c and 6d).

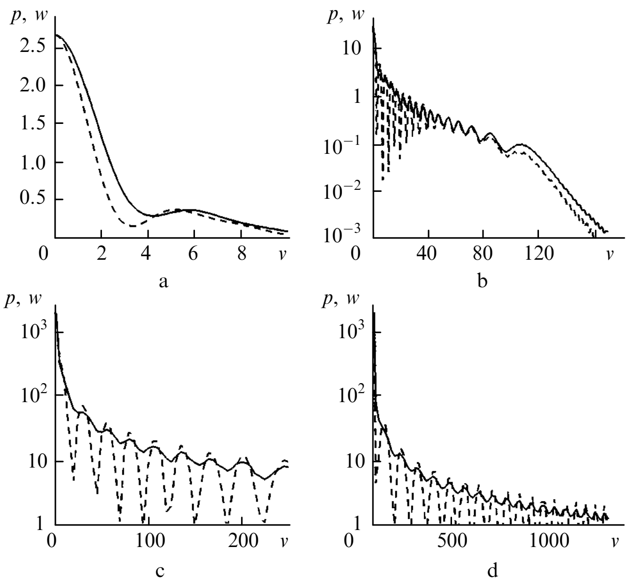

Figure 6. Dependences of w (continuous curves) and p (dashed curves) on v in the transverse plane $u = C$ for $\theta_m = 60°$ and $C = 5$ (a), 50 (b), 500 (c), and 5000 (d).

On moving away from the true focus ($u > C$), the field diverges and the degree of its localisation diminishes. However, this decrease is extremely slow for focused Bessel beams, i.e. in the case of filters with large values of C. In order to obtain a more detailed idea about the behaviour of the degree of concentration of the optical field beyond the geometric-optical focal plane, we shall consider the integral characteristics $K_{p,w}$, because the distribution of the quantities p and w characterises only qualitatively the spatial localisation of the radiation in the beam.

The dependences of the integral beam characteristics $K_{p,w}$ of two types were investigated. First, these are the dependences of $K_{p,w}$ on the optical coordinate v both in the geometric-optical focal plane and in other transverse planes ($u =$ const) at a specified distance from the geometric focus (Fig. 7). It was found that, for large values of C, the quantities $K_{p,w}$ reach

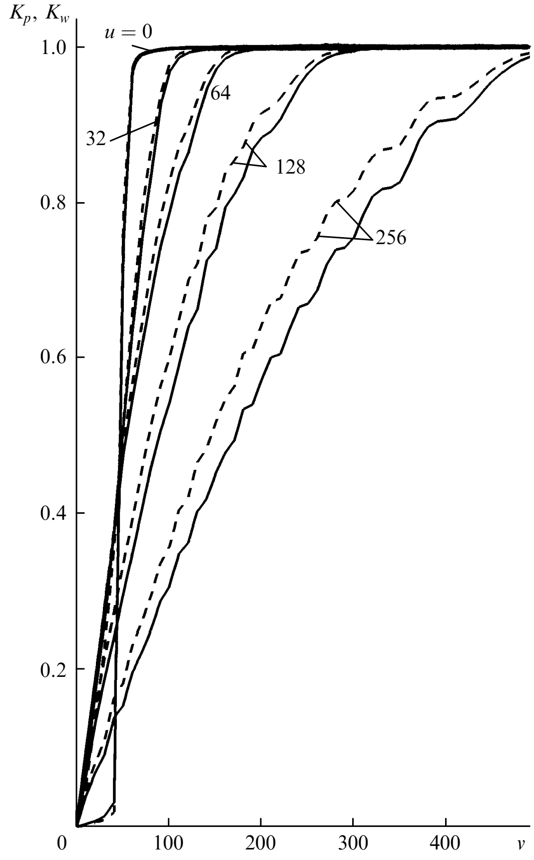

Figure 7. Dependences of K_w (continuous curves) and K_p (dashed curves) in different transverse planes $u =$ const on v for $C = 50$ and $\theta_m = 60°$.

0.5 for $v \approx C$ in the $u \lesssim C$ planes. In more remote planes ($u > C$), the increase in the functions $K_{p,w}$ is greater the larger the value of u.

Second, calculations were made also of the dependences of the degree of concentration of the focused optical field in a certain 'optical tube' with a radius v along the optic axis, i.e. the dependences of $K_{p,w}$ on u for fixed values of v (Fig. 8). The results show that, in the central part of the beam ($v < C$), in which less than half of the total beam energy ($K_{p,w} < 0.5$) is contained in the focal plane, the highest light concentration is attained outside the focal plane, approximately at a distance $u \approx C$ from the latter. For small values of C (but greater than 2.405), the degree of light concentration beyond this point (for $u > C$) decreases extremely rapidly (Fig. 8a), i.e. the diffraction broadening of the beam is pronounced. However, the greater the value of C the greater the stability of the beam and the slower its diffraction broadening in the region $u > C$ (Figs 8c and 8d). In 'optical tubes' with large radii ($v \gtrsim C$), the highest light concentration (the highest values of $K_{p,w}$) is attained in the geometric-optical focal plane, $K_{p,w}$ decreasing slowly and monotonically beyond this plane. Thus, the behaviour of the integral parameters $K_{p,w}$ of the beam also demonstrates that in the region $u > C$ the focused Bessel beam possesses diffraction properties similar to those of the corresponding properties of large-aperture Bessel beams considered in the first part of this study. We may also note that the difference between K_p and K_w is in principle insignificant, since it does not exceed several percent.

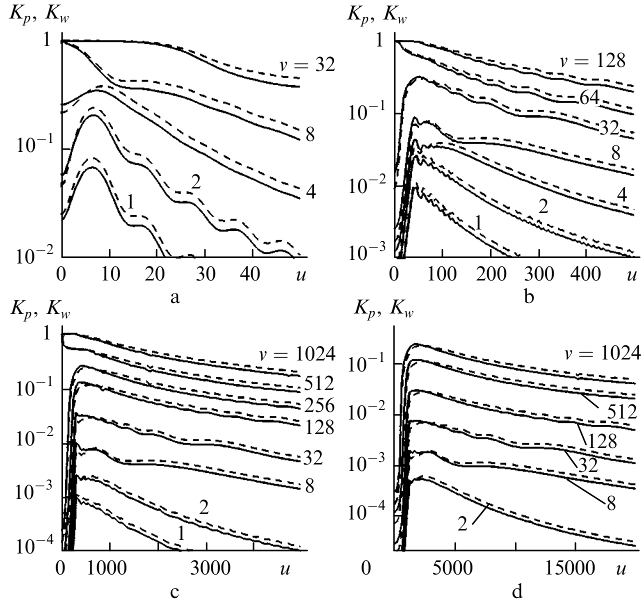

Figure 8. Dependences of K_w (continuous curves) and K_p (dashed curves) on u in 'optical tubes' with different optical elements v for $\theta_m = 60°$ and $C = 5$ (a), 50 (b), 500 (c), and 2000 (d).

Finally, we shall formulate the principal conclusions.

1. The radial distribution of the axial-flux density of the total electromagnetic energy and of the spatial electric-energy density in large-aperture Bessel beams differs appreciably from the corresponding transverse distributions for paraxial beams, which are satisfactorily fitted by the function $J_0^2(r)$. The main feature of the Bessel beams, which is that a very small part of the total power is concentrated in the main maximum, is definitely exhibited also by large-aperture Bessel beams, although the latter are on the whole much more strongly localised around their axes than paraxial Bessel beams.

2. When Bessel beams are intensely focused, the energy density at the centre of the beam begins to increase gradually on moving away from the geometric focus and reaches its maximum at a certain distance from this point. The energy density at the maximum (the true focus) is then larger by several orders of magnitude than at the geometric focus. On moving further away from the true focus, the field diverges and the degree of its localisation and the energy density in the beam decrease, albeit extremely slowly, remaining large for a long time.

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