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Optimal scaling of He – $Sr^+(Ca^+)$ recombination lasers

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Abstract. A method for scaling optimal parameters of the discharge-heated He – $Sr^+(Ca^+)$ recombination lasers is suggested. The method can be used for calculating the optimal excitation parameters and output characteristics of laser tubes of arbitrary dimensions from the known parameters of the optimised laser tube. The experimental verification of the scaling relations showed reasonable agreement between calculations and the experimental data, which is acceptable for practical applications.

He – $Sr^+(Ca^+)$ recombination lasers operate at short wavelengths and have comparatively high (for ion lasers) average power and efficiency [1 – 3], which makes them attractive for practical applications. For this reason, studies of the physical processes which occur in the active media of these lasers and optimisation of their operating regimes have been extensively developed. To optimise the operating regime of the laser, a proper choice of the excitation parameters of the laser tube is important. As for the possibility of predicting optimal parameters of gas-discharge lasers, there are two ways to solve this problem: the mathematical modelling of lasers based on the solution of a system of nonlinear differential equations that describe physical processes in a gas-discharge plasma [3 – 6], and the development of various scaling methods by using the properties of processes occuring in active media [6 – 8].

In this paper, we present a comparatively simple scaling method for discharge-heated He – $Sr^+(Ca^+)$ lasers. The method can be used to calculate optimal excitation parameters and output characteristics of laser tubes of arbitrary dimensions from the known parameters of the optimised laser tube with an accuracy sufficient for practical applications.

We developed this method by using some properties established during experimental optimisation and mathematical modelling of the He – Sr^+ laser. In the latter case, we used a model described in Refs [5, 9], which included a system of differential equations for concentrations of long-lived particles in a plasma and for parameters of a discharge circuit, as well as algebraic rate equations for the energy levels of the strontium ion. In particular, the following three features were found for the He – $Sr^+(Ca^+)$ lasers.

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(1) The storage capacity discharge in a discharge circuit consisting of capacity C, inductance L (consisting of the intrinsic inductance of a discharge tube and the inductance of the external circuits), and resistance R of the discharge tube proves to be close to the critical discharge, which corresponds to the passage of the discharge from the aperiodic regime to the oscillating one. This is due to the necessity of providing a steep trailing edge of the current pulse without undershoots. When the current pulse exhibits undershoots or when its trailing edge is delayed, the electron temperature increases at an early stage of the discharge afterglow, resulting in a sharp decrease in the recombination rate of $Sr^{++}(Ca^{++})$ ions and, hence, in the pumping rate and population of the upper laser level. The condition of the critical regime for the discharge circuit has the form

$$R = 2(L/C)^{1/2} . \quad (1)$$

Because $R = R(t)$, the resistance in Eqn (1) corresponds to the final high-current phase of the current pulse. The tube resistance is determined by the plasma conductivity σ:

$$R = \frac{l}{\sigma S} , \quad (2)$$

$$\sigma \propto \frac{n_e}{p\sqrt{T_e}} , \quad (3)$$

where l is the interelectrode distance; S is the tube cross section; p is the helium pressure; and n_e and T_e are the electron concentration and temperature [1, 2].

(2) The electron temperature in the current pulse is determined, with an acceptable accuracy, by the expression

$$T_e \propto E/p , \quad (4)$$

where E is the initial electric-field strength in the tube.

(3) The electron concentration in the current pulse in the optimal regime is approximately proportional to the specific input energy w,

$$n_e \propto w = \frac{CU^2}{2V} , \quad (5)$$

where U is the initial voltage across the storage capacity and V is the volume of the active medium.

Eqns (1) – (5) give scaling relations for the specific energy input, the initial voltage across the capacity, and the electric-field strength:

$$w \propto \frac{l}{S} p^{2/3} \frac{C^{1/3}}{L^{2/3}} , \quad (6)$$

$$U \propto l p^{1/3} \frac{1}{L^{1/3} C^{1/3}} , \tag{7}$$

$$E \propto p^{1/3} \frac{1}{L^{1/3} C^{1/3}} . \tag{8}$$

The time dependence of the current in the critical regime has the form

$$i(t) = \frac{U}{L} t \exp\left(-\frac{R}{2L} t \right) , \tag{9}$$

which yields for the current pulse amplitude

$$i_{\mathrm{m}} = \frac{2}{e} \frac{U}{R} . \tag{10}$$

Taking Eqns (1) and (7) into account, the scaling relations for the current pulse amplitude and the current density acquire the form

$$i_{\mathrm{m}} \propto l p^{1/3} \frac{C^{1/6}}{L^{5/6}} , \tag{11}$$

$$j_{\mathrm{m}} \propto \frac{l}{S} p^{1/3} \frac{C^{1/6}}{L^{5/6}} . \tag{12}$$

In the discharge-heated lasers, the generation regime optimal with respect to the average power is determined by the rate of heat removal at the temperature of the inner surface of the tube corresponding to the optimal pressure of metal vapour. The average lasing power P_{av} is determined by the efficiency η and the power P_{use} consumed from a power supply, which is equal to the total thermal power Q_{t} removed from the laser tube:

$$P_{\mathrm{av}} = \eta P_{\mathrm{use}} , \tag{13}$$

$$P_{\mathrm{use}} = Q_{\mathrm{t}} . \tag{14}$$

The linear thermal power removed from the tube by convection and radiation can be written in the form [1, 2]

$$Q = Q_{\mathrm{conv}} + Q_{\mathrm{rad}} = A d_{\mathrm{out}}^{0.75} + B d_{\mathrm{out}} , \tag{15}$$

where d_{out} is the external diameter of the laser tube. Coefficients A and B are functions of the tube temperature; however, because of a comparatively weak dependence of the optimal temperature on the helium pressure, these coefficients can be considered constant. Using Eqns (13)–(15) and assuming for simplicity that $d_{\mathrm{out}}^{0.75} \sim d_{\mathrm{out}}$, we can write simplified expressions for the linear and total thermal powers Q and Q_{t} removed from the tube:

$$Q = (A d_{\mathrm{out}}^{0.75} + B d_{\mathrm{out}}) \propto d_{\mathrm{out}} , \tag{16}$$

$$Q_{\mathrm{t}} = lQ = (A d_{\mathrm{out}}^{0.75} + B d_{\mathrm{out}}) l \propto l d_{\mathrm{out}} . \tag{17}$$

Taking into account that for typical laser tubes, $d_{\mathrm{out}} \propto d$, where d is the inner diameter of the laser tube, we obtain from Eqns (13), (14), and (17) the scaling relations for the average lasing power and the specific average power:

$$P_{\mathrm{av}} \propto \eta l d_{\mathrm{out}} \propto \eta l d , \tag{18}$$

$$P_{\mathrm{av}}^{\mathrm{sp}} = \frac{P_{\mathrm{av}}}{V} \propto \eta \frac{d_{\mathrm{out}}}{d^2} \propto \eta \frac{1}{d} . \tag{19}$$

On the other hand, $P_{\mathrm{av}}^{\mathrm{sp}}$ can be expressed in the form

$$P_{\mathrm{av}}^{\mathrm{sp}} = \varepsilon f , \tag{20}$$

where f is the pulse repetition rate and ε is the specific energy of the laser pulse, which can be defined as

$$\varepsilon = \eta w . \tag{21}$$

We obtain from Eqns (20) and (21)

$$P_{\mathrm{av}}^{\mathrm{sp}} = \eta w f , \tag{22}$$

which gives, taking Eqn (19) into account, the scaling relation for f:

$$f \propto \frac{d_{\mathrm{out}}}{w d^2} \propto \frac{1}{w d} , \tag{23}$$

which, after substitution of Eqn (6), takes the form

$$f \propto \frac{d}{l} \frac{1}{p^{2/3}} \frac{L^{2/3}}{C^{1/3}} . \tag{24}$$

The average current consumed by the tube is described by the expression

$$i_{\mathrm{av}} \propto \frac{P_{\mathrm{use}}}{U_{\mathrm{r}}} , \tag{25}$$

where U_{r} is the voltage of a power-supply rectifier. Taking into account that $U \propto U_{\mathrm{r}}$ (the proportionality coefficient is determined by the discharge excitation scheme), we obtain by using Eqns (7), (14), and (17) the scaling relation for the average current (for $d_{\mathrm{out}} \propto d$):

$$i_{\mathrm{av}} \propto d \frac{1}{p^{1/3}} L^{1/3} C^{1/3} . \tag{26}$$

On the other hand,

$$i_{\mathrm{av}} = i_{\mathrm{m}} \tau_i f , \tag{27}$$

which gives, taking into account Eqns (11) and (24), the scaling relation for the current pulse duration τ_i,

$$\tau_i \propto L^{1/2} C^{1/2} . \tag{28}$$

The relations obtained allow us to predict optimal parameters of recombination lasers based on the laser tube size and specified values of p and C. When the circuit inductance is determined mainly by the tube inductance, it can be found from the expression [6]

$$L = 2l \left[\ln \frac{4l}{d} - 1 \right] . \tag{29}$$

As our experiments showed, the values of η and P_{av} close to the maximum can be obtained by varying the capacity (by 3–5 times) within a wide range. The storage capacity can be also estimated, provided the scaling relation for C is determined. The additional condition required for this is $\sigma \approx \mathrm{const}$ because, as the measurements of the plasma conductivity during the current pulse and the results

Table 1.

Number of the active element	Laser	l/cm	d/cm	P_{av}/MW	P_{av}^{sp}/MW cm^{-3}	ε/μJ cm^{-3}	C/pF
1	He – Sr^+	45	1.5	1350	21	4.2	825
2	He – Sr^+	25	1.0	630	40	5.0	550
3	He – Sr^+	20	0.6	205	73	7.3	117.5
4	He – Sr^+	9	0.55	70	65	3.6	117.5
5	He – Ca^+	26.5	0.7	340	50	5.3	550

of numerical experiments showed, the conductivity is of the same order of magnitude for tubes of different geometry. Taking this into account, the scaling relation for the storage capacity can be obtained from Eqns (1) and (2) in the form

$$C \propto (S/l)^2 L \ . \tag{30}$$

It follows from Eqn (29) that we can assume with some accuracy that $L \propto l$. Therefore Eqn (30) can be written in the form

$$C \propto S^2/l \ . \tag{31}$$

Note that there are a number of restrictions on the regions of variation of parameters entering the scaling relations obtained, which are caused by the mechanisms of generation in the lasers under study.

In particular, analysis of the experimental data showed that in the optimal regimes of generation of discharge-heated recombination lasers with a longitudinal discharge, the specific energy output ε is 2 – 10 μJ cm^{-3} for $p \leqslant 1$ and the typical efficiency η is $\sim 0.1\%$. This yields the approximate range of variation of the specific energy input as $w \approx 2$– 10 μJ cm^{-3}.

The presence of the lower boundary of w is due to the existence of critical values of n_e and the rate of recombination pumping below which the inversion is broken [1, 2]. The upper boundary of w corresponds to the energy input at which almost complete double ionisation of strontium occurs. Above this boundary, the excess energy will be used mainly to

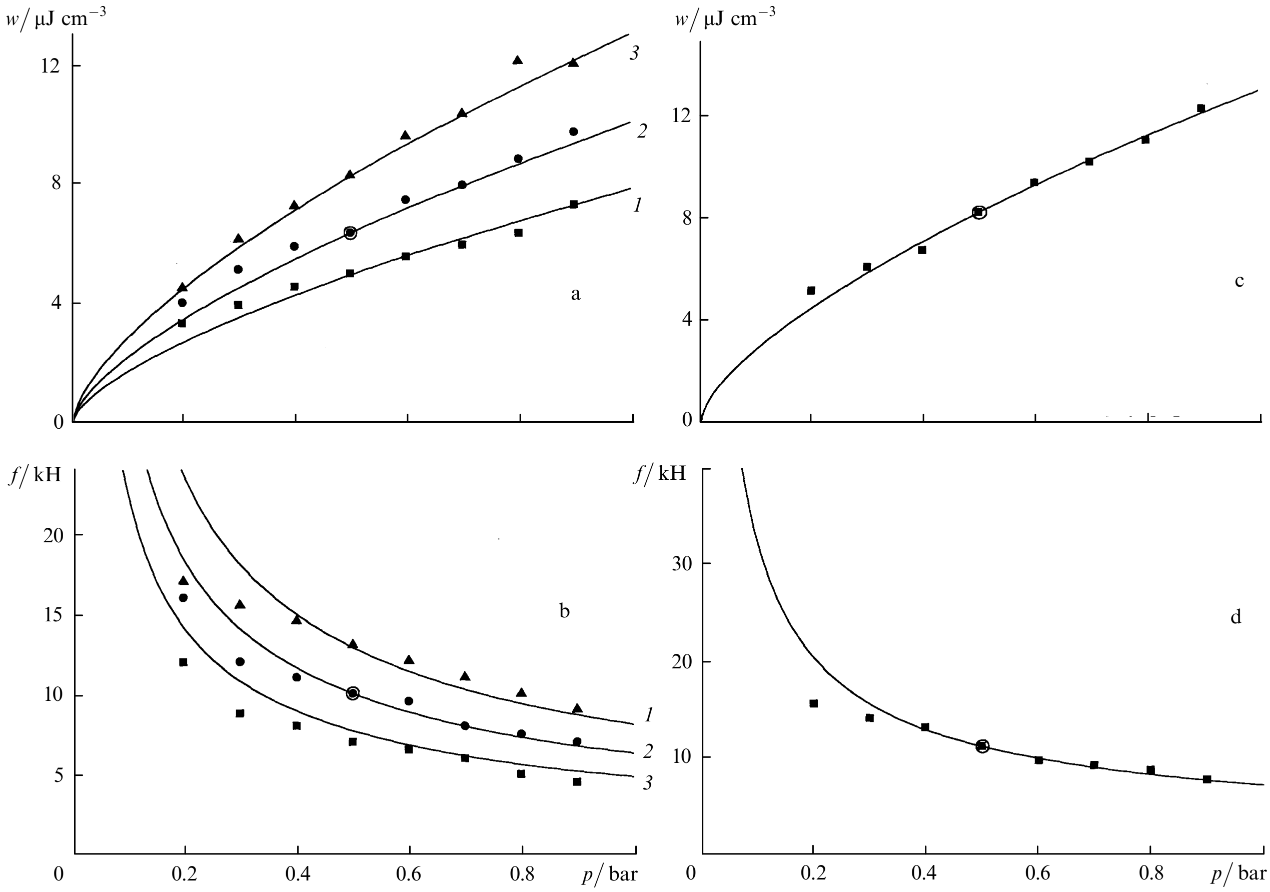

Figure 1. Dependences of the specific energy input (a, c) and pulse repetition rate (b, d) on helium pressure for a He – Sr^+ laser (a, b: tube 3) and a He – Ca^+ laser (c, d: tube 5). Triangles, circles, and squares show the experimental data. The curves are calculated by formulas (6) and (24) for $C = 117.5$ (1), 250 (2), and 550 pF (3 ; c, d).

ionise helium, resulting in the decrease in η and P_{av}, although ε can remain high.

Note that the scaling relations obtained are applicable only to the generation regimes in which the maximum average power is achieved. The experiments showed that the optimal regimes obtained after optimisation of lasers with respect to the input pulse characteristics, in particular, the laser pulse energy, can differ substantially from regimes with the maximum average power. In this case, the energy input achieved corresponds approximately to the upper boundary of the above range or somewhat exceeds it. Because the optimal pressure of the strontium vapour, which experiences in this case almost complete double ionisation, increases with increasing helium pressure [1 – 3], the value of w also increases.

Optimisation with respect to input pulse characteristics is performed, as a rule, for laser tubes with external heating, which operate in the single-pulse regime or at low pulse repetition rates. The maximum average and pulse energy characteristics can be simultaneously achieved only in rare cases. In these cases, the value of w determined by Eqn (6) should correspond to the upper boundary of the range of its variation for the given helium pressure.

As shown in Ref. [10], the optimal pulse repetition rate f is restricted by the limiting frequency determined by overheating of the active medium in near-axis regions of the discharge tube and described by the expression

$$f_{\lim} = \frac{55}{d^2}\,, \qquad (32)$$

where d is measured in centimetres, and $f_{\lim}$ in kilohertz.

Experiments with active elements of large diameter [11, 12] showed that the pulse repetition rate becomes restricted when $d \geqslant 2.5$ cm, which is explained by the formation of a strong radial inhomogeneity of the active medium. For this reason, the increase in the tube diameter up to 4 cm resulting in the corresponding increase in the active volume up to 1000 cm^3 no longer leads to the increase in the average output power, which remains at about 1 W. Therefore the scaling relations are valid only for $d < 2.5$ cm.

In addition, to provide efficient recombination pumping, the duration of the trailing edge and, hence, the duration of the current pulse should be short compared with the cooling time of electrons in the afterglow:

$$\tau_i \propto L^{1/2} C^{1/2} < \tau_{\rm cool} \propto \frac{1}{p}\,. \qquad (33)$$

To verify the above scaling relations, we performed experiments with four active elements of the $He - Sr^+$ laser of different sizes and one active element of the $He - Ca^+$ laser. The lasers were optimised with respect to the average power for different values of C and p. The geometrical dimensions of the laser tubes studied are presented in Table 1. Table 1 also gives the maximum values of the average output power, specific average power, and specific pulse energy achieved in the experiments, as well as the storage capacity at which these values were obtained. The specific average power and pulse energy were measured by taking into account the partial overlap of the discharge channel by metal pieces and electrodes.

The specific average powers 73 and 50 mW cm^{-3} achieved upon optimisation of the $He - Sr^+$ and $He - Ca^+$ lasers, respectively (see table), are maximum for conventional discharge-heated active elements of recombination lasers without forced cooling.

Fig. 1 shows the optimal specific energy input w and the pulse repetition rate f for the $He - Sr^+$ laser (tube 3) and $He - Ca^+$ laser (tube 5) measured in experiments. (Hereafter, the tube numbers correspond to the number of active elements in the table.) We chose the reference regimes (marked by circles in Fig. 1) from the optimal regimes determined experimentally and plotted for these regimes the dependences $w(p)$ and $f(p)$ predicted by scaling relations (6) and (24). One can see from Fig. 1 that experimental points are in good agreement with these dependences.

Fig. 2 presents experimental values of the current pulse amplitude and the average current for the same laser tubes and also the dependences $i_{\rm m}(p)$ and $i_{\rm av}(p)$ calculated by using expressions (11) and (26). One can see that the experimental and calculated dependences are in good agreement.

To verify dependences of the optimal values of $w, i_{\rm m}, f$, and $i_{\rm av}$ on the geometrical size of laser tubes, we optimised four active elements of the $He - Sr^+$ laser (tubes 1 – 4) for the same storage capacity and helium pressure equal to 550 pF and 0.5 bar, respectively. Using the experimental values of $w, i_{\rm m}$, and f obtained under these conditions for four laser tubes, we plotted dependences of these quantities on combinations of parameters in the right-hand sides of expressions (6), (11), (24), and (26) (except C and p, which are the same for all the tubes) (Fig. 3). We used the inductance of the discharge circuit consisting of the internal inductance of the tube (29) and the inductance of external circuits equal

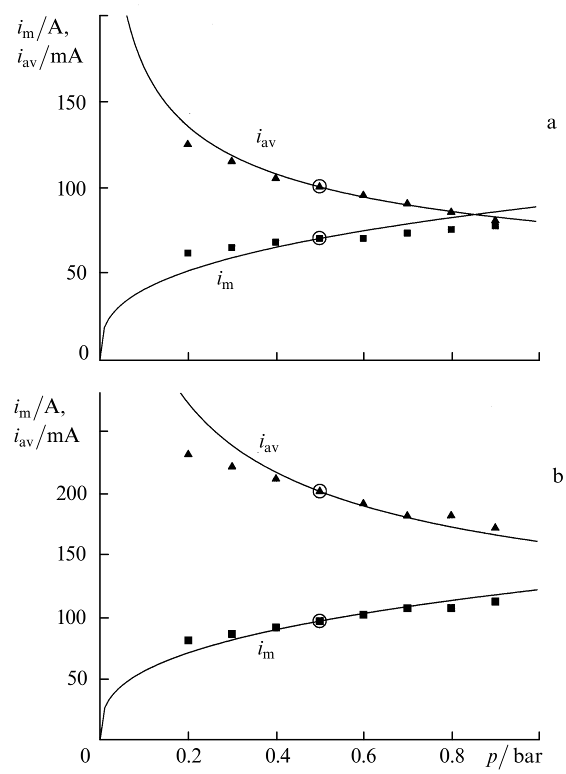

Figure 2. Experimental dependences (triangles and squares) and calculated dependences from expressions (11) and (26) (curves) of the current-pulse amplitude and average current on helium pressure for a $He - Sr^+$ laser (tube 3, a) and a $He - Ca^+$ laser (tube 5, b) for $C = 550$ pF.

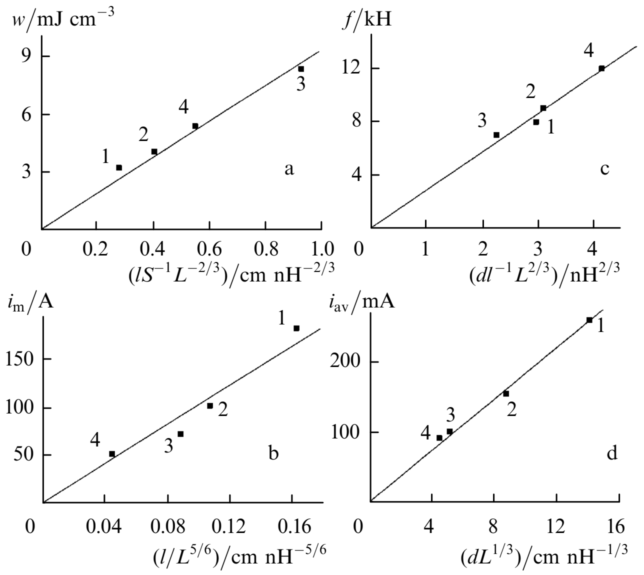

Figure 3. Dependences of the specific energy input (a), current-pulse amplitude (b), pulse repetition rate (c), and average current (d) on the geometrical size of laser tubes for a He – Sr$^+$ laser (tubes 1 – 4). The squares are experiment data for $C = 550$ pF, $p = 0.5$ bar (the numeration corresponds to that in the Table 1); the straight lines are linear approximations.

to 500 nH. One can see from Fig. 3 that the linear dependences given by these relations are valid with acceptable accuracy.

Fig. 4a shows the experimental dependence of the specific average power on the reciprocal diameter of the tube. This dependence is close to a linear dependence given by expression (19). Fig. 4b shows the dependence of the duration of the current pulse at its half-maximum for tubes 1 and 3 on the storage capacity. One can see that this dependence agrees with expression (28). It follows from Fig. 4c that, although the storage capacity can vary substantially, its dependence on the laser tube size is approximately described by expression (31).

Thus the method for scaling optimal parameters of the discharge-heated He – Sr$^+$(Ca$^+$) lasers suggested in this paper allows us to calculate optimal excitation parameters and output characteristics of laser tubes of arbitrary sizes with an accuracy acceptable for practical applications (the region of applicability of the method is determined by the operating regime of the optimised laser tube). In addition, this method can be used to predict the size and excitation parameters of laser tubes with specified output characteristics: the average power, energy, and pulse repetition rate. The scaling method can be also used for analysis of the experimental data and numerical simulations, and classification of the parameters of recombination lasers.

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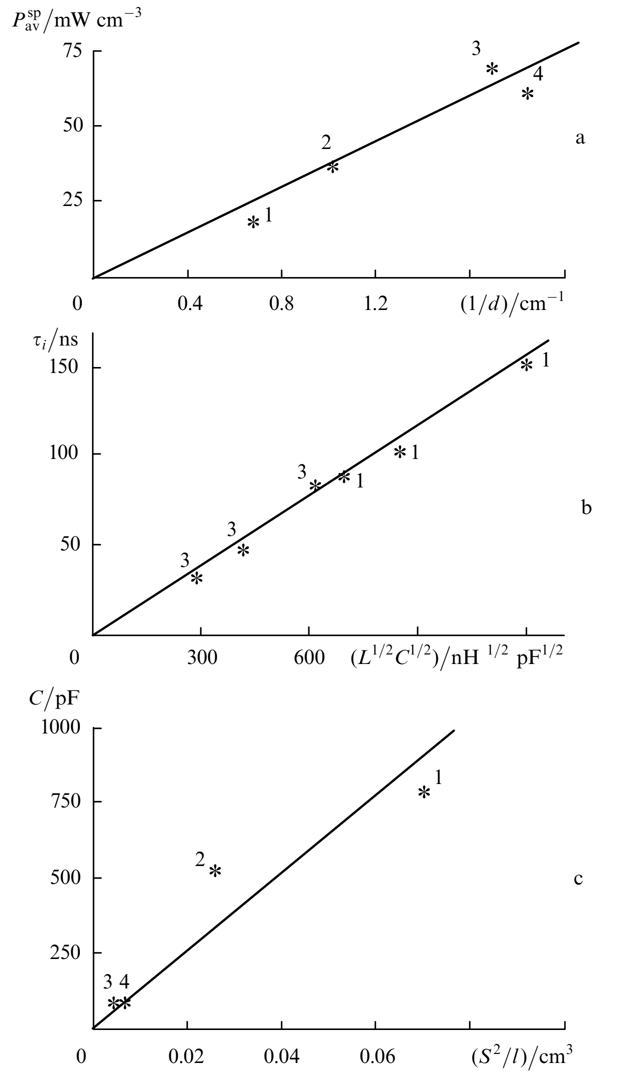

Figure 4. Dependences of the specific average power (a) and a storage capacity (c) on the geometrical size of laser tubes for a He – Sr$^+$ laser (tubes 1 – 4), and of the current-pulse duration (b) on $L^{1/2}C^{1/2}$ (tubes 1 and 3). The circles are experimental data (the numeration corresponds to that in the table); the straight lines are linear approximations.

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