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Specific features of partially coherent holography of dynamic objects

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Abstract. A method for recording partially coherent holographic interferograms is proposed, which makes it possible to control the sensitivity to the object point displacement by instrumental means. In this case, a decrease in sensitivity is accompanied by a decrease in the depth of the reconstructed image, which simplifies automated inputting of the resulting interferogram into a computer for further processing. The calculations showed that the partially coherent holograms using transverse scale transformation in one of the arms of a recording interferometer with a scale factor different from –1 possess the interferometric sensitivity, whereas the partially coherent holograms with the wavefront rotation in one of the arms of a recording interferometer lack the interferometric sensitivity to the displacement of object points during recording. The latter feature allows us to record objects whose displacement considerably exceeds the wavelength of recording radiation.

1. Introduction

The holograms recorded in partially coherent light possess some unique features, which are absent in photographic and coherent holographic recording [1 – 6]. Of particular interest is the degenerate case, which is described in detail in Refs [1, 3, 6] and referred to as the case of partially coherent holograms (PCH) with the plane-focusing effect (PCH-PFE). In this paper, we consider a more general case of PCH, and PCH-PFE represents its degenerate case. In Refs [1, 6], partially coherent Fourier holograms were divided into two classes: (1) PCH with transverse scale transformation of one of the wave fronts (PCFHI) and (2) PCH with rotation of one of the wave fronts (PCFHII). The latter holograms possess plane focusing [1, 6], which is of considerable interest.

In Refs [1, 3, 6], the rotation through 180° was analysed in detail. In this case, a twofold image magnification takes place, and polarised light is used with the highest efficiency. An interesting feature of this case is that it corresponds, according to the above classification, to PCFHII with the coefficient of scale transformation of the second beam equal to -1.

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In this case, the presence of a symmetry axis gives one more interesting property of PCH-PFE. This recording scheme possesses specific sensitivity to the object displacement during the hologram recording [3]. This is manifested in the fact that upon the reconstruction of a hologram of an object that moved during recording the image does not vanish, as opposed to conventional holography. The object motion during recording only causes a decrease in the resolution of the reconstructed image.

Note that PCFHI schemes with other scale transformation coefficients lack central symmetry, which is responsible for the specific character of the PCH-PFE sensitivity to the object displacement during recording.

In this paper, we analyse PCFHI schemes with the scale transformation coefficient different from –1 and PCFHII with the wave front rotation through an angle different from 180°. Note that the behaviour of the schemes with scale transformation coefficients different from –1 was previously unknown.

In particular, it was unknown whether this case is characterised by the holographic sensitivity of the recording method to the object displacements and whether it is possible, in principle, to record partially coherent interferograms (PCI). This problem is analysed in the first part of the paper. In the second part, we show that the PCFHII technique allows us to record objects whose displacement during recording considerably exceeds the wavelength of recording radiation. This allows one to increase the exposure time and obviate in this way the problems associated with noninterchangeability of actual sensitive media when recording fast processes.

2. Effect of object displacement during the exposure on the recording of PCFHII

To record PCFHII, we choose the scheme described in Ref. [1]. It represents a Mach – Zehnder interferometer with the wave-front rotation through 180° and scaling collimators placed in its arms. Each collimator consists of two confocal objectives with focal distances F_1, F_2 and F_2, F_1. Using this scheme, one can magnify one of the images with respect to the other when counterbalancing the optical path difference. This allows one to analyse a pure PCFHI rather than a mixture of PCFHI with a partially coherent Fresnel hologram [1].

To analyse the recording of PCFHI, we calculate the phase difference appearing upon propagation of a ray from an object to the recording plane through different interferometer arms. Let us write the magnification factor of the second interferometer arm with respect to the factor of the first one in the form $-(1-\alpha)$. Then, the difference of paths

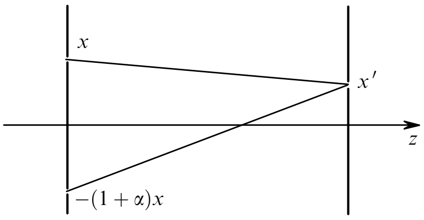

Figure 1. Schematic of the PCFHI technique: x is an object point seen from the point x' in the recording plane for light propagating in the first arm of the recording scheme; $-(1+\alpha)x$ is the object point seen from the point x' in the recording plane for light propagating in the second arm of the scheme.

from the point x in the object plane to the point x' in the recording plane (Fig. 1) for light travelling in different interferometer arms can be written in the form

$$\Delta x = \left\{\left[x' + (1+\alpha)x\right]^2 + F^2\right\}^{1/2} - \left[(x'-x)^2 + F^2\right]^{1/2}, \quad (1)$$

where F is the radius of curvature of the recording wave front on the axis of the recording scheme. Let us expand this expression in Taylor series and retain only the second-order terms. Introducing the dependence of the coordinate of an object point on the time t, we obtain

$$\Delta x(t) \approx \frac{2+\alpha}{2F}\left[\alpha x^2(t) + 2x(t)x'\right]. \quad (2)$$

The calculation of the phase difference at the point x caused by a change in the point position during time Δt gives

$$\delta(\Delta t) \approx \frac{1+\frac{1}{2}\alpha}{2\pi\lambda F}\left[2\alpha x(t_0)\xi(\Delta t) + \alpha\xi^2(\Delta t) + 2x'\xi(\Delta t)\right], \quad (3)$$

where

$$\xi(\Delta t) = x(t_0+\Delta t) - x(t_0)$$

is the displacement of a point in the object plane.

The second term in expression (3) may be comparable to the first one only in a small region in the neighbourhood of the axis of the scheme in the object plane. Moreover, the displacement should be smaller than the minimum resolvable element of an object being recorded. This is caused by a decrease in the resolution of the scheme because of the third term (this situation is similar to the case of PCH-PFE [3]). Therefore, even in the case where an object is found on the axis of the recording scheme, the second term will affect the recording in the region whose size is not resolved by the recording system. Therefore, one may neglect the second term. As a result, we have

$$\delta(\Delta t) \approx \frac{1+\frac{1}{2}\alpha}{2\pi\lambda F}\left[2\alpha x(t_0)\xi(\Delta t) + 2x'\xi(\Delta t)\right]. \quad (4)$$

The first term in expression (4) is independent of the coordinate in the hologram recording plane, and its contribution to the phase shift depends only on the displacement of an object point and its coordinate in the object plane. It accounts for the interferometric behaviour of an image in the hologram reconstruction process. The second term is independent of the coordinate in the object plane, but it linearly depends on the coordinate in the hologram recording plane and the displacement of the point in the object plane. It accounts for a decrease in resolution caused by the object motion during the hologram recording.

As a result, it is reasonable to specify two processes taking place in the recording of PCFHI. The first process is typical of holographic recording and is responsible for the interferometric sensitivity of the method to the displacement. The second process is typical of conventional photography and PCH-PFE [3], and it is responsible for a decrease in the resolution of the recording system. The interferometric sensitivity to the displacement is determined by the expression

$$\frac{1+\frac{1}{2}\alpha}{2\pi\lambda F}\alpha x(t_0)\xi(\Delta t) > \pi. \quad (5)$$

The distance between interference fringes should be larger than the displacement of any point of an object; otherwise, the fringes cannot be resolved because of a decrease in the effective recording aperture in the detection plane, which is caused by the second term in Eqn (4). Note that $x(t_0)/F$ is the sine of half the convergence angle of two recording beams. Thus, the sensitivity of the PCFHI interferometer depends (like in conventional coherent interferometry) on the wavelength λ of recording radiation and the convergence angle of recording beams, as well as (in contrast to coherent interferometry) on the difference of image magnification factors α in the interferometer arms.

Thus, we have an additional feasibility of decreasing sensitivity of PCH interferometry by instrumental means. Note that if a region being observed in the object plane is comparable in size to the distance from the centre of this region to the axis of the recording scheme, the sensitivity of the method at a certain point of an object of PCFHI interferometry, as in the case of coherent interferometry (where the angle between the ray going from object points and a reference ray substantially depends on the point position on an object), depends on the position of this point.

3. PCFHII without central symmetry

Consider now the recording of PCFHII in the case where the angle of the relative beam rotation differs from 180°. We will use the polar coordinate system in the recording plane and the object plane (Fig. 2). Let a point object be found in the object plane at the point r, φ. If one of the beams is rotated through the angle $-\alpha/2$ and the second one, through the angle $\alpha/2$, the coordinates of the points in the object plane are $r, \varphi - \alpha/2$ and $r, \varphi + \alpha/2$.

The path difference appearing in the recording plane at the point r', φ' for the rays propagated in different arms of the recording interferometer is

$$\Delta R = \left\{F^2 + \left[r'\cos\varphi' - r\cos\left(\varphi + \frac{\alpha}{2}\right)\right]^2 + \left[r'\sin\varphi' - r\sin\left(\varphi + \frac{\alpha}{2}\right)\right]^2\right\}^{1/2} - \left\{F^2 + \left[r'\cos\varphi' + r\cos\left(\varphi - \frac{\alpha}{2}\right)\right]^2 + \left[r'\sin\varphi' + r\sin\left(\varphi - \frac{\alpha}{2}\right)\right]^2\right\}^{1/2}. \quad (6)$$

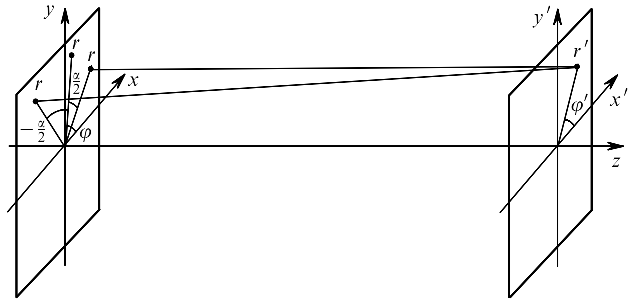

Figure 2. Schematic of the PCFHII technique: (r, φ) real polar coordinates of an object point; $(r, \varphi - \frac{1}{2}\alpha)$ object point coordinates seen from the point r' in the recording plane for light propagating in the first interferometer arm; $(r, \varphi + \frac{1}{2}\alpha)$ object point coordinates seen from the point r' in the recording plane for light propagating in the second interferometer arm.

Expanding (6) in a Taylor series and retaining terms up to the second order, we obtain

$$\Delta R \approx -\frac{rr'}{F}\left\{\cos\varphi'\left[\cos\left(\varphi+\frac{\alpha}{2}\right)+\cos\left(\varphi-\frac{\alpha}{2}\right)\right]\right.$$
$$\left.+\sin\varphi'\left[\sin\left(\varphi+\frac{\alpha}{2}\right)+\sin\left(\varphi-\frac{\alpha}{2}\right)\right]\right\}. \quad (7)$$

The corresponding phase difference is given by

$$\Delta\Phi \approx -\frac{rr'}{\pi\lambda F}\cos(\varphi'-\varphi)\cos\frac{\alpha}{2}. \quad (8)$$

If an object point is displaced by Δr and $\Delta\varphi$ during recording, the phase difference also changes:

$$\delta\Phi \approx -\frac{\cos\frac{1}{2}\alpha}{\pi\lambda F}r'[(r+\Delta r)\cos(\varphi'-\varphi+\Delta\varphi)$$
$$-r\cos(\varphi'-\varphi)]. \quad (9)$$

In the PCFHII scheme (in contrast to PCFHI), a phase change caused by the object displacement linearly depends, as in the degenerate case of PCH-PFE, on the coordinate in the recording plane. Because the term that is independent of the coordinate in the recording plane is absent, there always exists a region, at least a small one, near the rotation axis where recording will be realised for any object displacement. Interferogram recording in the PCFHII scheme is impossible. Therefore, this case, independently of the angle of relative beam rotation, is characterised by the same sensitivity to the displacement in the object plane as the degenerate case of PCF-PFE [3].

4. On the resolution and coherence in recording PCFHI and PCFHII

In the analysis of different versions of PCH, attention should be given to the relationship between characteristic dimensions, specifically, the coherence radius, the resolution of the recording scheme, and the displacement of object points. The resolution of a PCH recording system cannot be better than the coherence radius, which follows from the principle of the PCH method. On the other hand, it follows from (4) and (9) that the final resolution at a point in the object plane does not exceed the displacement at this point. To increase the diffraction efficiency of a hologram being recorded, it is desirable that the coherence radius be as close as possible to the required resolution. As for the coherence length, it should be no less than the maximum difference of optical paths from any point on an object to any point on a hologram for light propagating in different interferometer arms.

5. Conclusions

We analysed the use of PCFHI and PCFHII schemes for recording dynamic objects. It is shown that the PCFHI technique, with scale transformation coefficient different from –1, possesses the interferometric sensitivity. In this case, sensitivity to the displacement depends not only on the wavelength of recording radiation and the convergence angle of recording beams, but also on the scale transformation coefficient of a recording scheme. An additional feasibility appears to decrease the sensitivity of the PCH interferometry method by instrumental means. This decrease is accompanied by a decrease in the image depth, which may simplify automated interferogram processing because of a decrease in the number of in-depth scanning planes required for the desired resolution and, in some cases, allows one to abandon in-depth scanning at all.

Note that the PCFHII technique lacks the interferometric sensitivity to the object displacement during recording, and because of this, it allows the recording of objects whose displacement during the exposure considerably exceeds the wavelength of recording radiation. As for the size of a resolvable element, it cannot be smaller than the coherence radius of recording radiation and the displacement of an object plane during the recording for both schemes.

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