Quantum Electronics **30** (5) 457 – 458 (2000)

PACS numbers: 03.67.Lx; 42.79.Ta; 42.79.Yd
DOI: 10.1070/QE2000v030n05ABEH001742

On quantum computers

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Abstract. Attention is given to the fact that a computer whose functioning is based on the state superposition principle can be realised with classical, as well as quantum, elements.

The research in creation of a quantum computer has recently attracted much attention of researchers [1 – 12]. Under investigation are both the mathematical principles and the physical systems on which a practical implementation of such devices could be based.

It is well known that a conventional computer is based on binary (bistable) elements. The two states correspond to zero and unity, and a system composed of n such elements can be found in 2^n different states. This number of permissible states eventually determines the performance capabilities of the computer.

In a quantum computer, the role of the bistable elements can be played by a two-level quantum element ('two-level atom'). A system composed of n such elements can be also found in 2^n different states; however, these are only the basic states. In quantum mechanics, a wave function obeys the superposition principle; i.e., any linear combination of two permissible states of the system is a permissible state too. We will use the Dirac notation $|\alpha\rangle$ for the wave function of a two-level element, where α is either zero or unity. A linear combination

$$\psi^{(2)} = a_1|1\rangle + a_0|0\rangle \tag{1}$$

is then also a permissible state of the element. A system composed of n elements can be found in any of the following superposition states

$$\psi^{(n)} = \sum_{\alpha,\beta,\ldots,\mu} a_{\alpha,\beta,\ldots,\mu}|\alpha,\beta,\ldots,\mu\rangle, \quad \overbrace{\alpha,\beta,\ldots,\mu}^{n} = 0,\, 1. \tag{2}$$

Therefore, the set of the permissible states of a given quantum system virtually forms the continuum. How many of these states can be used for practical purposes depends on the overall resolution of the computer.

The aim of this brief is to call attention to the fact that the computers based on the superposition principle can be realised with classical systems. In particular, the electromagnetic field, polarisation, and magnetisation – all follow the superposition principle. Therefore, the computers operating with the electromagnetic field or polarisation (magnetisation) possess all the properties of a quantum computer. Since interference creates different spatial configurations only in the case of coherent fields, it is reasonable to introduce the term 'coherent computer'.

To illustrate the capabilities of coherent computing based on classical systems, we consider the electromagnetic field inside a cavity. The electric and the magnetic vectors of the field satisfy the wave equations. Let the field be represented in the form $E(\boldsymbol{r},t)\exp(-\mathrm{i}\omega t)$, where the amplitude $E(\boldsymbol{r},t)$ changes much slower than the exponential function. The wave equation can then be rewritten in the following way

$$\mathrm{i}\frac{\partial \boldsymbol{E}(\boldsymbol{r},t)}{\partial t} + \frac{1}{2\omega}\left(\omega^2 + c^2\nabla^2\right)\boldsymbol{E}(\boldsymbol{r},t) = -\frac{2\pi\omega}{\varepsilon}\chi(\boldsymbol{r},t)\boldsymbol{E}(\boldsymbol{r},t), \tag{3}$$

where the permittivity ε is constant in time and space and $\chi(\boldsymbol{r},t)$ is the medium polarisability induced by the control agent, e.g., the sound or electric (magnetic) field.

Eqn (3) is formally similar to the Schrödinger equation. The eigenfunctions $\boldsymbol{\Psi}_j(\boldsymbol{r})$ of the considered resonator satisfy the equation

$$\left(\omega_j^2 + c^2\nabla^2\right)\boldsymbol{\Psi}_j(\boldsymbol{r}) = 0. \tag{4}$$

Then, the solution of Eqn (3) can be represented as the series

$$\boldsymbol{E}(\boldsymbol{r},t) = \sum_j a_j(t)\boldsymbol{\Psi}_j(\boldsymbol{r}). \tag{5}$$

Next, we insert Eqn (5) into Eqn (3) and use Eqn (4) to obtain the following set of equations for the dynamics of the coefficients $a_j(t)$

$$\begin{gathered}\mathrm{i}\frac{\mathrm{d}a_j}{\mathrm{d}t} + (\omega - \omega_j)a_j = \sum_k V_{jk}(t)a_k\,, \\ V_{jk} = \frac{2\pi\omega}{\varepsilon}\int \boldsymbol{\Psi}_j(\boldsymbol{r})\chi(\boldsymbol{r},t)\boldsymbol{\Psi}_k(\boldsymbol{r})\mathrm{d}\boldsymbol{r}\,,\end{gathered} \tag{6}$$

where we assume that $(1/2\omega)(\omega^2 - \omega_j^2) \simeq \omega - \omega_j$. If nonzero nondiagonal elements are for some reason undesirable, one can make them zero by forming a suitable spatial dependence of the perturbation $\chi(\boldsymbol{r},t)$. The unavoidable damping can be readily accounted for in Eqns (6) by assuming the eigenfrequencies ω_j to be complex. The fluctuational Langevin forces can be included in Eqns (6) as well.

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Received 17 January 2000
Kvantovaya Elektronika **30** (5) 457 – 458 (2000)
Translated by I V Bargatin, edited by M N Sapozhnikov

Suppose that the resonator frequencies are chosen in the way that only two modes are excited. The set of Eqns (6) is then reduced to the following pair of equations

$$\mathrm{i}\frac{\mathrm{d}a_1}{\mathrm{d}t} + (\omega - \omega_1)a_1 = V_{11}a_1 + V_{12}a_2, \qquad (7)$$

$$\mathrm{i}\frac{\mathrm{d}a_2}{\mathrm{d}t} + (\omega - \omega_2)a_2 = V_{22}a_2 + V_{21}a_1. \qquad (8)$$

Eqns (7) and (8) are identical to the dynamics equations of a two-level quantum system: the resonator is then analogous to a quantum system with two stationary states.

As an example, we consider the modes of a dielectric sphere. As is known [13, 14], these modes can be of two types: E-modes and H-modes. Of all the modes of a dielectric sphere, the whispering gallery modes (WGM) have the highest quality factor. The frequencies of the E-modes and H-modes are given by the expressions [14, 15]

$$\omega_j^E \approx \frac{c}{a(\varepsilon\mu)^{1/2}}\left[v + 1.85576v^{1/3} - \frac{1}{\varepsilon}\left(\frac{\varepsilon\mu}{\varepsilon\mu - 1}\right)^{1/2}\right], \qquad (9)$$

$$\omega_j^H \approx \frac{c}{a(\varepsilon\mu)^{1/2}}\left[v + 1.85576v^{1/3} - \frac{1}{\mu}\left(\frac{\varepsilon\mu}{\varepsilon\mu - 1}\right)^{1/2}\right], \qquad (10)$$

where $v = j + 1/2$; n is the mode index; ε and μ are the permittivity and the magnetic permeability of the sphere medium, respectively; and c is the speed of light in vacuum. A WGM of the E-type and a WGM of the H-type can then represent the two required states. Alternatively, two modes of the same type can be chosen, provided that they correspond to different values of the index. If the index values are large enough, the two modes can be separately excited, leading to different ratios of their amplitudes. For a quartz microsphere ($\varepsilon = 2.37$, $\mu = 1$) with a diameter of 100 μm, the frequencies of E-modes and H-modes differ by approximately 2×10^{12} Hz.

A chain of microspheres joined by a common waveguide system can be viewed as a prototype of the computer processor. The spheres interact via the common waveguide and the surface fields (if the spheres are close enough). This interaction splits every eigenfrequency into n different frequencies, each of which corresponds to one of the basic field configurations. One can therefore excite various field configurations of the system by choosing the right frequency and amplitude of the control signal. The Q-factor of a WGM can reach approximately 10^9 [16]. For the eigenfrequency of the order of 10^{15} Hz, this corresponds to the field lifetime within microspheres equal to $\sim 10^{-6}$ s. This means that computing should be performed for a shorter time, approximately for 10^{-7} s. High-quality microresonators can be used instead of the dielectric microspheres. Note that the use of WGM and other types of the resonator modes in computers has been discussed in the literature [7], but only in the contex of a 'quantum computer'. The assumption that a computer based on microresonators can be a classical system has not been discussed in Ref. [7].

The author does not want to give the false impression that a proposal for the implementation of a coherent computer is presented here. The development of such a computer requires further investigation. What the author wants to stress is that the term 'quantum' is inadequate for the computers based on the state superposition principle. In fact, this term obscures the matter. The coherent computer can be realised with classical as well as quantum elements, and this strongly widens the field of research for coherent computers. The time will show which elements — classical or fundamentally quantum — will become the foundation of a functioning coherent computing device.

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