Quantum Electronics **30** (6) 555 – 558 (2000)

PACS numbers: 42.55.Lt; 42.79.Gn
DOI: 10.1070/QE2000v030n06ABEH001764

Violation of the symmetry of waveguide laser modes caused by the lens effect

P L Rubin, F J Blok, W J Witteman

Abstract. It is shown that the formation of a negative gas lens in a waveguide gas laser of medium pressure can break the symmetry and the number (classification) of waveguide modes. The violation of symmetry can be caused by imperfect waveguide construction or can occur spontaneously. The spontaneous violation of symmetry is caused by the optical nonlinearity of an active medium (the saturation effect) and leads to the appearance of new modes, which are absent in the linear waveguide with the same optical characteristics. The violation of symmetry of waveguide modes can cause, in particular, a decrease in coherence of laser radiation.

1. Introduction

Recent studies (see, e.g., Ref. [1]) showed that gas heating in a slab waveguide laser of medium pressure can result in a well-pronounced lens effect. If the discharge exciting the gas and the optical laser system are symmetric about the middle plane of the slit, it is natural to expect the same symmetry for optical waveguide modes. However, this assumption is not always valid in reality because of a negative gas lens formed in the waveguide. As the lens power increases, the frequencies of waveguide modes approach one another in pairs (corresponding to successive even and odd modes), and each pair is gradually divided into two parallel radiation fluxes, which are separated by the region where nearly total internal reflection of radiation takes place. The larger the lens power, the larger the number of pairs formed in the system. Nevertheless, in a symmetric waveguide, each mode of a pair retains symmetry and coherence of radiation throughout the waveguide section.

However, as the lens power is increased, the coupling between these two fluxes becomes so weak that even weak engineering (or technological) deviations from the waveguide symmetry, which are inevitable, are able to cause an almost complete breakdown of symmetry of waveguide modes and the disintegration of the aforementioned pairs of modes into other modes, which are almost totally spatially separated. In this case, mutual coherence of radiation in two halves of the waveguide (two radiation fluxes) is absent.

P L Rubin P N Lebedev Physical Institute, Russian Academy of Sciences, Leninskii prosp. 53, 117924 Moscow, Russia
F J Blok, W J Witteman University Twente, 759 AE Enschede, Netherlands

Received 16 December 1999
Kvantovaya Elektronika **30** (6) 555 – 558 (2000)
Translated by A N Kirkin, edited by M N Sapozhnikov

As will be shown below, the violation of mode symmetry can take place even in an exactly symmetric waveguide, which is associated with an inevitable optical nonlinearity (saturation) in a laser. Because of this, it is reasonable to speak about a kind of spontaneous violation of the waveguide symmetry.

2. Modes of a nonlinear waveguide

Let the radiation field be dependent on two Cartesian coordinates x and y. The x axis is directed along the normal to the waveguide boundaries, and the z axis specifies the direction in which radiation travels. The waveguide thickness is $2l$, and its boundaries lie in the planes $x = \pm l$. The equation describing the propagation of the light field in the waveguide has the form

$$\Delta f + (1 + \delta\varepsilon) k_0^2 f = 0 \ . \tag{1}$$

Here,

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$$

is the Laplace operator; k_0 is the wave number of radiation in vacuum; f is one of the field components; and the dielectric constant of a gas at the frequency under consideration $1 + \delta\varepsilon$ is a function of x and a functional of the radiation field. The dependence of $\delta\varepsilon$ on x describes refraction and gain of a medium, and the dependence of $\delta\varepsilon$ on the field intensity describes saturation.

For simplicity, the gain is assumed to have a local dependence on the light field strength, and it is given by the formula (Ref. [2])

$$\mathrm{Im}\delta\varepsilon(x) = \frac{1}{k_0} \frac{g(x)}{1 + |f|^2/I} \ . \tag{2}$$

Here, $g(x)$ is the unsaturated gain and I is the saturation parameter. For simplicity, we assume that $I = \text{const}$.

Let (see also Ref. [3])

$$f = A \exp(\mathrm{i}k_0 z) \ ,$$

where $A = A(x, z)$ is the slowly varying field amplitude satisfying the modified wave equation

$$\Delta A + 2\mathrm{i}k_0 \frac{\partial A}{\partial z} + \delta\varepsilon k_0^2 A = 0 \ . \tag{3}$$

Waveguide modes (in the nonlinear regime) are described by the solutions of the last equation found by the method of separation of variables, i.e., the solutions of the form

$$A(x, z) = a(x)\alpha(z). \tag{4}$$

Substituting Eqn (4) in Eqn (3), we have

$$\frac{1}{a(x)}\frac{\mathrm{d}^2 a(x)}{\mathrm{d}x^2}+\frac{1}{\alpha(z)}\frac{\mathrm{d}^2\alpha(z)}{\mathrm{d}z^2}+2\mathrm{i}k_0\frac{1}{\alpha(z)}\frac{\mathrm{d}\alpha(z)}{\mathrm{d}z}$$
$$+k_0^2\delta\varepsilon[x,a(x)\alpha(z)]=0\ .$$

Let us fix the quantity α and consider the equation

$$\frac{1}{a(x)}\frac{\mathrm{d}^2 a(x)}{\mathrm{d}x^2}+\left\{k_0^2\delta\varepsilon[x,a(x)\alpha]+q(\alpha)\right\}=0 \qquad (5)$$

with additional (boundary) conditions

$$a(-l)=a(l)=0\ . \qquad (6)$$

This is a nonlinear analogue of the problem of waveguide modes.

Strictly speaking, the procedure described above is not really correct. From Eqn (5), it follows that the function a depends on α and, therefore, on z. However, when the unsaturated gain is small (this case often takes place in practice), one may neglect the dependence of a on z (for detail, see [3]). This situation will be considered further. The quantity q will be called, as in the linear case, the eigenvalue of the problem, which is now nonlinear. The discrete set of values of q with the corresponding functions $a(x)$ (nonlinear modes) can be found from Eqn (5) and the boundary conditions. In this case, α is treated as a parameter.

Then, one should solve the equation

$$\frac{1}{\alpha(z)}\left[\frac{\mathrm{d}^2\alpha(z)}{\mathrm{d}z^2}+2\mathrm{i}k_0\frac{\mathrm{d}\alpha(z)}{\mathrm{d}z}\right]=q(\alpha)\ ,$$

which describes a change in amplitude of the field travelling along the waveguide. One may neglect in this equation the second derivative of α because $1/k_0$ is much smaller than the characteristic distance on which the amplitude changes. Generally speaking, the dependence of the field on z in the nonlinear waveguide is no longer exponential, but it may be rather close to it, provided the saturated gain (depending now on the cavity loss) is low, which was already assumed above.

3. Violation of the symmetry of modes

Consider Eqn (5) with boundary conditions (6). It is convenient to rewrite it in a more customary form

$$\frac{\mathrm{d}^2 f(x)}{\mathrm{d}x^2}+\left\{k_0^2\delta\varepsilon[x,\,f(x)]+q\right\}f(x)=0\ . \qquad (7)$$

Here, z and α are fixed, and (see above) $f(x)=a(x)\alpha$. It is an intricate problem to analyse this equation in the general case. However, one can analyse it in sufficient detail in certain specific cases, which give knowledge of basic features of the problem.

Consider the fundamental mode and ignore, first, both the gain and its saturation. In this case, we have a conventional linear problem. Let

$$\varphi=\int_0^l\left[-q_0-k_0^2\delta\varepsilon(x,0\right]^{1/2}\mathrm{d}x,\qquad \varphi\gg 1\ . \qquad (8)$$

Here, q_0 is the eigenvalue of the fundamental mode, which is assumed to be negative and give a positive radicand in Eqn (8). In this case, radiation undergoes strong reflection near the middle of the waveguide and, as noted above, is divided into two fluxes, which are coupled very weakly. The decay of the field toward the waveguide centre, as well as the difference of eigenvalues q for the zero and first modes, is given, in order of magnitude, by the factor $\exp(-\varphi)$.

However, the fields of both modes under consideration (with indices $n=0$ and 1) remain symmetric about the middle waveguide plane. For both modes, $f^2(x)$ are even functions of x. This directly follows from the linearity of the problem. Indeed, if $f(x)$ were an asymmetric solution of the boundary problem (6), (7), then, according to the superposition principle for the solutions of linear problems, $f(x)+f(-x)$ and $f(x)-f(-x)$ would be two linearly independent degenerate solutions of the initial problem, which is impossible.

Assume that the saturated gain is small and that the quantity $g(x)/k_0(1+|f(x)|^2 I^{-1})$ [see Eqn (2)] may be treated as a small correction to the real part of the dielectric constant $\mathrm{Re}\,\delta\varepsilon$. In this case, one may use the perturbation theory. It is convenient to carry out further analysis in a more compact general form and subsequently come back to the specific problem under consideration.

Let $\hat{L}$ be a linear Hermitian operator with a discrete spectrum in the space H, and $\phi(f)$ a nonlinear operator in the same space ($f\in H$). The nonlinear problem on eigenvalue perturbations is formulated in the following way. One should find approximate solutions of the equation

$$\hat{L}f+\varepsilon\phi(f)=\lambda f \qquad (9)$$

for $\varepsilon\to 0$. In addition to the vector f, one calculates λ in a similar linear problem. It will be clear that, in the nonlinear problem, λ represents a parameter that should be determined from certain additional conditions.

As in the linear case, we will seek the solution of the problem in the form

$$f=f_0+\varepsilon f_1+\ldots\ ,\qquad \lambda=\lambda_0+\varepsilon\lambda_1+\ldots\ . \qquad (10)$$

Here, f_0 is one of the eigenvectors of the operator $\hat{L}$, and λ_0 is the corresponding eigenvalue. The case of degenerate eigenvalues is not a priori excluded. Substituting Eqn (10) in Eqn (9), we have in the first order with respect to ε

$$\left(\hat{L}-\lambda_0\right)f_1=\lambda_1 f_0-\phi(f_0)\ . \qquad (11)$$

The solvability of this equation requires that the vector in its right-hand side be orthogonal to the subspace of eigenvectors corresponding to the eigenvalue λ_0. For simplicity, consider the case of doubly degenerate λ_0 because it will be needed in the further analysis. Let the eigenvector space have the orthonormal basis $\{f_\mathrm{e},\,f_\mathrm{o}\}$ (the choice of notations will be explained below). We also assume that

$$f_0=c_\mathrm{e}f_\mathrm{e}+c_\mathrm{o}f_\mathrm{o}\ .$$

The orthogonality condition takes the form

$$\lambda_1 c_\mathrm{e}=\langle f_\mathrm{e}|\phi(c_\mathrm{e}f_\mathrm{e}+c_\mathrm{o}f_\mathrm{o})\rangle\ ,$$
$$\lambda_1 c_\mathrm{o}=\langle f_\mathrm{o}|\phi(c_\mathrm{e}f_\mathrm{e}+c_\mathrm{o}f_\mathrm{o})\rangle\ , \qquad (12)$$

where the angle brackets denote the scalar product in the space H. Because now ϕ is a nonlinear operator, the last equations are also nonlinear and have a solution for different, generally speaking, arbitrary λ_1. Thus, the nonlinear problem (9) is radically different from the corresponding linear problem. Now, the eigenvalue is not determined by the problem itself, but it should be set as an additional condition. In the problem of waveguide laser modes, this condition represents the equality of saturated gain for a round cavity trip to the cavity loss. If the modes of a nonlinear waveguide are considered as such, one should simply assume $\mathrm{Im}q$ to be specified.

Now, let us turn back to the waveguide mode problem. We make one more simplification and assume that the gain itself and the degree of saturation are small. Then

$$\frac{1}{1+|f|^2/I} \simeq 1 - \frac{|f|^2}{I} \, .$$

The space $H = L^2(-l, l)$ is a conventional Hilbert space of functions on the interval $(-l, l)$. In this case,

$$\hat{L} = \frac{\mathrm{d}^2}{\mathrm{d}x^2} + \alpha(x), \quad \phi(f) = \mathrm{i}\beta(x)\left(1 - \frac{|f(x)|^2}{I}\right) f(x) \, ,$$

where $\alpha(x) = k_0^2 \mathrm{Re}\delta\varepsilon(x)$, $\beta(x) = k_0^2 \mathrm{Im}\delta\varepsilon(x)$, and it is assumed that

$$\max_x |\alpha(x)| \gg \max_x |\beta(x)| \, .$$

As noted above, in a symmetric cavity, modes of the linear approximation should be symmetric. In particular, the fundamental (zero) mode f_e is even, and the next (in order of increasing eigenvalue) mode f_o is odd. For clearness, the mode indices are chosen by analogy with the notation used in spectroscopy. Because the difference of eigenvalues of the linear problem q_e and q_o exponentially decreases with increasing φ (see above), we consider the case where one may neglect this difference and assume the modes f_e and f_o to be degenerate with a rather high accuracy. In this case, Eqns (12) take the form

$$q_1 c_e = \mathrm{i}\langle f_e | \beta f_e \rangle c_e - \mathrm{i}\left\langle f_e \middle| \beta \frac{|c_e f_e + c_o f_o|^2}{I} (c_e f_e + c_o f_o) \right\rangle \, , \tag{13}$$

$$q_1 c_o = \mathrm{i}\langle f_o | \beta f_o \rangle c_o - \mathrm{i}\left\langle f_o \middle| \beta \frac{|c_e f_e + c_o f_o|^2}{I} (c_e f_e + c_o f_o) \right\rangle . \tag{14}$$

Here, we took into account that α, β, and f_e are even functions of x, whereas f_o is an odd function.

Further, in the same degree as q_e and q_o may be assumed identical, we neglect the difference of the functions $f_e^2(x)$ and $f_o^2(x)$ (one may assume that both functions are real because they are eigenfunctions of the real Hermitian operator $\hat{L}$). Formally analysing system of Eqns (13), (14) as a linear system whose coefficients, however, depend on c_e and c_o, one can easily see that q_1 should be purely imaginary, and the coefficients c_e and c_o themselves may be assumed real (in actuality, they are determined within an arbitrary common phase factor).

Taking everything said above into account, one can write the system of equations under consideration in the form

$$c_e^3 + 3c_e c_o^2 = r c_e \, , \qquad c_o^3 + 3c_o c_e^2 = r c_o \, . \tag{15}$$

Here, $r = (\langle f | \beta f \rangle + \mathrm{i}q_1)/\langle f^2 | \beta f^2 \rangle$, and f means any of the functions f_e and f_o. The parameter r, as mentioned above, should be assumed to be fixed and dependent on external conditions. System of Eqns (15) has the following set of solutions: the solution for the fundamental even mode in the form

$$c_o = 0 \, , \quad c_e^2 = r \, ,$$

the solution for the first odd mode in the form,

$$c_e = 0 \, , \quad c_o^2 = r$$

and a new solution

$$c_e^2 = c_o^2 = r/4 \, .$$

In the latter case, $c_e = \pm c_o$, and therefore we have two more modes. One of them is almost completely concentrated in the region $x < 0$, and the other one is concentrated in the region $x > 0$. These two modes are asymmetric.

In the above example, we considered perturbations of a virtually degenerate pair of modes. Note that numerical simulation of the problem under conditions close to those used in Ref. [1] made it possible to observe the violation of symmetry for modes that are far from degeneracy. Fig. 1 presents one of such modes (solid curve) and the symmetric mode (dashed curve), which remained in addition to the first one. The calculation was made for a pressure of 100 Torr (133 mbar) in the gas mixture Ar : He : Xe = 59.5 : 40 : 0.5, with a temperature difference of 300 K between the wall and the waveguide center.

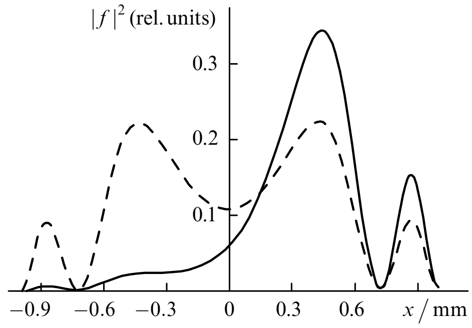

Figure 1. Spontaneous violation of symmetry of waveguide modes.

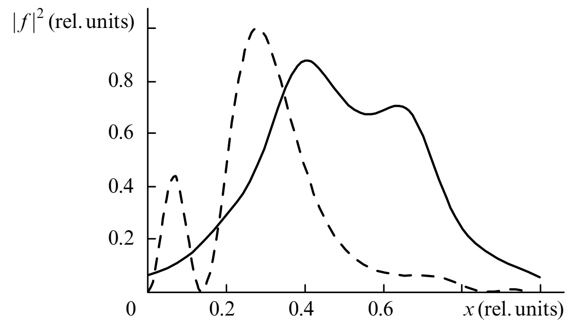

Figure 2. Comparison of one of the theoretically obtained asymmetric modes (dashed curve) with the experimental data.

The asymmetry of the radiation field in a laser with the lens effect was observed experimentally. In Fig. 2, the same asymmetric mode (dashed curve) is compared with the experimental data [4]. Here, the waveguide is 2 mm thick, as before (now the coordinate x is given in relative units). The measurements were made at a pressure of 150 mbar for the aforementioned composition of a gas mixture. Most likely, lasing in this experiment was multimode.

4. Conclusions

Thus, the optical nonlinearity of an active medium in the presence of a negative gas lens, even in an ideally symmetric waveguide, is able to cause the formation of asymmetric modes, with symmetric modes being retained. In addition to the spontaneous violation of the waveguide symmetry, which was described above, the violation of the field symmetry can be caused by an inevitable imperfection of the waveguide construction. This asymmetry is of particular importance in the case of nearly degenerate pairs of modes considered above. In any case, the appearance of asymmetric modes can cause a decrease in coherence of laser radiation because the smaller the spatial overlap of modes forming laser radiation, the lower the mutual coherence of the radiation field in two parts of a waveguide (at $x < 0$ and $x > 0$).

Acknowledgements. The authors thank V N Ochkin and A A Kuznetsov for discussion of the results. This work was partially supported by the Fundamental Spectroscopy Subprogram, the Russian Foundation for Basic Research (Project No. 99-02-16339) and the NWO grant (Netherlands).

References

1. Ilyukhin B I, Ochkin V N, et al. Kvantovaya Elektron. (Moscow) **25** 512 (1998) [Quantum Electron. **28** 497 (1998)]
2. Svelto O Principles of Lasers (New York: Plenum, 1982)
3. Kuznetsov A A, Ochkin V N, Rubin P L, Blok F, Witteman W J, J. Rus. Laser Research **20** 386 (1999)
4. Blok F J, Kochetov I V, Napartovich A P, Ochkin V N, Peters P J M, Starostin S A, Udalov Y B, Witteman W J Proc. IEEE (EOS Symposium, Enschede, 1997) p. 157