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Babinet principle and diffraction losses in laser resonators

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Abstract. A simple analytical technique, based on the Babinet principle, for calculating low diffraction losses of different kinds in stable resonators is described. The technique was verified by comparison with the known numerical and analytical calculations of the losses in specific diffraction problems.

1. Introduction

Diffraction losses in a laser resonator are frequently a constituent of its internal losses. These losses arise from various perturbations inside the resonator: openings in the mirrors, 'scrapers' (mobile mirrors intended for controllable radiation extraction from the outer beam area), various diaphragms, polarisers, etc. The losses due to finite mirror apertures in open laser resonators also belong to the category of diffraction losses.

Resonators with low diffraction losses hold the greatest practical interest. It is widely believed that there is no way to adequately calculate these losses by a simple perturbation technique, because the inaccuracy of their determination allegedly proves to be of the order of the losses themselves. For this reason, it is standard practice to calculate diffraction losses employing rather complex and cumbersome methods of solution of integral equations. The aim of this paper is to show that the simplest perturbation technique introduces, when applied correctly to stable resonators, an error much smaller than the losses to be determined [1].

2. Formulation of the technique

We will investigate a single-mode laser operation by one of the lowest modes of an open or waveguide resonator. Consider perturbations having a characteristic linear dimension δ which is small compared to the gradient dimension of the operating mode a and yet large compared to the wavelength λ. Because of the smoothness of the field distribution in the mode, this requirement means, as a rule (exceptions to this rule are described below), the smallness of the perturbation itself. The unperturbed mode, i.e., the resonator mode in the absence of perturbations, will be adopted as the zero-order approximation.

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This mode is usually well known, and in the first-order approximation, the perturbation-induced losses may be represented as a sum of two terms. The first term is 'geometric' losses c_{g}, which represent the fraction of radiation of the unperturbed mode that falls within the perturbation cross section:

$$c_{\mathrm{g}} = \frac{\int_{S_{\mathrm{p}}} I(x, y)\mathrm{d}x\mathrm{d}y}{\int_{S_{\mathrm{m}}} I(x, y)\mathrm{d}x\mathrm{d}y}, \qquad (1)$$

where $I(x, y)$ is the unperturbed mode intensity; S_{p} and S_{m} are the respective cross sections of the perturbation and the mode, respectively; and x and y are lateral coordinates. Apart from geometric losses, additional diffraction losses arising from the scattering of the remaining part of radiation should be taken into account.

The perturbation cross section and the remaining part of the beam section are mutually complementary screens. According to the Babinet principle [2], the fraction of scattered radiation will therefore be equal to the fraction of radiation intercepted by the perturbation cross section, i.e., to geometric losses, while the angular distribution of this radiation will be the same as for the geometric losses.

Therefore, the angular width $\sim\lambda/\delta$ of the scattered radiation will be much greater than the maximum characteristic radiation divergence $\sim\lambda/a$ of the operating mode. This implies that virtually all the scattered radiation escapes from the mode and is lost. As a result, the total loss by a small perturbation is

$$c = 1 - (1 - c_{\mathrm{g}})^2 \approx 2c_{\mathrm{g}}. \qquad (2)$$

3. Verification of the technique

Let us verify the technique and determine the range of its applicability taking advantage of the well-known special cases where the diffraction problem was solved exactly with the aid of integral equations. Consider the perturbations of two types, whose special cases have such solutions. First, this is a perturbation arising from the cut-off of the outer field area due to a limited mirror aperture of an arbitrary shape, or conventional diffraction losses of an open resonator. As the second example, we take a perturbation caused by the presence of some coupling apertures in the resonator mirrors. Mirrors of this type are quite often optimal for optically pumped lasers [3], electric-discharge DCN lasers operating in the specific mode of mutual amplification of different transverse modes [4], submillimeter lasers with a low-gain active medium [5], free-electron lasers, etc.

3.1. Diffraction losses on the outer aperture of the mirrors of an open resonator

Consider the TEM_{00} mode of a symmetric open resonator with large spherical mirrors. This mode is known to possess a Gaussian radial intensity distribution [6]

$$I = I_0 \exp\left[-(r/a)^2\right],\ a = (l\lambda/2\pi)^{1/2}\left(1-g^2\right)^{-1/4},$$

where l is the resonator length; $g = 1 - l/R$; and R is the radius of curvature of the mirrors. The geometric losses for round mirrors of a finite radius r_m have the form

$$c_g = \exp\left[-2\pi N_m\left(1-g^2\right)^{1/2}\right], \tag{3}$$

where $N_m = r_m^2/\lambda l$ is the Fresnel number for the resonator mirrors. The ratio of the characteristic width Δr of the field ring behind the mirror aperture to the characteristic diameter of the TEM_{00} mode is

$$\frac{\Delta r}{2a} = \left[\pi(\ln 2)N_m\left(1-g^2\right)^{1/2}\right]^{1/2} \times\left\{1-\left[1-\frac{\ln 2}{2\pi N_m\left(1-g^2\right)^{1/2}}\right]^{1/2}\right\}, \tag{4}$$

where Δr is defined by the equality $I(r_m + \Delta r) = e^{-1} I(r_m)$. The requirement that the losses be low $2c_g < 0.1$ leads, in accordance with expression (3), to the inequality $N_m(1-g^2)^{1/2} > 0.5$. Substituting this inequality in formula (4) gives $\Delta r/2a < 0.1$, i.e., the width of the ring will be far less than the characteristic dimension of the mode.

By applying the Babinet principle to the circular perturbation under study, we obtain that for low losses the radiation scattered at the edge of the mirrors is virtually not 'captured' by the operating TEM_{00} mode and is lost. The total loss c_a at this perturbation may be calculated by formula (2), in which we should substitute the geometric losses from expression (3).

Fig. 1 compares the losses calculated by formulas (2) and (3) with the data of 'classical' paper [7], in which these losses were calculated by numerically solving the integral equation. A good agreement is observed for all the plots of the losses, with the exception of the curves for a confocal ($g = 0$) and plane-parallel (Fabry–Perot) ($g = 1$) resonators.

The resonators of two last types are at the stability boundary of the known g_1, g_2-diagram. For them, the perturbation under consideration cannot be treated as small (for a plane-parallel resonator, by definition). However, these resonators find limited use in lasers because of their well-known disadvantages. Resonators with a parameter $g = 0.5 - 0.9$ are used much more widely. For them, the average departure of the curves calculated by formulas (2) and (3) from the accurate losses is no greater than 15% (Fig. 1).

Also plotted in Fig. 1 are the results of calculation of these losses by the familiar analytical formula from Ref. [8] [formula (32)], which was derived using extremely complicated and cumbersome transformations of the integral equation and finding its approximate solution. The losses calculated by formulas (2) and (3) virtually coincide with the data of Ref. [8] for all values of the parameters g and N_m. It is interesting to note that both techniques give the same deviations from the exact solution of Ref. [7] for a confocal resonator ($g = 0$) and upon an approach to the plane-parallel resonator ($g = 0.99$). This suggests that these techniques describe, despite a radical difference in approaches, virtually similar approximate models of the problem under study.

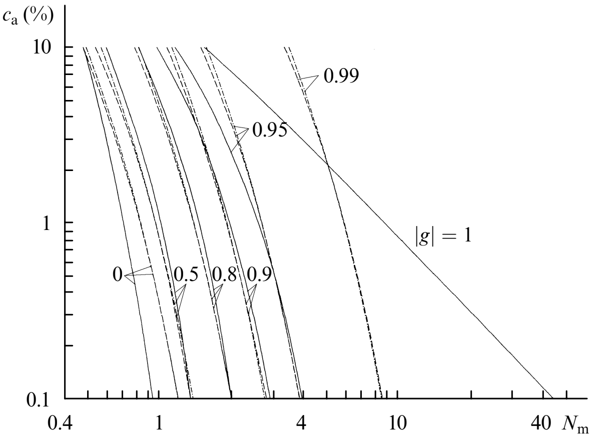

Figure 1. Single-pass aperture losses of the TEM_{00} mode of a symmetric resonator with round mirrors as functions of the aperture Fresnel number for different values of the parameter g: the data of Ref. [7] (solid lines), calculation by formulas (2) and (3) (dashed lines), and the data of Ref. [8] (dash-dotted lines).

3.2. Diffraction losses on a coupling aperture

The diffraction losses for the special case of a confocal resonator with round coupling apertures at the centres of its round mirrors were calculated by McCumber [9] employing a rather complex method of solution of the integral equation. Fig. 2 shows the total diffraction losses c_s, borrowed from Ref. [9], as functions of the Fresnel number $N_m = r_m^2/\lambda l$ of the resonator mirrors for different Fresnel numbers $N_0 = r_0^2/\lambda l$ (r_0 is the aperture radius) of the aperture for the TEM_{00}, TEM_{01}, and TEM_{10} modes. The diffraction losses associated with the coupling aperture correspond to the regions in Fig. 2 where the curves flatten

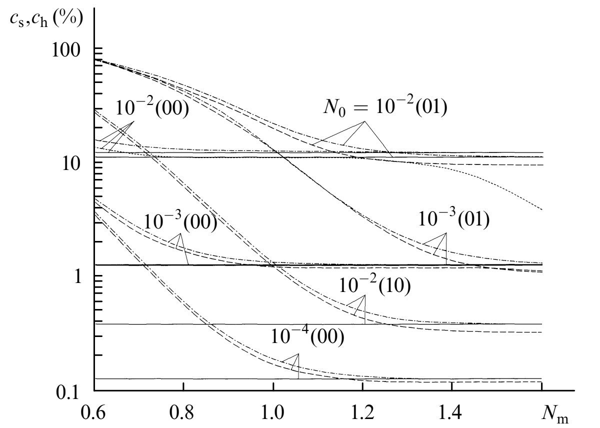

Figure 2. Total single-pass diffraction losses c_s of the resonator borrowed from Ref. [9] (dashed lines), losses c_h on the coupling aperture calculated by formulas (2) and (5)–(7) (solid lines), and total losses $c_s = 1 - (1 - c_a)(1 - c_h)$, where c_a stands for the aperture losses borrowed from Ref. [9] (dash-dotted lines), as functions of the aperture Fresnel number for different Fresnel numbers of the coupling aperture N_0 and different TEM_{ij} modes (the numbers in parentheses denote the mode indices).

out, because here these losses significantly exceed the losses c_a on the outer aperture of the mirrors. Also shown in Fig. 2 are the losses $c_h = 1 - (1 - c_g)^2$ on the coupling aperture calculated by the simplest formulas for the geometric losses c_g^{ij} for the TEM_{ij} modes:

$$c_g^{00} = 2\pi N_0, \tag{5}$$

$$c_g^{01} = 1 - \left[1 + (2\pi N_0)^2\right] \exp(-2\pi N_0), \tag{6}$$

$$c_g^{10} = 1 - (1 + 2\pi N_0) \exp(-2\pi N_0). \tag{7}$$

In the notation of the modes, recourse is made to the same order of indices as in Ref. [9], i.e., the azimuth index comes first and the radial one comes second.

One can see that the calculations by the technique under discussion agree nicely with the data of Ref. [9], with the exception of the range of excessively high N_m (see Fig. 2) for high N_0 for the TEM_{00} mode. This discrepancy is inherent in the confocal resonator only and is explained by its boundary 'instability' (see Section 3.1). For resonators away from the stability boundary ($g = 0.4 - 0.95$), this discrepancy should not take place. The losses calculated by the technique outlined above agree closely with the data of Ref. [9] also for other combinations of the parameters (the mode, N_0) not given in Fig. 2. For instance, for $N_0 = 10^{-3}$, we obtain from formulas (2) and (7) that $c_h^{10} = 0.0039\%$, which is close to $c_h^{10} = 0.0036\%$ from Ref. [9]. For $N_0 = 10^{-4}$, expressions (2) and (6) give $c_h^{01} = 0.126\%$, which is close to $c_h^{01} = 0.107\%$ from Ref. [9]. By and large, the results of calculations employing formulas (2) and (5)–(7) depart from the data of Ref. [9] by 6–10% for the TEM_{00} mode, by 10–15% for the TEM_{10} mode, and 12–15% for the TEM_{01} mode. As would be expected, this departure increases slightly as the characteristic gradient scale dimension of the mode decreases.

The dash-dotted lines in Fig. 2 show the total diffraction losses $c_s = 1 - (1 - c_a)(1 - c_h) \approx c_a + c_h$, which include the diffraction losses c_a on the outer aperture of the mirrors in the absence of coupling apertures borrowed from Ref. [9] (the method under discussion is inappropriate for calculating the aperture losses in a confocal resonator; see Section 3.1) and the diffraction losses c_h on the coupling aperture, which were calculated using formulas (2) and (5)–(7). The proximity of these curves to the curves from Ref. [9] suggest that the losses initiated by a small perturbation possess, as would be expected, the property of additivity.

Note that the analytical formula for calculating the losses of the TEM_{00} mode arising from round coupling apertures at the centre of round mirrors of a confocal resonator, coinciding with expressions (2), (5), can be obtained by elementary rearrangements of formula (26b) from Ref. [9].

4. Conclusions

The technique for elementary calculation of low (less than 10 % in one pass; Sections 3.1 and 3.2) diffraction losses in stable laser resonators described above is based on general principles. First, this allows a radical simplification of the calculations. Second, the method can be easily applied to different, yet unsolved diffraction problems. The author has repeatedly employed this technique to calculate the losses due to different perturbations of the outer beam area and the losses due to a coupling aperture in the mirrors of gas waveguide lasers [4, 5].

The technique was also used to calculate the resonators of a high-power submillimetre free-electron laser and a compact free-electron laser with a planar waveguide [10, 11]. In all the cases, the calculation was carried out employing general formulas (1) and (2). The technique can also be easily applied to various asymmetric problems that cannot be solved by the methods developed in Refs [8, 9].

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