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Cooperative interaction of dressed atoms with a quantised mode of the electromagnetic field

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Abstract. The nonlinear dynamics of an open quantum system containing an arbitrary number of two-level atoms coupled with a classical polychromatic electromagnetic field and a quantised mode of the electromagnetic field is studied. Two particular cases of the elastic and inelastic interactions are considered. In the first case, the quantised mode is resonant with the transition between the quasi-energy levels corresponding to the same quasi-energy state, while in the second case, the levels involved in the interaction correspond to different quasi-energy states. For the elastic interaction, which can appear only in open quantum systems, an analytic solution of the Heisenberg equations is obtained. The time dependences of the population of quasi-energy states, the number of photons in the quantised mode, and photon statistics are numerically analysed for the inelastic interaction and at the crossing point of quasi-levels, when both types of the interaction are simultaneously present.

The number of papers devoted to the fundamental physical models of quantum optics, the Jaynes – Cummings one-atom model [1] and the Tavis – Cummings model [2] describing the cooperative interaction of the polyatomic system with electromagnetic radiation, rapidly increases in recent years. Various modifications of these models have been proposed that consider several modes of the electromagnetic field, multilevel atoms, and multiphoton transitions (see, for example, Refs [3 – 7] and references therein).

Interest in physical models of this type is caused by a rapid progress in experimental quantum optics, in particular, by creation of a one-atom maser [8, 9] and laser [10] using beams of cooled atoms and high-Q superconducting cavities. In experiments with a one-atom maser, one of the most interesting effects predicted by the Jaynes – Cummings model was observed – collapses and revivals of oscillations of the population inversion of atoms [9]. Various states of a quantised mode of the electromagnetic field were investigated, including squeezed states. A detailed review of theoretical and experimental studies in this field is presented in book [11].

In Refs. [12, 13], an open modification of the Jaynes – Cummings model was suggested that considers a dressed atom and a quantised mode of the electromagnetic field. By a dressed atom is meant a two-level atom interacting with a classical electromagnetic field with the equidistant spectrum. The absorption and luminescence spectra of such an atom are determined by the quasi-energy level diagram. The characteristic feature of the open model is the possibility to control the positions of quasi-energy levels by varying parameters of a classical field. In particular, in the region of crossing and anti-crossing of these levels, the model dynamics exhibits a number of specific features [14]. Another feature of this model is the possibility of the elastic interaction of a dressed atom with a quantised mode when the populations of quasi-energy levels do not change [12, 14].

Both these features are also inherent in an open polyatomic model, which is considered in this paper.

It was shown in Refs [13, 15] that the spectroscopic properties of a dressed atom, i.e., the atom located in a polychromatic classical field with the equidistant spectrum, can be described by the diagram of quasi-energy states (QESs) and the spectrum of the quasi-energy levels. Each QES corresponds to an infinite sequence of the equidistant quasi-energy levels separated by the energy gap $\hbar\omega'$, where ω' is the difference of the frequencies of adjacent spectral components of a classical field. When the parameters of a classical field are changed, the quasi-energy levels are shifted and they may cross with each other.

When a dressed atom is not subjected to any external perturbation, the operators of transitions between QESs are integrals of motion in the Heisenberg representation. The interaction between a dressed atom and a quantised mode of the electromagnetic field results in the time dependence of these operators. In this case, the timed dependences of QES populations and the number of photons in the quantised mode substantially depend on the type of quasi-energy levels coupled by the transitions that are resonant with the quantised mode. If the quasi-energy levels correspond to different QESs, the interaction dynamics is qualitatively the same as that in the absence of a classical field. Emission and absorption of photons is accompanied by the corresponding change in the QES populations. In this case, a term ‘inelastic’ interaction can be used.

However, if the involved quasi-energy levels correspond to the same QES, the QES populations do not change upon absorption and emission of photons. The interaction of this type, which can be called ‘elastic’, is possible only for open quantum systems, such as a dressed atom. In the region of quasi-energy level crossing (approach), these two types of interaction interfere, which can result in a number of interesting phenomena, for example, the generation of squeezed quantum states [13].

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Unlike previous papers, in this paper, a system of many dressed atoms is considered that interact with a quantised mode of the electromagnetic field. It is assumed that the atoms have two levels and are located in a region whose size is small compared to the wavelength [16]. A general case is considered, when the effective Hamiltonian may contain the terms describing both the inelastic and elastic interaction.

The expression for the effective Hamiltonian can be obtained by summing the corresponding expression for the one-atom Hamiltonian from Ref. [13] over the atoms of the system under study. This expression can be conveniently represented in terms of cooperative atomic operators

$$\hat{J}_+ = \sum_{k=1}^{N} \hat{c}_k^+,\ \hat{J}_- = \sum_{k=1}^{N} \hat{c}_k,\ \hat{J}_3 = \frac{1}{2}\sum_{k=1}^{N}\left(\hat{c}_k^+\hat{c}_k - \hat{c}_k\hat{c}_k^+\right) \quad (1)$$

in the form

$$\hat{H}_{\rm ef} = {\rm i}\big[\alpha\hat{a}\hat{J}_+ - \alpha^*\hat{a}^+\hat{J}_- + \gamma\hat{a}^+\hat{J}_+ - \gamma^*\hat{a}\hat{J}_- - (\beta^*\hat{a}^+ - \beta\hat{a})\hat{J}_3\big]. \quad (2)$$

Here, k is the number of an atom; N is the number of atoms; $\hat{a}$ and $\hat{a}^+$ are the annihilation and creation operators for photons of the quantised mode; $\hat{c}_k$ and $\hat{c}_k^+$ are operators of the transition of the kth atom from the quasi-energy state $|\theta_1\rangle_k$ to the state $|\theta_0\rangle_k$ and back (the 'plus' sign means Hermitian conjugation); α and γ are constants of the 'elastic' interaction; β is a constant of 'inelastic' interaction. All the notations correspond to Ref. [13]. The constants of interaction of different atoms with a quantised mode are assumed equal to each other.

By introducing the time evolution operator

$$\hat{u}(\tau) = \exp\left(-{\rm i}\hat{H}_{\rm ef}\tau\right), \quad (3)$$

we consider the time dependence of the following average quantities, which characterise a coupled system of dressed atoms and a quantised mode: the average number of photons in the quantised mode

$$\bar{n}(\tau) = \langle\hat{a}^+(\tau)\hat{a}(\tau)\rangle = \langle\hat{u}^+(\tau)\hat{a}^+\hat{a}\hat{u}(\tau)\rangle, \quad (4)$$

the average square of the number of photons

$$\overline{n^2}(\tau) = \langle\hat{a}^+(\tau)\hat{a}(\tau)\hat{a}^+(\tau)\hat{a}(\tau)\rangle = \langle\hat{u}^+(\tau)\hat{a}^+\hat{a}\hat{a}^+\hat{a}\hat{u}(\tau)\rangle, \quad (5)$$

the Fano factor

$$F(\tau) = \frac{\overline{n^2}(\tau) - [\bar{n}(\tau)]^2}{\bar{n}(\tau)} = \frac{\Delta n^2(\tau)}{\bar{n}(\tau)}, \quad (6)$$

and populations of the quasi-energy states $|\theta_0\rangle$ and $|\theta_1\rangle$

$$N_0(\tau) = \left\langle\sum_{k=1}^{N}\hat{c}_k(\tau)\hat{c}_k^+(\tau)\right\rangle = \frac{N}{2} - \langle\hat{J}_3(\tau)\rangle = \frac{N}{2} - \langle\hat{u}^+(\tau)\hat{J}_3\hat{u}(\tau)\rangle,$$

$$N_1(\tau) = \left\langle\sum_{k=1}^{N}\hat{c}_k^+(\tau)\hat{c}_k(\tau)\right\rangle = \frac{N}{2} + \langle\hat{J}_3(\tau)\rangle = \frac{N}{2} + \langle\hat{u}^+(\tau)\hat{J}_3\hat{u}(\tau)\rangle. \quad (7)$$

The angle brackets in Eqns (3)–(7) denote quantum-mechanical averaging over the initial quantum state of a coupled system of dressed atoms and a quantised mode; $\tau = \varkappa_{\rm c}\omega' t$ is the dimensionless time; and $\varkappa_{\rm c}$ is a constant of the interaction between an atom and the quantised mode [13].

Below, we will consider only those quantum states of dressed atoms that are symmetrical relative to the atom interchange. These symmetric states can be represented in the form (cf. [16, 17])

$$|m\rangle_{\rm s} \equiv \left[\frac{(N/2+m)!(N/2-m)!}{n!}\right]^{1/2} \times\sum_{\sigma}|\theta_1\rangle_{\sigma_1}\ldots|\theta_1\rangle_{\sigma_{N/2+m}}|\theta_0\rangle_{\sigma_{N/2+m+1}}\ldots|\theta_0\rangle_{\sigma_N}, \quad (8)$$

where the index m runs all the values from $-N/2$ to $+N/2$ with a unit step. Summation in Eqn (8) is performed over all permutations $\sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_N\}$ of numbers $1, 2, \ldots, N$. In the Hilbert space of dressed atoms and a quantised mode, the basis of the states

$$|m\rangle_{\rm s}|n\rangle,\ m = -N/2,\ -N/2+1, \ldots, N/2,\ n = 0,\ 1,\ 2\ldots, \quad (9)$$

can be chosen, where $|n\rangle$ are the Fock states of a quantised mode containing n photons.

The right-hand sides of equations (4)–(7) can be calculated analytically in the case of purely 'elastic' interaction when we have $\alpha = \gamma = 0$, $\beta \neq 0$ in expression (2) for the effective Hamiltonian. Consider a quantised mode excited initially to the coherent state $|v\rangle$ (so that $\hat{a}|v\rangle = v|v\rangle$) and a system of dressed atoms excited to the symmetric state $|m\rangle_{\rm s}$, so that the wave function of the total system has the form $|m\rangle_{\rm s}|v\rangle$.

The operators under the sign of quantum-mechanical averaging in expressions (4), (5), and (7) can be rewritten in the normally ordered form by using known methods of operator algebra [18]. As a result, we obtain the following relations

$$\bar{n}(\tau) = |v - 2m\beta^*\tau|^2, \quad (10)$$

$$\overline{\Delta n^2}(\tau) = \bar{n}(\tau), \quad F(\tau) = 1, \quad (11)$$

$$N_0(\tau) = \frac{N}{2} - m, \quad N_1(\tau) = \frac{N}{2} + m. \quad (12)$$

One can see from the solution obtained that photons in the quantised mode are always described by the Poisson statistics, while the QES populations do not change with time.

In the general case, the dynamics of a polyatomic model can be studied by numerical methods by using the matrix representation of the effective Hamiltonian in the basis of states (9). As an example, Fig. 1 shows the time dependences of the number $\langle n\rangle$ of photons in the quantised mode (solid curves), the population N_1 of the quasi-energy states $|\theta_1\rangle$ (the dashed curve in Fig. 1a), and the Fano factor F (the dashed line in Fig. 1b) for a system of dressed atoms excited initially to the $|\theta_1\rangle$ state and for a quantised mode excited initially to the coherent $|v\rangle$ state.

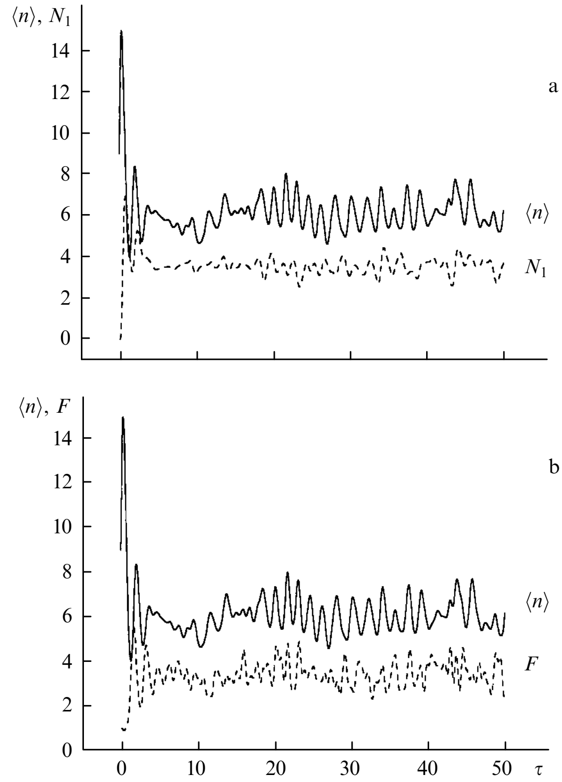

Figure 1.

One can see from Fig. 1 that after a few first oscillations, the dependence on the initial conditions virtually vanishes and the system transfers to the state that can be treated as equilibrium. In this state, the relative fluctuations of the number of photons and populations of quasi-energy levels are small and irregular.

The use of dressed atoms, whose quasi-energy levels can be controlled by an external classical field, represents a natural development of the modern technique of atomic beam masers and lasers. By changing the amplitude of the classical field, one can produce lasing on a variety of quantum transitions between quasi-energy levels and observe the resonances caused by crossing and anti-crossing of these levels. If the upper and lower quasi-energy levels correspond to the same quasi-energy state, the interaction of a quantised mode with a dressed atom will be elastic. In this case, as follows from relations (10) – (12), the energy efficiently transfers from a classical field to the quantised mode, the populations of quasi-energy states being invariable. Depending on the atomic beam density, one or several atoms can interact simultaneously with the quantised mode. In the latter case, the efficiency of energy transfer can substantially increase.

To describe adequately different experiments with atomic beam masers and lasers, it is necessary to develop further the theory of an open polyatomic model. In particular, one should consider relaxation both of the atoms and a quantised mode, analyse excitation fluctuations, which depend on the atomic statistics in a beam, study the squeezing of quadrature components of the quantised field, etc.

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