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Dynamics of injection locking in a solid-state laser with intracavity second-harmonic generation

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Abstract. The dynamics of oscillation in a solid-state laser with intracavity second-harmonic generation under the influence of an external signal at the second-harmonic frequency injected into its cavity in the presence of feedback at the double frequency is theoretically studied. Boundaries of the regions of injection locking for three stationary laser states differing in the nonlinear phase incursion caused by radiation conversion into the second harmonic are found. Relaxation oscillations in the stationary state of injection locking are studied. It is shown that the second relaxation frequency, which is related to phase perturbations of the second harmonic and perturbations of the phase difference of waves in a nonlinear crystal, is excited in a single-mode solid-state laser in addition to the fundamental frequency of relaxation oscillations. Conditions are found under which relaxation oscillations at the second relaxation frequency are excited.

1. Introduction

The physics of phenomena accompanying the injection of external radiation into a laser is well studied. These phenomena are of considerable importance for the study of fundamental problems of the nonlinear dynamics (such as dynamic chaos, locking of regular and chaotic oscillations, multistability, etc) [1 – 3] and the formation of squeezed light [3 – 6]. Injection locking is of applied interest in improving emission characteristics of high-power lasers. Here, we study previously unsolved aspects of these phenomena related to injection of optical radiation into a solid-state laser generating the second harmonic in a doubly resonant cavity (by a doubly resonant cavity is meant a cavity having a high Q factor both at the fundamental frequency ω and the double frequency 2ω).

Previous studies on frequency doubling [7 – 12] and self-doubling [13 – 16] in a cavity have been mainly performed for systems in which the second harmonic was outcoupled from a cavity upon a single round trip (feedback at the frequency 2ω was absent). In papers [17 – 20], SHG in a doubly resonant laser cavity has been theoretically studied. Such lasers have three stationary states [20], which differ in the nonlinear phase incursion associated with conversion into the second harmonic and in oscillation frequencies. Optical injection into such lasers may be useful to study nonlinear shifts of radiation frequency. Moreover, as will be shown, nonlinear frequency conversion in a cavity affects the dynamics of phase locking of radiation by an injected signal and leads to the appearance of one more frequency of relaxation oscillations in a single-mode solid-state laser.

Injection locking in a laser with intracavity SHG can be realised by two methods. In addition to the conventional method, which uses injection of an external signal at the frequency $\omega_{\rm in}$ close to the laser frequency ω ($\omega_{\rm in} \approx \omega$), one can use another method with an injection signal at the frequency $\omega_{\rm in} \approx 2\omega$. Here, we analyse the second method.

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2. System of equations and the stationary regime of injection locking

The system of rate equations describing the dynamics of injection locking in a solid-state laser with SHG in a doubly resonant cavity will be written in the form

$$\frac{{\rm d}a_1}{{\rm d}t} = \frac{a_1}{2T_{\rm c}}[k_1(N-1) - \sqrt{\varepsilon}\,a_2\sin\psi], \quad (1)$$

$$\frac{{\rm d}a_2}{{\rm d}t} = -\frac{k_2}{2T_{\rm c}}a_2 + \frac{\sqrt{\varepsilon}}{2T_{\rm c}}a_1^2\sin\psi + \frac{k_{\rm in}}{T_{\rm c}}E_{\rm in}\cos\varphi_2, \quad (2)$$

$$\frac{{\rm d}\psi}{{\rm d}t} = \frac{\sqrt{\varepsilon}}{2T_{\rm c}}\left(\frac{a_1^2}{a_2} - 2a_2\right)\cos\psi + \omega_{2{\rm c}} - 2\omega_{1{\rm c}} + \frac{k_{\rm in}E_{\rm in}}{T_{\rm c}a_2}\sin\varphi_2, \quad (3)$$

$$\frac{{\rm d}\varphi_2}{{\rm d}t} = \omega_{\rm in} - \omega_{2{\rm c}} - \frac{\sqrt{\varepsilon}}{2T_{\rm c}}\frac{a_1^2}{a_2}\cos\psi - \frac{k_{\rm in}E_{\rm in}}{T_{\rm c}a_2}\sin\varphi_2, \quad (4)$$

$$\frac{{\rm d}N}{{\rm d}t} = \frac{1}{T_1}\left[N_0 - N(1+a_1^2)\right]. \quad (5)$$

Here, $a_{1,2} = (I_{1,2}/I_{\rm s})^{1/2}$ and $\varphi_{1,2}$ are dimensionless amplitudes and phases of the intracavity fields at the fundamental and double frequencies, respectively; $I_{1,2}$ are their intensities; $I_{\rm s}$ is the saturation intensity of an active medium; $\psi = 2\varphi_1 - \varphi_2$; $E_{\rm in} = (I_{\rm in}/I_{\rm s})^{1/2}$ is the dimensionless amplitude of the injected signal; $k_{1,2}$ are linear losses in the doubly resonant cavity; $\omega_{1{\rm c}}$, and $\omega_{2{\rm c}}$ are natural frequencies of the doubly resonant cavity; $k_{\rm in}$ is the transmittance of the mirror through which an external signal is injected; $T_{\rm c}$ is the round-trip time for light travelling in the cavity; $\varepsilon = (\chi l)^2 I_{\rm s}$

is the nonlinearity parameter; χ is the coefficient of nonlinearity; l is the length of an active (nonlinear element); T_1 is the relaxation time for inverse population; N is inverse population normalised to its threshold value; and N_0 is the value of N in the absence of saturation.

The phase-matching condition in a nonlinear crystal is assumed to be fulfilled. For a single-mode solid-state laser, the relative detuning of the fundamental frequency from the gain line centre is small, and its effect will be ignored. First, we consider the stationary regime of injection locking in the specific case when the injection signal frequency $\omega_{\rm in}$ coincides with the natural frequency of the cavity for the second harmonic $\omega_{2\rm c}$, which is equal to its double frequency for fundamental radiation $2\omega_{1\rm c}$.

It follows from equations (1)–(5) that stationary solutions in this case should satisfy the conditions

$$\cos\psi = 0, \ \sin\varphi_2 = 0. \tag{6}$$

The problem may have several stationary solutions, which differ in signs of $\sin\psi$ and $\cos\varphi_2$, but only one of them may be stable, and it is given by

$$\psi = \pi/2, \ \varphi_2 = \pi. \tag{7}$$

If phase relations (7) are fulfilled, the intracavity field amplitudes are determined by the formulas

$$a_1^2 = \frac{-B + \left\{B^2 + 4[A(N_0-1) + 2k_{\rm in}E_{\rm in}/\sqrt{\varepsilon}]\right\}^{1/2}}{2},$$

$$a_2 = \sqrt{\varepsilon}\,\frac{a_1^2}{k_2} - 2\frac{E_{\rm in}k_{\rm in}}{k_2}, \tag{8}$$

where $A = k_1k_2/\varepsilon$; $B = 1 + A - 2k_{\rm in}E_{\rm in}/\sqrt{\varepsilon}$.

Let us analyse the stability of solution (7), (8). One can easily show that the characteristic equation for small perturbations is divided into two, and we obtain a cubic equation for perturbations of the variables a_1, a_2 and N relative to their stationary values (8) and a quadratic equation for perturbations of φ_2 and ψ relative to the stationary values (7). The analysis of these equations shows that the injection of an external signal produces the second (additional) frequency of relaxation oscillations in a single-mode solid-state laser with SHG in a doubly resonant cavity. The fundamental frequency of relaxation oscillations ($\omega_{\rm r1}$) is associated with perturbations of radiation intensity and population difference, whereas the additional frequency ($\omega_{\rm r2}$) is related to phase perturbations of the second harmonic and perturbations of phase difference in a nonlinear crystal.

The cubic equation determines damped perturbations at the fundamental frequency of relaxation oscillations $\omega_{\rm r1}$, and the quadratic equation gives the following expressions for the frequency $\omega_{\rm r2}$ and the damping factor (or the enhancement factor) γ of relaxation oscillations:

$$\omega_{\rm r2} = \left(\frac{k_{\rm in}E_{\rm in}\sqrt{\varepsilon}}{T_{\rm c}^2} - \gamma^2\right)^{1/2}, \ \ \gamma = \frac{\sqrt{\varepsilon}}{4T_{\rm c}}\left(\frac{a_1^2}{a_2} - 2a_2\right). \tag{9}$$

The stationary values of amplitudes a_1 and a_2 in (9) are determined by formulas (8). Note that formulas (6)–(9) describe the stationary mode of operation and its stability for any value of N_0 (for any pump). Depending on N_0, they describe a degenerate optical parametric oscillator ($N_0 = 0$), a degenerate parametric oscillator with regenerative amplification at the frequency ω ($0 < N_0 < 1$), and a laser with intracavity SHG and optical injection at the frequency 2ω ($N_0 > 1$).

Fig. 1 presents the dependences of the dimensionless output intensities $k_1a_1^2$ and $k_2a_2^2$ and the frequency of relaxation oscillations $\omega_{\rm r2}$ on the dimensionless injection signal intensity $k_{\rm in}E_{\rm in}^2$ for different N_0. Stable solutions are shown by solid curves, and unstable solutions are shown by dotted curves. The calculations were performed assuming that $\varepsilon = 5\times10^{-5}$, $T_{\rm c} = 0.2$ ns, $k_1 = 0.01$, and $k_2 = 0.01$. These parameters (except k_2) correspond to parameters of the Nd : YAG laser with SHG in a KTP crystal 5 mm long, which was used in [9–11]. Dependences in Fig. 1a correspond to a parametric oscillator ($N_0 = 0$). In this case, $k_2a_2^2$ first linearly increases with increasing $k_{\rm in}E_{\rm in}^2$ and then, above the threshold of parametric oscillation, an increase in $k_2a_2^2$ sharply slows down and the conversion of the second harmonic to the fundamental frequency causes an increase in $k_1a_1^2$. The threshold of parametric oscillation considerably lowers with increasing N_0, and when $N_0 > 1$, it is absent (parametric oscillation is excited for the amplitude $E_{\rm in}$ as small as is desired).

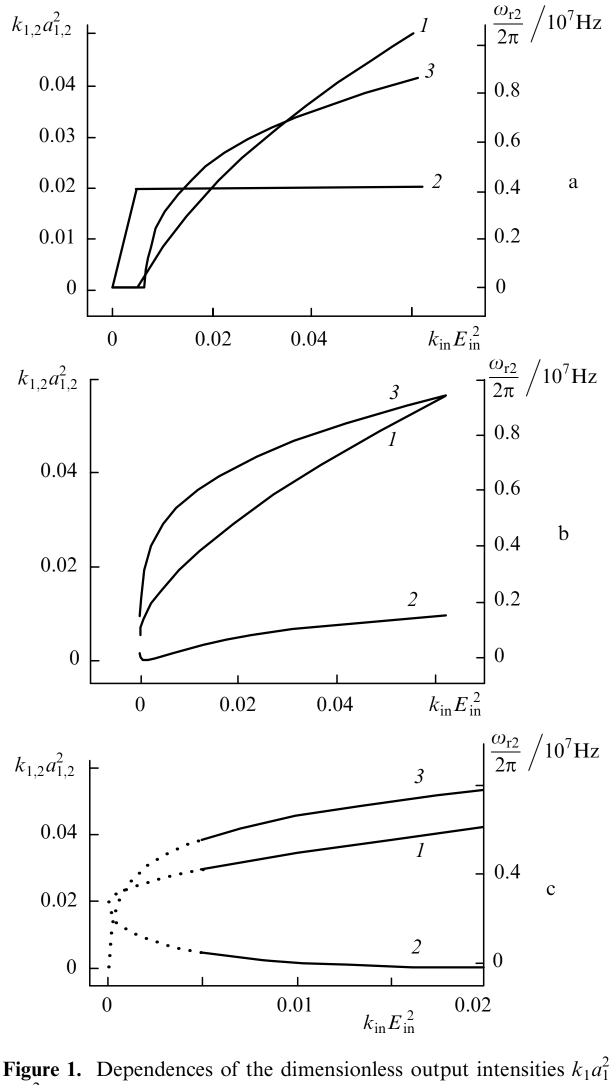

Figure 1. Dependences of the dimensionless output intensities $k_1a_1^2$ (*1*), $k_2a_2^2$ (*2*) and the frequency of relaxation oscillations $\omega_{\rm r2}$ (*3*) on the dimensionless intensity of an injected signal $k_{\rm in}E_{\rm in}^2$ for $\varepsilon = 5\times10^{-5}$, $T_{\rm c} = 0.2$ ns, $k_1 = 0.01, k_2 = 0.01$, and $N_0 = 0$ (a), 2 (b), and 6 (c). Stable solutions are shown by solid curves, and unstable solutions are shown by dotted curves.

In the case of a laser ($N_0 > 1$), the second-harmonic intensity $k_2 a_2^2$ nonmonotonically varies with increasing $k_{\text{in}} E_{\text{in}}^2$. First, it monotonically decreases down to zero and then increases (Figs 1b and 1c). The laser radiation intensity $k_1 a_1^2$ monotonically increases with increasing $k_{\text{in}} E_{\text{in}}^2$, which is explained by parametric conversion of the injected signal into laser radiation.

One can see from Fig. 1 that the frequency ω_{r2} weakly depends on the pump power. This directly follows from (9). If the decrement is small ($\gamma \ll \omega_{\text{r2}}$), formula (9) for ω_{r2} takes the form

$$\omega_{\text{r2}} = \left(k_{\text{in}} E_{\text{in}} \sqrt{\varepsilon}\right)^{1/2} / T_{\text{c}}.$$

For a specified amplitude of the injection signal, the frequency ω_{r2} for a laser ($N_0 > 1$) and for a parametric oscillator ($N_0 = 0$) has close values. For a parametric oscillator, relaxation oscillations have been studied earlier in [21–23].

The stability of the stationary regime of injection locking (7), (8) depends on the pump level. Let N_0 be written in the form $N_0 = 1 + \eta$, where η is the excess of pump power over the threshold. For η below the critical value

$$\eta_0 = \left(\frac{k_2}{2}\right)^2 \frac{2 + k_2/k_1}{\varepsilon} + \frac{k_2}{2k_1}, \tag{10}$$

the stationary solution (7), (8) is stable for any injection signal amplitude. But if $\eta > \eta_0$ and an injection signal has a low amplitude satisfying the inequality

$$k_{\text{in}} E_{\text{in}} < \frac{\sqrt{\varepsilon}(\eta - \eta_0)}{2 + k_2/k_1}, \tag{11}$$

a Hopf bifurcation arises. As a result, relaxation oscillations with the frequency ω_{r2} are self-excited, and stationary injection locking changes to the self-modulation regime.

3. Boundaries of injection locking regions

We have already noted that the analytical expressions presented above describe a particular case of injection locking. In the general case (for arbitrary detunings $\omega_{\text{in}} - \omega_{2\text{c}}$ and $\omega_{2\text{c}} - 2\omega_{1\text{c}}$), one can easily obtain from (1)–(5) the following formulas for stationary injection locking:

$$N = \frac{1 + \eta}{1 + a_1^2},$$

$$a_2 = \frac{\left[k_1^2 (N-1)^2 + T_{\text{c}}^2 (\omega_{\text{in}} - \omega_1)^2\right]^{1/2}}{\sqrt{\varepsilon}},$$

$$\cos\psi = \frac{(\omega_{\text{in}} - \omega_{1\text{c}}) T_{\text{c}}}{\sqrt{\varepsilon} a_2}, \quad \sin\psi = \frac{(N-1)k_1}{\sqrt{\varepsilon} a_2},$$

$$\cos\varphi_2 = \frac{k_2 a_2 - \sqrt{\varepsilon} a_1^2 \sin\psi}{2 k_{\text{in}} E_{\text{in}}}, \tag{12}$$

$$\sin\varphi_2 = \frac{(\omega_{\text{in}} - \omega_{2\text{c}}) T_{\text{c}} / a_2 - \sqrt{\varepsilon} a_1^2 \cos\psi / 2}{k_{\text{in}} E_{\text{in}}},$$

$$k_{\text{in}}^2 E_{\text{in}}^2 = \frac{(k_2 a_2 - \sqrt{\varepsilon} a_1^2)^2}{4} + \left[\frac{(\omega_{\text{in}} - \omega_{2\text{c}}) T_{\text{c}}}{a_2} - \frac{\sqrt{\varepsilon} a_1^2}{2}\right]^2.$$

These formulas determine parametric dependences of N, a_2, ψ and E_{in}^2 on a_1^2. Specifying the parameter a_1^2, one can calculate N, a_2, ψ and E_{in}^2 from expression (12).

Using formulas (12), we studied the region of existence of injection locking. Injection locking is possible in a finite region of frequency detunings $\omega_{\text{in}} - \omega_{2\text{c}}$, which is determined by the inequalities $\Omega_1 < \omega_{\text{in}} - \omega_{2\text{c}} < \Omega_2$, where $\Omega_{1,2}$ are the boundaries of the region of injection locking.

If an injection signal is absent and the cavity frequency detuning $\Delta = \omega_{2\text{c}} - 2\omega_{1\text{c}} = 0$, a laser with SHG in a double cavity may have three stationary states [20], which differ in nonlinear phase incursion associated with conversion into the second harmonic, and in the oscillation frequency. In the presence of an injected signal, these states are characterised by three regions of injection locking. For η below the value η_0, which is determined by formula (10), there exists only one state, which has $\cos\psi = 0$. This state has a zero nonlinear frequency shift, and its oscillation frequency coincides with the mode frequency $\omega_{1\text{c}}$ of the cavity. In this case, we have only one region of injection locking. For $\varepsilon = 5 \times 10^{-5}$, $k_1 = 0.01$, and $k_2 = 0.01$, the quantity η has the critical value $\eta_0 = 2$. The dependence of the width of the injection locking region on the dimensionless intensity of an injected signal $k_{\text{in}} E_{\text{in}}^2$ for $\eta < \eta_0$ and $\Delta = \omega_{2\text{c}} - 2\omega_{1\text{c}} = 0$ is presented in Fig. 2a.

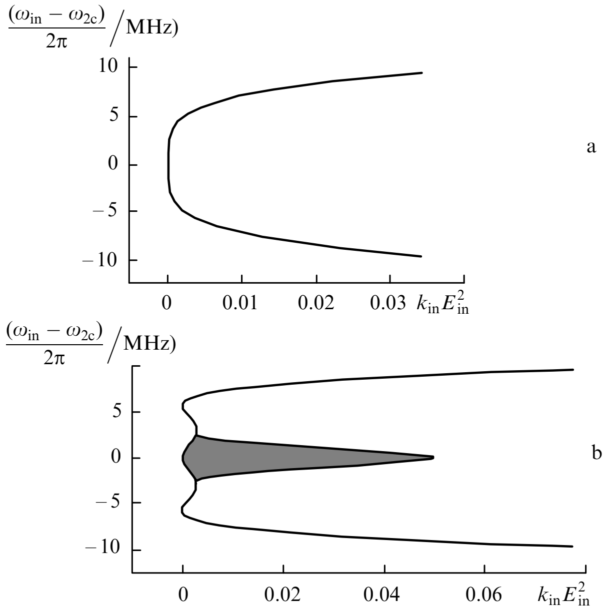

Figure 2. Dependences of the boundaries of injection locking regions on the dimensionless intensity of an injected signal $k_{\text{in}} E_{\text{in}}^2$ for $\varepsilon = 5 \times 10^{-5}$, $T_{\text{c}} = 0.2$ ns, $k_1 = 0.01, k_2 = 0.01$, and $\eta = 1$ (a) and 5 (b). The instability re

For $\eta > \eta_0$ and $E_{\text{in}}^2 = 0$, the state with $\cos\psi = 0$ becomes unstable and two other states appear. Their oscillation frequencies $\omega_{a,b}$ are shifted because of nonlinear phase shifts:

$$\omega_{a,b} = \omega_{1\text{c}} \pm \omega_{\text{nl}}, \tag{13}$$

where ω_{nl} is the nonlinear shift of the oscillation frequency, which is determined by the formula

$$\omega_{\text{nl}} = \frac{1}{T_{\text{c}}} \left[\frac{\varepsilon(\eta - k_2/2k_1)}{2 + k_2/k_1} - \left(\frac{k_2}{2}\right)^2\right]^{1/2}. \tag{14}$$

The dependence of the boundaries of injection locking regions on the dimensionless intensity of an injected signal $k_{\text{in}}E_{\text{in}}^2$ for $\eta > \eta_0$ and $\Delta = \omega_{2c} - 2\omega_{1c} = 0$ is presented in Fig. 2b. In this case, we have three injection locking regions, whose centres correspond to the oscillation frequencies ω_{1c} and $\omega_{a,b}$ in the states considered above. For $\eta > \eta_0$ and low intensities of an injected signal, an instability takes place inside the region centered at the frequency ω_{2c}, which is caused by self-excitation of relaxation oscillations at the frequency ω_{r2} (the instability region in Fig. 2b is shaded).

4. Conclusions

Our analysis of injection locking in a solid-state laser with intracavity SHG in the presence of an injected signal at the second harmonic showed the existence of three regions of injection locking, which correspond to three stationary lasing regimes. Centres of these regions correspond to laser oscillation frequencies in the absence of an injected signal, and the difference in their central frequencies directly gives the nonlinear shift of laser radiation frequency.

We showed that the second relaxation frequency may appear in a single-mode solid-state laser in addition to the fundamental frequency of relaxation oscillations, and this additional frequency is associated with phase perturbations of the second harmonic φ_2 and perturbations of phase difference ψ in a nonlinear crystal. Conditions of excitation of relaxation oscillations at the second relaxation frequency are found. For pump levels both above and below the laser threshold, the fundamental frequency output increases with increasing injected signal amplitude, which is related to the parametric conversion of the injected signal frequency into the fundamental frequency.

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