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Effect of regular wavefront perturbations on the structural transformation of laser beams

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Abstract. The effect of a regular phase modulation on the dynamics of variation in spatial characteristics of laser radiation is studied. The conditions and processes of the transformation of phase aberrations to topological wavefront perturbations resulting in the appearance of edge and screw dislocations are considered. The relationship between the caustic and dislocation elements of the laser beam is discussed. It is shown that the phase modulation leads to the formation of narrow channels with an increased intensity inside the laser beam.

Output laser beams often have a quasi-regularly modulated wavefront. In gas lasers with the circulating active medium, this modulation can be induced by the inhomogeneities caused by a periodic nozzle grating [1], the influence of the strata and domains of the gas discharge [2], the superposition of shock waves [3, 4], or other factors. In the literature, the wavefront modulation of output laser beams is mostly viewed as a factor affecting the beam divergence.

Much less attention is paid to the analysis of the transformations of the wavefront structure, the conditions for the appearance of caustic and phase dislocations in the laser beam, and the interplay between them. Such formations are detected in the emission of the lasers of all cavity types [5, 6]. In this work, we consider qualitative changes in the amplitude-phase structure of the laser beams whose wavefront initially has a smooth regular modulation.

One can get the general idea about the nature of the considered processes by considering the well-known problem [7] on the propagation of an infinite wave whose wavefront is initially harmonically modulated. The amplitude of this wave has the form

$$\Psi = \exp\left(\frac{\mathrm{i}m}{2}\sin\frac{2\pi x}{a}\right), \qquad (1)$$

where the parameter m describes the depth of the phase modulation; x is a transverse coordinate; and a is the modulation period. The field at the distance z from the initial plane can be represented as the superposition of plane waves [7]

$$\Psi = \sum_{q=-\infty}^{\infty} J_q(m/2)\exp\{\mathrm{i}[kz\cos(q\alpha) + kx\sin(q\alpha)]\}, \qquad (2)$$

where J_q is the qth-order Bessel function; $\alpha = \lambda/a$; and k is the wave number. This field is a special case of the self-reproducing fields, whose properties have found applications in the laser technology [8,9].

Using the plane-wave decomposition (2) and the ray method of Ref. [10] to calculate the field characteristics, we can determine the main features of the transformation of the initial amplitude and phase distributions. The calculations show that, even in the case of a small degree of the phase modulation and a homogeneous distribution of the intensity in the initial plane, the diffractional effects lead to a considerable spatial redistribution of the intensity.

The redistribution is most noticeable near the planes $z = (2n+1)a^2/2\lambda$ ($n = 0,1,2,...$), which are situated between the planes where, in accordance with the Talbot effect, the intensity distribution reproduces the original homogeneous one. For example, in the case of $m = 0.1$, the contrast of the intensity distribution $K = I_{\max}/I_{\min} = 1.28$, and in the case of $m = 0.5$, the contrast $K = 2.82$. When the modulation degree exceeds a certain threshold value, the structure of the wave undergoes qualitative changes. For $m > 1$, beak-shaped caustics appear in the wave [10], and for $m > \pi/2$, edge dislocations (EDs) of the wavefront begin to form near the generating caustics.

Fig. 1 shows the distributions of the amplitude A and the phase Φ in the planes $z = (2n+1)a^2/2\lambda$ for various degrees of the initial phase modulation. One can see that, when the mod-

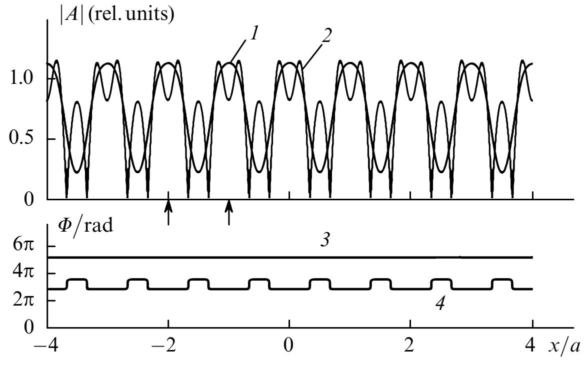

Figure 1. Distributions of the amplitude A (*1*, *2*) and phase Φ (*3*, *4*) of the infinite wave along the transverse coordinate x in the planes $z = (2n+1) \times a^2/2\lambda$ for $m = 1.2$ (*1*, *3*) and π (*2*, *4*). The arrows indicate the positions of caustic beaks.

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ulation degree exceeds the threshold value, zero-amplitude lines appear in the amplitude distribution. In the phase distribution, these lines correspond to the EDs where the phase abruptly changes by π. The EDs are symmetrically situated with respect to the axes of the beak-shaped caustics. Upon the further increase in the phase modulation degree, the beaks of the caustics, initially located near the planes $z = (2n+1)a^2/2\lambda$, approach the planes of the initial structure reproduction. The number of EDs also increases in this case.

Their arrangement with respect to the generating caustic corresponds to the phase structure of the field calculated in Ref. [11] using the Percy integral. Fig. 2 shows the longitudinal structure of the intensity distribution for $m = 1.2$. As can be seen from the figure, the phase modulation induces the formation of channels that are oriented along the propagation direction and where the radiation intensity is considerably greater than the average one. The axes of these formations coincide with the symmetry axes of the beak-shaped caustics.

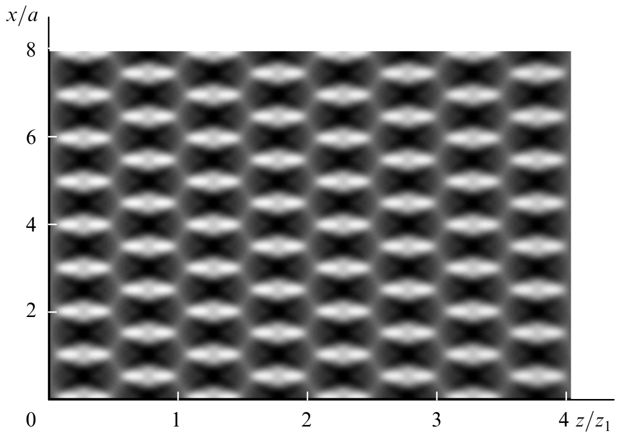

Figure 2. Longitudinal distribution of the intensity of the infinite wave for $m = 1.2$ and $z_1 = 2a^2/\lambda$.

If the phase in the initial plane is modulated along two, rather than one, transverse coordinates, screw dislocations (SDs) of the wavefront can form in the field. The SDs differ from the ED by the qualitatively different topological structure: the phase incursion along a circuit enclosing an SD is 2π. Fig. 3a shows the structure of the equiphase lines for the initial wavefront of the field distribution

$$\Psi(x, y) = \Psi(x, 0) + \mathrm{i}\Psi(y, 0) + C, \tag{3}$$

where the functional dependence of $\Psi(y, 0)$ and $\Psi(x, 0)$ on the transverse coordinates x and y is the same; and C is a constant.

The equiphase line structure shown in Fig. 3a was constructed using formula (3) for $C = 0.2$ and $m = 2$. The shape of the lines indicates the presence of smooth regular wavefront perturbations. Fig. 3b shows the structure of the equiphase lines in the planes $z = (2n+1)a^2/2\lambda$. The SDs are located at the points of intersection of equiphase lines. They form a kind of quadrupoles consisting of four SDs, two of which are positive (right-handed) and the other two are negative (left-handed). The quadrupoles surround the caustic axes.

Unlike EDs, each of which is formed in a definite plane $z = \text{const}$, the SDs have a certain longitudinal dimension.

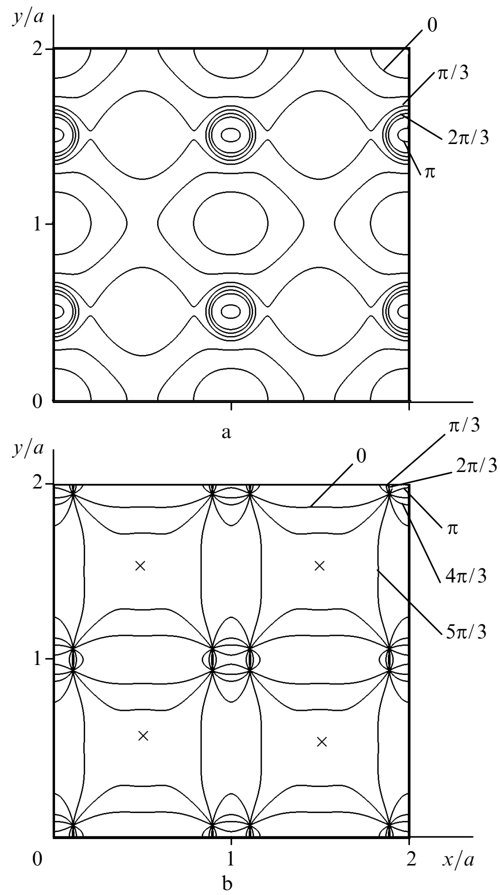

Figure 3. Structure of the equiphase lines in the initial plane (a) and the planes $z = (2n+1)a^2/2\lambda$ (b). The crosses indicate the positions of the caustic axes.

But like EDs, screw dislocations appear only if the modulation degree of the original wavefront exceeds a certain threshold value. If $\Delta\Phi$ is the difference between the maximum and the minimum phase in the initial plane (for the modulation along a single coordinate, $\Delta\Phi$ coincides with m), the SDs appear for $\Delta\Phi > \pi/2$.

We have analysed all the mentioned effects for a finite beam with a Gaussian intensity profile. The calculations were based on formula (2), in which the superposition of plane waves was replaced by the system of nonparallel Gaussian modes of the free space [12]. The waists of the modes lied in the initial plane.

The calculations have shown that the passage to the more rigorous model of a Gaussian beam with a periodic modulation of the wavefront does not introduce any significant qualitative changes to the properties of the transformation of the amplitude-phase distribution, at least at the distances comparable to the characteristic length a^2/λ. Like in the case of the infinite wave, the wavefront dislocations begin to form in the near-field zone when the phase modulation degree exceeds $\pi/2$. This can be seen from Fig. 4, which is the analogue of Fig. 1 for the Gaussian beam. The ratio of the waist radius w_0 to the modulation period a is five.

Comparing Figs 1 and 4, we see that the difference consists in the diffractional blurring of some of the dislocations. The difference becomes more pronounced with increasing coordinate z, as the periodic reproduction of the initial field structure deteriorates. This can be seen, for example, from

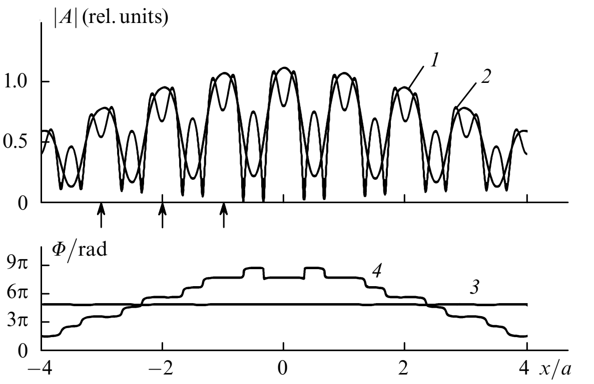

Figure 4. Distributions of the amplitude A (*1*, *2*) and phase Φ (*3*, *4*) of a Gaussian beam along the transverse coordinate x ($w_0 = 5a$) in the plane $z = a^2/2\lambda$ for $m = 1.2$ (*1*, *3*) and π (*2*, *4*).

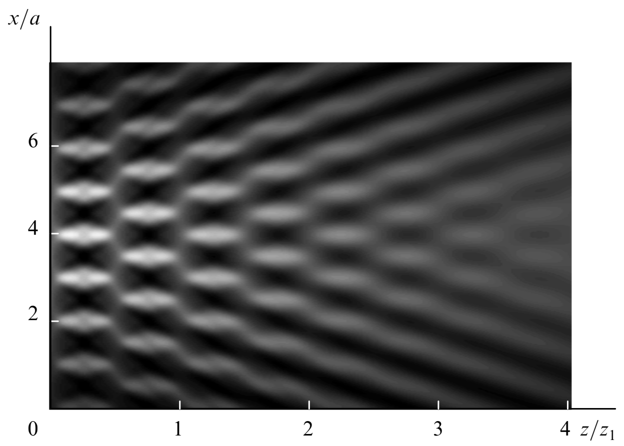

Figure 5. Longitudinal distribution of the intensity of a Gaussian beam for $m = 1.2$ and $w_0 = 5a$.

Fig. 5, which shows the longitudinal intensity distribution for the parameter values $m = 1.2$ and $w_0/a = 5$.

The distributions shown in Figs 2 and 5 are similar only in the near-field zone, where the energy of the light beam concentrates in narrow regions. In the far-field diffraction zone, the overlap between the angular Gaussian components of the field decreases, and the field structure begins to differ qualitatively from the structure of the infinite wave. The field now consists of a pencil of beams whose intensity decreases with increasing angle of inclination.

If the Gaussian beam is modulated in the two transverse directions and the modulation degree exceeds the mentioned critical value, SDs appear in the wavefront. As in the case of the infinite wave, they have a certain longitudinal dimension, which grows with increasing modulation degree. This property of SDs considerably facilitates their experimental detection. The phase modulation in the both transverse directions of the initial plane leads to the formation of two beam pencils in the far-field diffraction zone, which lie in mutually orthogonal planes.

Note in conclusion that the results of the performed analysis are partially applicable to the case of irregular smooth modulation of the wavefront provided that the dimension of the considered spatial region is comparable to the quantity a_n^2/λ, where a_n is the characteristic scale of irregular wavefront perturbations. In particular, the increased intensity channels will form in the field and the wavefront dislocations will appear only if the amplitude of the phase perturbations exceeds a certain value.

Thus, smooth perturbations of the wavefront play an important role for the transformation of the amplitude-phase field profile and the formation of caustics and dislocations. The appearance of wavefront caustics and dislocations exhibits a threshold behaviour and is directly related to the degree of the initial phase modulation. From the practical viewpoint, it is important that the appearance of these structures in the laser beam is accompanied by the formation of narrow channels where the radiation intensity is considerably greater than the average one.

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