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Self-sustaining exothermic reaction of anti-Stokes gamma transitions in long-lived isomeric nuclei. II

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Abstract. The conditions for the combustion chain reaction in a system of long-lived metastable isomers and a quasi-equilibrium high-temperature plasma radiation are considered. Estimates are presented for the ignition energy, the combustion pulse duration, the reaction zone dimensions, etc.

1. Introduction

The release of the energy stored in metastable states of long-lived isomeric nuclei is one of the topical problems of modern quantum nucleonics [1].

A straightforward way to solve this problem would be to realise a resonant chain reaction on radiative anti-Stokes transitions of an ensemble of nuclei. In this case, the energy of the metastable state m would be released as a result of the resonant excitation of an upper-lying quickly relaxing trigger level t, circumventing the forbiddance of direct transitions from the metastable state to lower levels. Such triggering transitions were studied experimentally in Refs [2, 3].

For this resonant reaction to be realisable there must be two pairs of equidistant levels in the nucleus (Fig. 1a). This means that the energy $E_{\rm tm}$ of the triggering transition from the metastable level m to the upper-lying trigger level t must coincide with the energy $E_{\rm pq}$ of one of the radiative cascade transitions ($E_{\rm tm} = E_{\rm pq}$). Provided that the relevant threshold conditions are satisfied, the nuclear chain reaction would be maintained upon the initial ignition by an infinite sequence of resonant absorptions and emissions $\rm m \to t$, $\rm p \to q$, $\rm m \to t, \ldots$, and the energy would be released as a result of the other transitions of the cascade.

Unfortunately, we are not aware of any isomeric nuclei that contain two pairs of equidistant levels and are therefore suitable for realising the described resonant scheme. Alternatively, the working substance can be a mixture of two different nuclei with equidistant levels (Fig. 1b). The reaction would then be initiated by an anti-Stokes transition of the metastable nucleus M, a radiative $\rm t \to s$ transition of its cascade being resonant with the upward $\rm q \to p$ transition of the auxiliary nucleus A. The auxiliary nucleus A must in turn have a radiative $\rm p \to q$ transition resonant with an $\rm m \to t$ transition of the metastable nucleus M. So far, however, we have not been able to find such a pair of nuclei either.

In this connection, the nonresonant version of the self-sustaining exothermic reaction [1] looks more promising. In this version, the triggering upward transition is induced by the wideband hard radiation that fills the reaction zone and which is spectrally similar to the blackbody radiation. The high temperature necessary for the reaction would be maintained in the reaction zone by the absorption of the energy released in other nuclear transitions, including gamma-ray quanta.

Thus, the reaction energy cycle forms the following closed chain: triggering initiation of the anti-Stokes nuclear transition — release of the energy of metastable isomeric states — absorption of the energy in the reaction zone and the heat-up of the reaction zone — broadband emission by the hot reaction zone — triggering of the anti-Stokes nuclear transition.

Below, we estimate qualitatively the parameters of the exothermic reaction, without going into the details of the processes taking place in the reaction zone.

2. Reaction zone

The reaction zone is a plasma with the particle concentration n and the dimensions that provide efficient absorption of the energy released in the reaction, in particular, gamma-ray quanta. The absorption mechanisms of gamma-ray quanta by the plasma are rather complex, but phenomenologically we can estimate the probability of this process as

$$\eta = \sigma_{\rm a} n_0 L \leqslant 1, \quad (1)$$

where $\sigma_{\rm a} = \sigma_{\rm a}(n,T,t)$ is the effective absorption cross section of gamma-ray quanta in the reaction zone, which has a characteristic dimension L. This cross section takes into account all processes (interaction with the electron shells of ions and atoms, interaction with free plasma electrons etc.) that take place in the plasma with the nucleus concentration n_0 and the temperature T. We need to know η, to determine the normalisation constant n^* (see formula (13) in [1]):

$$n^* = \frac{2\sigma}{\pi\eta}\frac{S}{V}\left(\frac{\hbar\omega_0}{k}\right)^4\frac{\tau_0}{\hbar\omega_{\rm mg}}, \quad (2)$$

where σ is the Stefan constant; k is the Boltzmann constant; τ_0 is the characteristic transition time; $\hbar\omega_{\rm mg}$ is the energy of

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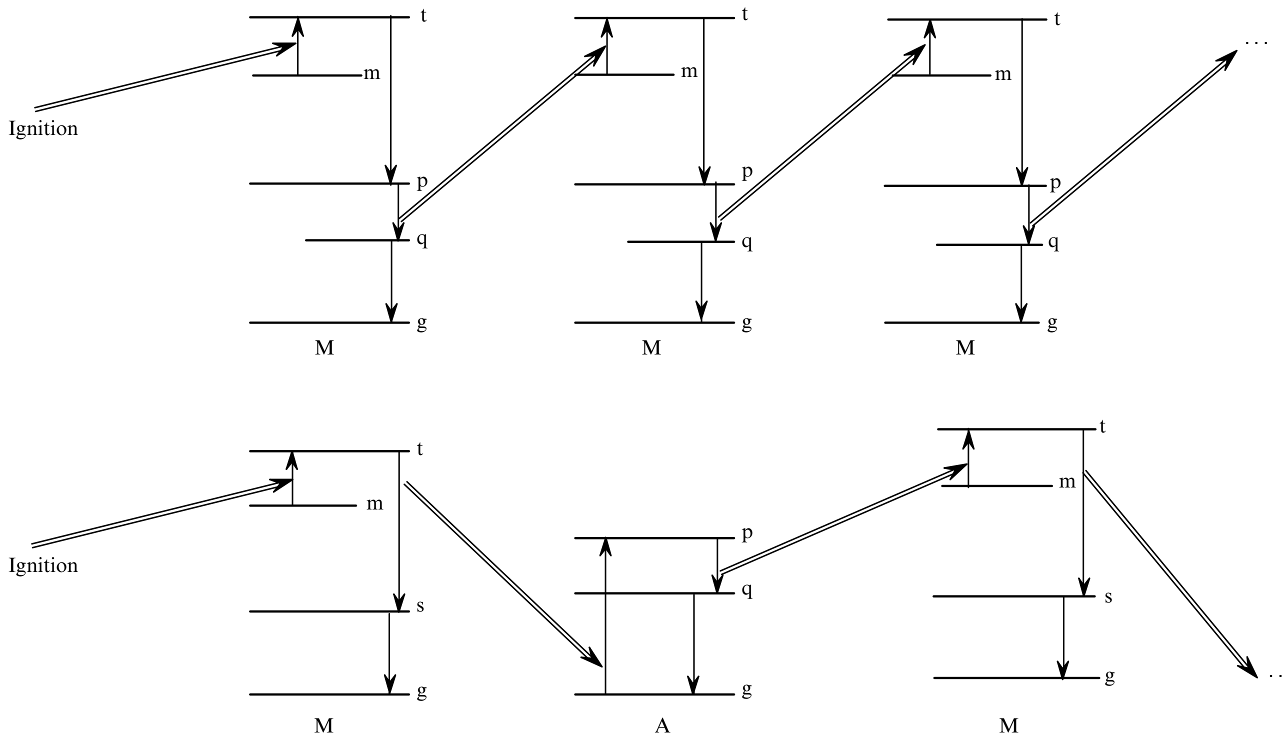

Figure 1.

the metastable state; $\hbar\omega_0$ is the energy of the triggering transition, and V and S are the volume and the surface area of the reaction zone, respectively. We can assume that, before the combustion reaction, the plasma consists exclusively of metastable isomeric nuclei, i. e., $n = n_0$.

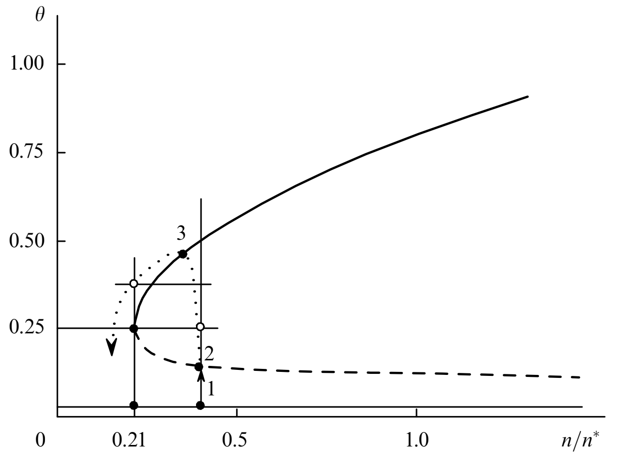

Figure 2.

The normalised plasma temperature $\theta = kT/\hbar\omega_0$ depends on the ratio n/n^*. Fig. 2 shows this S-shaped dependence. The critical point in this curve beyond which the chain reaction starts has the co-ordinates $n/n^* = 0.21$ and $\theta = 0.25$.

In other words, the chain reaction will start only if

$$\frac{n}{n^*} \equiv a > 0.21, \tag{3}$$

i. e., if

$$n_0 = an^* = an_1^* \frac{S}{V\eta}, \tag{4}$$

where $n_1^* = n^*$ for $\eta = 1$ and $V/S = 1$ cm. Taking into account Eqn. (1), we obtain

$$n_0 L_0 = \left(\frac{an_1^*}{\sigma_a}\right)^{1/2}, \tag{5}$$

where

$$L_0 = \left(\frac{VL}{S}\right)^{1/2} \tag{6}$$

is the characteristic linear dimension of the plasma formation. For example, in the case of a plasma sphere of radius R, we have $L_0 = R/\sqrt{3}$. For $n_1^* = 0.8 \times 10^{19}$ cm^{-3} (see Ref. [1]), $a = 0.4 > 0.21$, and $\sigma_a = 10^{-21}$ cm^2, we obtain $n_0 L_0 = 0.58 \times 10^{20}$ cm^{-2}.

The triggering nuclear reaction under study results in a complex dynamics of the created plasma formation. Its short-term stability can be ensured either by the own inertia of the plasma or by an external magnetic confinement. The stability problem is beyond the scope of the present discussion, and in the following, we will assume that the lifetime of the plasma formation is large enough for the entire reaction to take place.

3. Ignition energy

The starting condition for the reaction is an increase in reaction zone temperature from T_1 to T_2, when the representing point (see Fig. 2) undergoes a transition from the stable lower branch of the S-shaped curve (point 1) to the unstable branch that defines the reaction threshold (point 2). This transition corresponds to the temperature difference

$$\Delta T = T_2 - T_1 = \frac{\hbar\omega_0}{k}(\theta_2 - \theta_1), \quad (7)$$

where

$$\theta_2 - \theta_1 < 0.25, \quad (8)$$

and the energy supply

$$\Delta W = \chi \Delta T n_0 V = \frac{\hbar\omega_0}{k}\chi(\theta_2 - \theta_1) n_0 V < \frac{\hbar\omega_0}{4k}\chi n_0 V. \quad (9)$$

Here, $\chi = \chi(n_0, T, t)$ is the effective per-particle heat capacity of the reaction-zone plasma, which depends in a complex way on the concentration n, temperature T, time t, etc. For the following rough estimates, we neglect these dependences and assume that $\chi = \text{const}$. For example, in the case of $\hbar\omega_0 = 1$ keV, $n_0 = 10^{19}$ cm^{-3}, and $\chi = 10k$, we have $\Delta W <$ 4 kJ cm^{-3}.

It follows from relationship (2) that the energy of the triggering ignition photon $\hbar\omega_0$ depends on many parameters and is not defined by the choice of the isomer alone, whatever is its energy structure. As noted earlier, inequality (3) is the condition for the chain reaction. Together with Eqns (1) and (6), it yields

$$\hbar\omega_0 < k\left[\frac{\pi\sigma_a}{2a\sigma}(n_0 L_0)^2 \frac{\hbar\omega_{mg}}{\tau_0}\right]^{1/4}. \quad (10)$$

Thus, the energy of the triggering photon is proportional to $n_0^{1/2}$. If we neglect the possibilities of plasma compression, which are known from the experiments on inertial thermonuclear fusion, the greatest value of n_0 is the solid-state concentration of the plasma that is created at the initial stage from the condensed target. Then, assuming that $n_0 = 3 \times 10^{22}$ cm^{-3}, $L_0 = 1$ cm, $\sigma_a = 10^{-21}$ cm^2, $\hbar\omega_{mg} =$ 500 keV, and $\tau_0 = 100$ ns, we have $\hbar\omega_0 < 3$ keV.

4. Estimated duration of the pulsed combustion reaction

The pulsed combustion reaction evolves in time according to the energy-balance equation [1]

$$\frac{dQ}{dt} = -\sigma(T^4 - T_0^4)S + w_{mtg}\hbar\omega_{mg} n V \eta. \quad (11)$$

This equation should be complemented by the equation for the metastable isomer concentration n in the reaction zone, which is depleted in the combustion process,

$$\frac{dn}{dt} = -n w_{mtg} = -\frac{\pi n}{2\tau_0}\left(\exp\frac{\hbar\omega_0}{kT} - 1\right)^{-1}, \quad (12)$$

as well as by the dependence of temperature T on the energy Q accumulated in the reaction zone,

$$T = Q/\chi n_0 V. \quad (13)$$

Here, T_0 is the temperature of the external shell of the reaction zone and w_{mtg} is the probability of the anti-Stokes transition m $\rightarrow$ t $\rightarrow$ g [1].

In the dimensionless variables

$$\theta = \frac{kT}{\hbar\omega_0},\ \theta_0 = \frac{kT_0}{\hbar\omega_0},\ \varkappa = \frac{t}{t_0} \quad (14)$$

these equations assume the form

$$\frac{d\theta}{d\varkappa} = b\frac{n^*}{n_0}\left[-(\theta^4 - \theta_0^4) + \frac{n/n^*}{\exp(1/\theta) - 1}\right], \quad (15)$$

$$\frac{dn}{d\varkappa} = -\frac{\pi}{2} n\left[\exp(1/\theta) - 1\right]^{-1}, \quad (16)$$

where

$$b = \frac{\pi}{2}\frac{k}{\chi}\frac{\hbar\omega_{mg}}{\hbar\omega_0}\eta. \quad (17)$$

Eliminating time $\varkappa$ from the system of equations (15) and (16), we derive the trajectory of the point that represents the combustion pulse on the diagram $\theta(n/n^*)$ (see Fig. 2)

$$\frac{d\theta}{dn} = \frac{2b}{\pi n_0}\left\{\frac{n^*}{n}(\theta^4 - \theta_0^4)\left[\exp(1/\theta) - 1\right] - 1\right\}. \quad (18)$$

For a rough estimate of the combustion pulse duration, we can assume that $\chi = \text{const}$ and that the time during which the plasma formation is relatively stable is no shorter than the combustion pulse duration. The general character of the trajectory representing the combustion pulse (the dashed curve in Fig. 2) is obvious: it starts from the initial point 2 on the middle unstable branch of the S-shaped curve $\theta(n/n^*)$ and then ascends, deviating from the vertical line. This latter heating stage corresponds to increasing temperature θ and decreasing concentration n of the metastable nuclei that burn away.

The maximum temperature on the trajectory is reached at point 3, which lies on the upper stable branch of the S-shaped curve and where the derivatives $d\theta/d\varkappa = dn/d\varkappa = 0$.

Then, the cooling stage follows as the trajectory enters the region where $d\theta/d\varkappa < 0$.

A numerical estimate of the total duration of the two stages gives the combustion pulse duration on the order of a few characteristic transition times τ_0.

Another way to estimate the combustion pulse duration is to calculate the time Δt it takes for the isomeric nuclei to be burnt completely. In this time period, the probability that every metastable nucleus makes the anti-Stokes triggering gamma transition reaches unity:

$$w_{mtg}\Delta t = 1, \quad (19)$$

where

$$w_{mtg} = \frac{\pi}{2\tau_0}\left(\exp\frac{\hbar\omega_0}{kT} - 1\right)^{-1} \quad (20)$$

is the probability of the triggering transition per unit time in the field of the equilibrium radiation of temperature T (see equation (9) of Ref. [1]). Equations (19) and (20) yield the dependence of the normalised plasma temperature θ on $\Delta t/\tau_0$ (Fig. 3):

$$\theta = \ln^{-1}\left(\frac{\pi\Delta t}{2\tau_0} + 1\right).$$

One can see from the figure that the time Δt of the complete burnout equals a few characteristic transition times τ_0 if the plasma temperature does not exceed the energy of the triggering photon ($\theta < 1$). An attempt to further reduce the time of the complete burnout is hopeless because it requires an unjustifiably high increase in the plasma temperature θ.

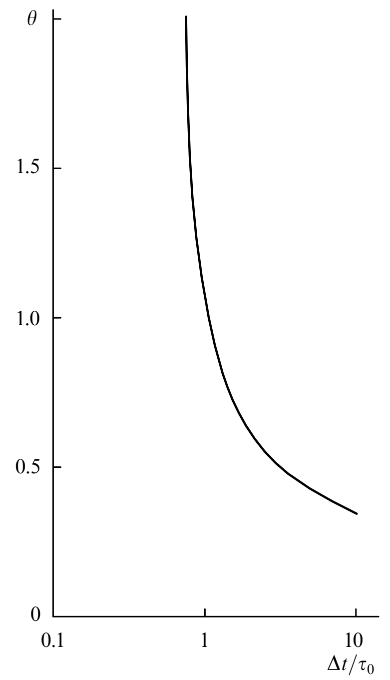

Figure 3.

Obviously, the time of the relative stability of the plasma formation should be no less than the estimated combustion duration. This imposes restrictions on the choice of the isomer, and, therefore, τ_0 is determined by the plasma confinement abilities.

Note that, in this discussion, we ignored other (nonradiative) channels of nucleus excitation in a dense hot plasma (e. g., an electron impact, inverse electron conversion, etc.), which can considerably increase the probability of the triggering transitions.

Note also that the model of blackbody radiation can only provide a rough estimate because the spectral distribution of the plasma radiation can be far from equilibrium. In particular, it may contain characteristic x-ray lines of plasma ions that are significantly more intensive than the continuous background. If the emission lines of these ions, which can as well be intentionally introduced into the plasma, happen to coincide with the isomeric triggering transition [4], the efficiency of the anti-Stokes process can be substantially increased.

5. Conclusions

To conclude, we note that the problem of the controlled release of the nuclear energy stored in long-lived metastable isomeric states is methodologically similar to the problems of inertial thermonuclear fusion and gamma-ray laser creation. On the other hand, the specific difficulties one faces when dealing with this problem are partially redeemed by the absence of the main difficulties inherent in the two mentioned related problems: the isomeric chain reaction requires neither the deep cooling of the nuclear medium that is necessary to create a gamma-ray laser [5] nor the extreme target compression required for the inertial thermonuclear fusion.

We can identify three key problems that require further analysis: the choice of the optimal isomer, the study of the plasma-zone reaction dynamics as well as the development of the system for its relative stabilisation, and the development of efficient methods for the reaction ignition. With a reasonable share of optimism, we can hope for the successful solution of these problems.

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ERRATA

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Erratum: Interrelation of the laser-induced damage characteristics in statistical theory

M F Koldunov, A A Manenkov, I L Poko(lo)tilo

In the paper through the fault of the translator an error was commited in the name of the third author:
instead of wrong I L Pokolotilo one should read I L POKOTILO.
The collective of the editorial office gives its apologies to the author.