

Theory of generation of sub-10-fs pulses in a cw solid-state laser with a semiconductor passive switch and self-focusing under coherent-interaction conditions

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Abstract. An analysis made on the basis of the theory of the self-consistent field shows the possibility of production of ultrashort sub-10-fs soliton-like pulses in a solid-state laser with a coherent semiconductor switch in the absence of mode-locking caused by self-focusing. Mode-locking caused by self-focusing leads to the formation of an extremely short sech-shaped pulse. In this case, the restriction imposed on the minimum modulation degree in a passive semiconductor switch that provides self-starting USP generation is lifted.

1. Introduction

Owing to a recent rapid progress in the development of methods of generation of ultrashort pulses (USPs), sub-10-fs pulses with duration close to the theoretical limit determined by the wave period in the visible region were produced [1]. The generation of such pulses is based on the use of self-focusing of laser radiation in an active medium, which causes inversionless ‘bleaching’ of diffraction loss [2] in combination with inertial saturation of interband and excitonic transitions in a semiconductor switch [3] and the ‘soliton’ mode-locking mechanism [4].

Semiconductor switches are essential elements of modern femtosecond lasers. They provide laser stability and make possible self-starting mode locking. Because of this, one should take into account the physics of interaction of USPs with semiconductor structures, which can substantially change characteristics of mode locking [5–7]. Because the USP duration is smaller than the characteristic coherence times of a semiconductor switch, which are as long as several tens of femtoseconds, the coherence of the interaction of a pulse with a semiconductor structure is a factor of particular importance among the factors affecting mode locking.

Mode locking in lasers with a coherent absorber has been studied in Refs [8–11]. In particular, the analysis made on the basis of the theory of the self-consistent field showed the key role of dynamic gain saturation in the generation and stabilisation of soliton-like 2π pulses [8–10]. However, the dynamic saturation of gain within the limits of a femtosecond laser

pulse is negligibly weak, whereas self-focusing, self-phase modulation (SPM), and group velocity dispersion become the dominant nonlinear factors in this case.

The numerical analysis of the dynamics of the USP generation in a cw solid-state femtosecond laser in the absence of self-focusing [11] showed the possibility of forming self-induced transparency in a semiconductor switch, which has a decisive effect on the laser dynamics. However, as shown in [12], the neglect of the contribution from self-focusing to the USP formation is inconsistent with the conditions realised in the majority of cw solid-state lasers.

Here, we study, on the basis of the theory of the self-consistent field, the characteristics of stationary USPs under conditions of their coherent interaction with a semiconductor switch in a cw solid-state laser, taking into account self-focusing and self-phase modulation (SPM) in an active medium. The stability of the generation of a soliton-like 2π pulse and the possibility of self-starting under conditions of self-induced transparency in a semiconductor switch are studied.

2. Model

Lasing was studied on the basis of the distributed model (see, e.g., [5]) taking into account the amplification of a slowly varying laser field a in a quasi-two-level active medium with the gain α saturated by the full USP energy, linear loss γ , the group velocity dispersion d , the action of a spectral filter with the reciprocal spectral width t_f , SPM and the ‘instantaneous’ saturation of loss through self-focusing with coefficients β and σ , respectively, and the coherent interaction of USPs with a semiconductor switch. The solution of the system of laser equations was sought in the analytic form using the two-level energy diagram of a semiconductor switch, which well corresponds to actual switches based on quantum-well semiconductor structures.

If the duration of a bandwidth-limited USP is much smaller than the transverse relaxation time t_{coh} of a semiconductor switch and the carrier pulse frequency is not detuned from the resonance frequency, the interaction with a switch can be described by the well-known system of Bloch equations [13]

$$\begin{aligned} \frac{du(t)}{dt} &= qa(t)w(t), \\ \frac{dv(t)}{dt} &= 0, \\ \frac{dw(t)}{dt} &= -qa(t)u(t), \end{aligned} \quad (1)$$

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where u, v and w are the slowly varying amplitudes of quadrature polarisation components and the population difference, respectively; $q = \mu/\hbar$; $\mu = e \times 0.28$ C nm is the dipole moment corresponding to the saturation energy $E_a = 50$ $\mu\text{J cm}^{-2}$ for a GaAs/AlGaAs semiconductor switch and $t_{\text{coh}} = 50$ fs; and e is the elementary charge. The initial loss for a small incoherent signal in such a switch, with the semiconductor layer thickness $z_a = 10$ nm and carrier density $N = 2 \times 10^{18}$ cm^{-3} , is $\gamma_a = 0.01$.

Taking into account the interaction with a semiconductor switch, the laser equation can be written in the form

$$\frac{\partial a(z, t)}{\partial z} = \left[\alpha - \gamma + i\phi + \delta \frac{\partial}{\partial t} + (t_f^2 + id) \frac{\partial^2}{\partial t^2} + (\sigma - i\beta) |a(z, t)|^2 \right] a(z, t) - \frac{2\pi N z_a d \omega}{c} \sin[\psi(z, t)], \quad (2)$$

where ω is the laser field frequency; c is the speed of light; t is the local time; z is the longitudinal coordinate normalised to the cavity length, i.e., the transit number; $\psi(z, t) = q \int_0^t a(z, t') dt'$ is the pulse 'area'; and δ and ϕ are the time and phase delays of a pulse on a cavity period, respectively.

Below, we consider the propagation of USPs under stationary conditions, i.e., in the absence of the dependence on z . The fields and the time will be normalised to q and t_f , respectively. In this case, the parameters β and σ should be normalised to the quantity $2(q t_f)^2 / n c \epsilon_0 = 5 \times 10^{-12}$ $\text{cm}^2 \text{W}^{-1}$, where ϵ_0 is the dielectric constant and $t_f = 2.5$ fs for a Ti:sapphire laser. For a semiconductor switch with the parameters presented above, the dimensionless parameter $\sigma = 0.14$ corresponds to a power of 10^7 W for the saturation of an effective switch caused by self-focusing, with a laser mode in the section of an active medium being 30 μm in size; and the dimensionless parameter $\beta = 0.26$ corresponds to a Ti:sapphire crystal 1 mm thick. In what follows, we use the dimensionless parameters d, β , and σ . The dimensional USP duration is given only in the figures to give a better understanding of the results presented there.

Moreover, it makes sense to introduce the practically important parameter λ , which represents either the ratio of cross sections of a laser mode in an active medium and in a semiconductor switch or the amplitude reflectivity of the multilayer mirror under which the semiconductor switch is positioned. The variation of λ in real experiments corresponds to a change in the contribution of SPM and the saturation of an effective switch based on self-focusing to the field-switch interaction. Passing from the nonlinear integro-differential equation in the field amplitude to the differential equation in the USP area, we have

$$\left[(\alpha - \gamma) \frac{d}{dt} + \delta \frac{d^2}{dt^2} + (1 + id) \frac{d^3}{dt^3} + \frac{\sigma - i\beta}{\lambda^2} \left(\frac{d\psi(t)}{dt} \right)^2 \frac{d}{dt} \right] \psi(t) - \frac{\gamma_a}{t_{\text{coh}}} \sin[\psi(\tau)] = 0. \quad (3)$$

3. Coherent laser USP in the absence of group velocity dispersion and SPM

For simplicity, we assume that the system has no SPM, which is valid for large cross sections of a laser mode in an active medium, and that the group velocity dispersion is zero.

In the absence of laser factors, the solution of Eqn (3) represents a 2π pulse of shape $a(t) = a_0 \text{sech}(t/t_p)$ (in what follows, it will be termed a sech-shaped pulse) [13], where a_0 is the USP amplitude and t_p is its duration. One can easily verify by direct substitution that it does not represent an exact solution of Eqn (3) in the absence of self-focusing ($\sigma = 0$).

To study the character of pulsed solutions of Eqn (3), we make the substitution $\psi(t) = x, d\psi(t)/dt = y(x)$. Then, one can lower the order of Eqn (3):

$$\left[\left(\frac{d^2 y}{dx^2} \right) y + \left(\frac{dy}{dx} \right)^2 \delta \frac{dy}{dx} + (\alpha - \gamma) \right] y - \frac{\gamma_a}{t_{\text{coh}}} \sin x = 0. \quad (4)$$

The numerical solutions of Eqn (4) contain 2π -pulse solutions, which, however, differ from a sech-shaped pulse (see Fig. 1, which presents one of such solutions and, for comparison, a sech-shaped solution of Eqn (3) with the same amplitude in the absence of laser factors, i.e., in the absence of gain, linear loss, and frequency filtering).

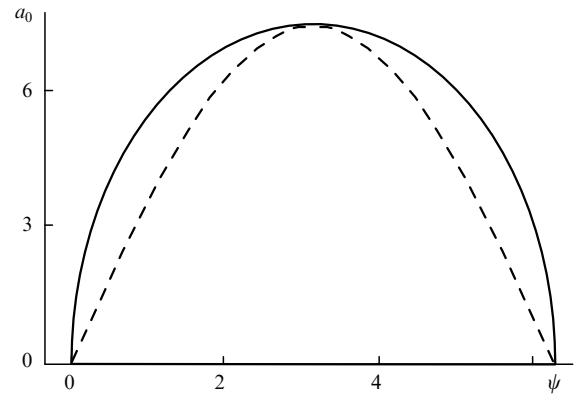


Figure 1. USP envelope calculated in the field-pulse area coordinates by the numerical solution of Eqn (4) (solid curve) and the sech-shaped profile of a soliton-like pulse (dashed curve) for $\gamma = 0.04, \gamma_a = 0.01$, and $\delta = 0.042$.

It is of interest to carry out an analytical study of the soliton-like solution obtained above. For this purpose, we will seek approximate solutions of Eqn (4) on the basis of the harmonic approximation of the form $y(x) = a_1 \sin(x/2) + a_2 \sin x + \dots$. We restrict our consideration to the first term, which corresponds to a sech-shaped USP. In the absence of gain, linear loss, and frequency filtering, the sech-shaped solution of Eqn (3) corresponds to the solution $a_1 = 2(\gamma_a/t_{\text{coh}}\delta)^{1/2}$. In the presence of these factors, the relationship between the amplitude of a pulse and its duration is retained, but they become dependent on laser parameters (gain, linear loss and, taking into account normalisation, the reciprocal bandwidth of a frequency filter): $a_1 = 2[2(\alpha - \gamma)]^{1/2}, \delta = \gamma_a/[2(\alpha - \gamma)t_{\text{coh}}]$. The reverse change of variables enables one to determine the dimensionless USP duration: $t_p = 2/a_1$.

The parameter α entering in Eqns (1)–(4) represents the gain. We assumed that the inverse population in an active medium decreased under the action of a full USP energy. The gain α can be conveniently expressed in terms of the dimensionless pump intensity $P = \sigma_{14} T_{\text{cav}} I_p / \hbar \nu$, where I_p is the dimensional pump intensity; ν is the pump field frequency; T_{cav} is the cavity period; and σ_{14} is the absorption

cross section at the pump frequency. Then, the stationary USP generation in a quasi-two-level active medium is described by the expression

$$\alpha = \frac{P\alpha_{\max}}{P + \tau E + 1/T_r},$$

where α_{\max} is the gain for the total inversion; T_r is the relaxation time reduced to the cavity period; E is the total USP energy; and τ is the dimensionless reciprocal saturation energy. For the parameters and the normalisation chosen above, we have for the Ti:sapphire medium $\tau = 6.25 \times 10^{-4}$.

Fig. 2 presents the dependence of the USP duration for two physical solutions, obtained taking into account the gain saturation, on the dimensionless intensity (solid curves). One can see that the laser pulse duration becomes shorter than 10 fs with increasing pump intensity, which is typical of systems with a passive switch (see, e.g., [14]).

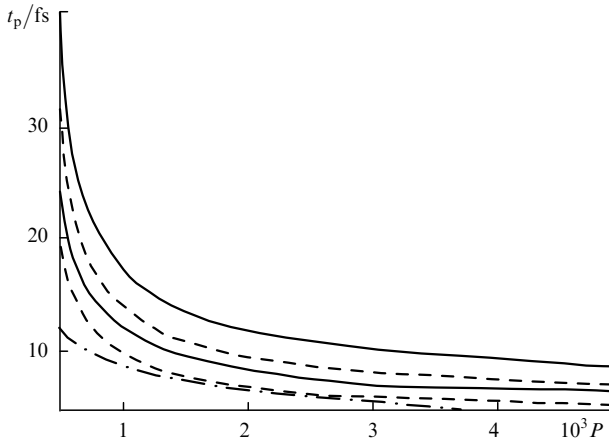


Figure 2. Dependences of the dimensional USP duration on the dimensionless pump intensity. Two approximate physical solutions of Eqn (4) (solid curves) and the result of their correction on the basis of the energy conservation law (dashed curves) for $\alpha_{\max} = 0.1$, $T_r = 3 \mu\text{s}$, $T_{\text{cav}} = 10 \text{ ns}$, $\tau = 6.25 \times 10^{-4}$, $\gamma = 0.01$, and $\lambda = 1$; and the duration of a sech-shaped pulse under conditions of self-focusing and coherent interaction with a semiconductor switch for $\lambda = 0.5$ (dot-and-dash curve).

However, the result obtained is approximate. Nevertheless, these pulse durations can be corrected on the basis of the energy conservation law. Multiplying Eqn (2) by the field strength, integrating the result over time within infinite limits, and taking into account the boundary conditions, we obtain the energy conservation law in the form

$$(\alpha - \gamma)E - \int_{-\infty}^{\infty} \left[\frac{da(t')}{dt'} \right]^2 dt = 0.$$

The calculation of the last term requires the knowledge of the pulse shape. Assuming, as was made above, that an USP is sech-shaped, one can obtain from the energy conservation law the following expression for the corrected pulse duration:

$$t_p = \frac{1}{[3(\alpha - \gamma)]^{1/2}}.$$

The resulting duration is somewhat smaller than the value obtained above (see the dashed curves in Fig. 2).

Note that the expressions obtained for the USP parameters show that the total saturated gain is positive. This im-

poses certain restrictions on the pulse stability. To be specific, an USP is stable against laser noise in the case when the total saturated gain in front of it and behind it is negative [2]: $\alpha - \gamma - \gamma_a < 0$. This condition restricts the minimum loss in a semiconductor switch.

Fig. 3 presents the dependences of the minimum bleachable loss in a semiconductor switch that are necessary for USP stabilisation on the pump intensity for two physical solutions presented in Fig. 2 (the dashed curve corresponds to the threshold loss, and the lower curve corresponds to the solution with larger durations). One can see from Fig. 3 that the USP stabilisation is possible in the hatched domain, i.e., the stability against laser noise is obtained for the solution with a larger duration. In this case, the width of the stability domain considerably decreases with lowering initial loss in a semiconductor switch, whereas the expansion of the stability domain requires an increase in initial loss, which is accompanied by

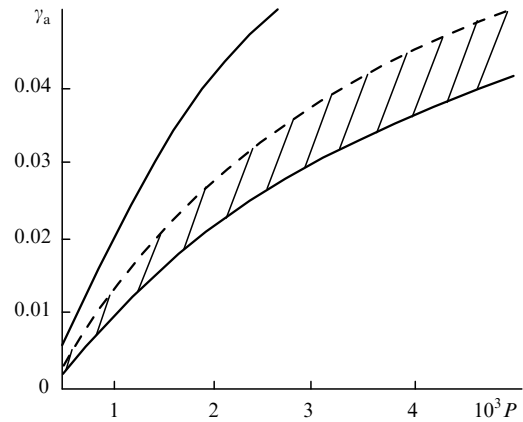


Figure 3. Minimum initial loss in a semiconductor switch required for USP stabilisation for two approximate solutions of Eqn (4) (solid curves) and the threshold initial losses in a semiconductor switch (dashed curve) for the same parameters as in Fig. 2. The hatched region specifies the stability domain.

the growth of the laser threshold.

For a further generalisation, we consider a coherent USP in a laser in the presence of an effective switch based on self-focusing. A substantial specific feature of this situation is that it allows the formation of sech-shaped pulses for a certain relationship between parameters of a semiconductor switch and an effective switch based on self-focusing. The parameters of such USPs are given by the relations

$$t_p = \frac{1}{(\gamma - \alpha)^{1/2}}, \quad \delta = \frac{\gamma_a}{t_{\text{coh}}(\gamma - \alpha)}, \quad \sigma = \frac{\lambda^2}{2}. \quad (5)$$

We note two specific features of lasing in the presence of self-focusing: (1) an USP is produced under conditions of a negative total gain, which lifts restrictions imposed on the minimum saturable loss in a semiconductor switch that are required for laser pulse stabilisation; (2) the USP duration is determined by a formula that is similar to the formula for an instantaneous switch (see, e.g., [2]), which gives evidence of the fact that self-focusing is the mechanism making the dominant contribution to USP formation. In this case, an USP is shorter than in the absence of self-focusing (the dot-and-dash curve in Fig. 2). A decrease in intensity is best pronounced for low and high pump intensities.

The physical sense of the restriction on the relationship between the parameters of a semiconductor switch and an instantaneous switch based on self-focusing consists in the following. Because of its physical nature, self-focusing imposes no limits on the USP area in contrast to the coherent interaction with a semiconductor switch. Therefore, when both these factors are present in the system, an USP of a certain form (here, a sech-shaped pulse) can appear only in the case of its definite area (here, we consider only 2π pulses; although stationary pulses with a different area, in particular, π pulses and chirped pulses with a variable area can be formed too), from which follows an additional restriction on the allowable values of σ .

As shown in [15], a factor of primary importance impeding USP generation is the modulation instability of a laser pulse. To estimate the modulation instability, we use the aberration-free approximation, which assumes the possibility of variation of USP parameters, with its shape being unchanged. Substituting a sech-shaped pulse into the laser equation, using an expansion in a power series in t , and equating the factors multiplying the terms with the same orders of t , we obtain the following equation for the evolution of USP parameters:

$$\begin{aligned} \frac{da_0}{dz} &= 2 \frac{(\alpha - \gamma)\lambda^2 t_p^2(z) - \lambda^2 + 4\sigma}{\lambda^2 t_p^3(z)}, \\ \frac{dt_p(z)}{dz} &= 4 \frac{\lambda^2 - 2\sigma}{a_0(z)\lambda^2 t_p^2(z)}, \\ \delta(z) &= 2 \frac{\gamma_a t_p(z)}{t_{\text{coh}} a_0(z)}. \end{aligned} \quad (6)$$

When choosing the value of σ providing the USP generation, we obtain for the evolution of pulse duration the condition of marginal stability, i.e., the zero increment for the damping of its perturbations. Note that the damping condition for amplitude perturbations $-4(\gamma - \alpha)^2 < 0$ (the negative value of the derivative of the right-hand side of the first equation in (6) with respect to duration) is fulfilled automatically.

4. Coherent laser USP in the presence of group velocity dispersion and SPM

First we analyse the influence of these laser factors on an USP in the absence of self-focusing for bandwidth-limited pulses, which are of most importance for practice. In this case, the character of the field makes it possible to lower the order of Eqn (3) by using the change of variables mentioned above:

$$\begin{aligned} y(x) \frac{dy(x)}{dx} \delta + \frac{\beta}{d} y^3(x) \\ + \left(\alpha - \gamma - \frac{\phi}{d} \right) y(x) - \frac{\gamma_a}{t_{\text{coh}}} \sin x = 0. \end{aligned} \quad (7)$$

An approximate solution of Eqn (7) in the form $y(x) = a_1 \sin(x/2)$ is given by

$$\phi = 3\beta a_1^2 + d(\alpha - \gamma), \quad \delta = \frac{4\gamma_a}{a_1^2}, \quad a_1 = 2[3(\alpha - \gamma)]^{1/2}.$$

The USP duration is $t_p = 2/a_1$. The gain saturation, when taken into account (similarly to the above analysis), leads to an insignificant increase in the USP duration on the solution

branch that is stable against loss and to a noticeable decrease in duration on the unstable branch.

If the same factors are taken into account in the presence of self-focusing, no changes in USP parameters are observed. However, as in the presence of self-focusing, there appears a phase delay $\phi = -2\beta(\alpha - \gamma)/\lambda^2$ and an additional limitation of dispersion required for the formation of a bandwidth-limited USP: $d = -2\beta/\lambda^2$.

The analysis of the automodulation stability, made by the technique described above, shows that when one chooses the dispersion $d = -2\beta/\lambda^2$ required to counterbalance the chirp, the character of USP stability is not different from the one described above, except the appearance of the additional phase delay $\phi = 2(d + 4\beta)/a_0(z)t_p^3(z)$.

5. Self-starting USP generation

From the viewpoint of practical applications, one of the most important features of solid-state lasers with a semiconductor switch is their ability of self-starting USP generation. To analyse this ability, we considered the evolution of a noise sech-shaped field spike whose duration is much greater than the longitudinal relaxation time for excitation in a semiconductor switch $T_a = 1$ ps. In this case, the action of a semiconductor switch may be described as the action of an instantaneous switch, and the action of SPM and self-focusing at the initial lasing stages may be neglected. It is convenient to change the normalisation: the times will be normalised to the cavity period; the intensities will be normalised to E_a/T_{cav} ; and the gain saturation energy will be normalised to E_a .

As a result, the equation of spike evolution takes the form

$$\begin{aligned} \frac{\partial a(z, t)}{\partial z} &= \left[\frac{P\alpha_{\text{max}} T_r}{1 + 2\tau T_r a_0^2(z) t_p(z)/\lambda^2 + P T_r} \right. \\ &\quad \left. - \frac{\gamma_a}{1 + 2a(z, t) T_a} - \gamma + \frac{t_f^2}{T_{\text{cav}}^2} \frac{\partial^2}{\partial t^2} \right] a(z, t). \end{aligned} \quad (8)$$

In the aberration-free approximation, as above, we can obtain equations for the evolution of spike parameters. The spike damping (an increase in duration and a decrease in amplitude) will correspond to the absence of self-starting USP generation. On the contrary, an asymptotic growth of spike intensity, accompanied by a decrease in duration, will correspond to the self-starting USP generation.

The black region in Fig. 4 specifies the domain of initial spike parameters corresponding to the self-starting USP generation for the dimensionless pump intensity $P = 8.5 \times 10^{-4}$. A decrease in the pump intensity below the specified value hampers the self-start of the USP generation, whereas its increase up to $P = 8.8 \times 10^{-4}$ provides self-starting for the spike parameters lying in the whole domain (black and hatched domains).

6. Conclusions

Thus, the analysis of USP generation in a cw solid-state laser under conditions of coherent interaction with a semiconductor switch, made on the basis of the theory of the self-consistent field suggests some conclusions.

In the absence of self-focusing, a sub-10-fs 2π pulse can be formed, which, however, differs from a sech-shaped pulse.

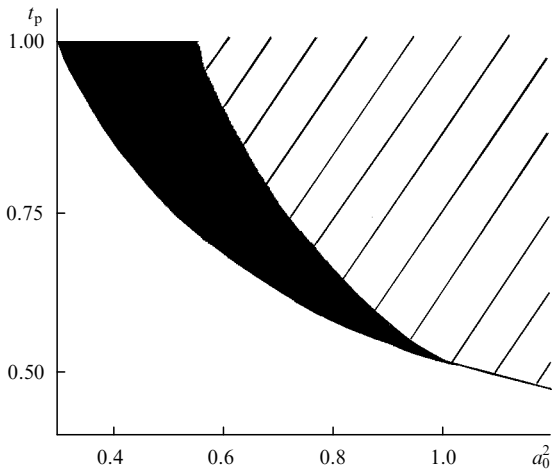


Figure 4. The domain of self-starting USP generation on the plane ‘initial spike intensity a_0^2 – initial pulse duration t_p ’. Self-starting at $P = 8.5 \times 10^{-4}$ (black region) and $P = 8.8 \times 10^{-4}$ (black and hatched regions) for $T_a = 1$ ps, $\gamma = 0.01$, $\lambda = 1$, $t_f/T_{cav} = 2.5 \times 10^{-7}$, $\tau = 6.25 \times 10^{-5}$; the rest parameters are the same as in Fig. 2.

The requirement of stability against laser noise imposes limits on the minimum bleachable loss in a semiconductor switch. A decrease in diffraction loss through self-focusing has a radical effect on character of USP generation. It becomes possible to produce self-modulation-stable sech-shaped USPs by choosing a certain relationship between parameters of a semiconductor switch and self-focusing. The USP duration can be substantially decreased down to the limit determined by the approximation of slowly varying field amplitudes and polarisation; no restrictions are imposed on initial losses in a semiconductor switch, which enables one to lower the laser threshold. In this case, USP characteristics are determined by the self-focusing mechanism of mode locking, whereas a semiconductor switch imposes limits on the pulse area. SPM and group velocity dispersion cause no substantial change in USP characteristics, but impose additional restrictions on the dispersion required for the formation of a bandwidth-limited USP. Of most importance from the viewpoint of practical applications is the possibility of self-starting USP generation in a cw solid-state laser with a semiconductor switch for a certain pump intensity above the threshold value.

All mathematical calculations in this paper were made in the Maple V environment. The corresponding program and detailed comments can be found on the web page <http://www.geocities.com/optomaplev>.

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