 

PACS numbers: 42.55.Wd; 42.55.Ye
DOI: 10.1070/QE2000v030n10ABEH001840

# A simple analytic model of a cw multicascade fibre Raman laser

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**Abstract. A simple model of a multicascade fibre Raman laser is considered and analytic expressions are obtained for its output characteristics. The efficiency of the Raman laser is shown to be determined to a considerable extent by the lumped optical loss of the cavity. A proposal is made to estimate the quality of optical fibres as an active medium for multicascade fibre Raman lasers from the efficiency of the model Raman laser.**

## 1. Introduction

In the last years considerable advances have been achieved in the development of multicascade fibre Raman lasers. By pumping such lasers by radiation in the 1-μm region, highly efficient lasing (about 50 %) can be obtained at almost any wavelength in the region from the pump wavelength to $\sim$ 1700 nm. Since the first demonstration of such lasers in 1994 [1] they attract increasing attention because of their unique properties.

These lasers have found one of the most important applications in fibre optic communication systems where they are used as pump sources for fibre erbium and Raman amplifiers. Because the position of the amplification band of a Raman amplifier is determined by the pump wavelength, fibre Raman lasers are promising for the use in broad-band wavelength-division multiplexing communication systems within the entire transparency region of optical fibres.

The efficiency of a frequency converter, whose role is in fact played by a multicascade fibre Raman laser, is one of the most important parameters. It depends on many properties of a fibre, which represents the Raman-active medium, and of other elements of the laser, including a pump source. Thus, the use of germanosilicate fibres as Raman-active media, which are commercially available, restricts the Stokes shift of the radiation frequency in each cascade of a Raman laser by the magnitude $\sim$400 cm$^{-1}$. In optic communication systems, it is necessary to obtain frequency shifts of about 1300 – 3000 cm$^{-1}$ (optical Raman amplifiers operating at 1300 nm require the pumping at $\lambda \approx 1240$ nm, while Raman amplifiers operating in the range from 1400 to 1650 nm, as well as erbium fibre amplifiers should be pumped even at longer wavelengths). For this reason, to obtain a large radiation frequency shift in a germanosilicate fibre, it is necessary to use a multicascade Raman laser containing from three to six cascades, whose conversion efficiency decreases with increasing number of cascades.

In Ref. [2], the use of a phosphosilicate glass as a Raman-active medium in one- and two-cascade lasers was demonstrated. The SRS spectrum of a phosphosilicate fibre exhibits a narrow line with a large Stokes shift of about 1300 cm$^{-1}$, which exceeds the Stokes shift in a germanosilicate fibre by a factor of three. This allows one to reduce the required number of SRS conversion cascades by a corresponding factor. However, optical loss in phosphosilicate fibres are somewhat higher, on the average (at the modern technology level), than that in a germanosilicate fibres. Therefore, to obtain the most efficient configuration of a Raman laser, one should perform simulations of lasing in each specific case.

Lasing in an $N$-cascade fibre Raman laser is mathematically described by a system of $2(N+1)$ ordinary differential equations with the corresponding boundary conditions at cavity mirrors. The results of numerical analysis of some lasers of this type are reported in a number of papers (see, for example, [3, 4]). However, because of a great number of parameters affecting the laser operation, the numerical methods cannot give, as a rule, a clear understanding about the ways of improving the laser design. In particular, a comparative analysis of the efficiencies of a standard three-cascade germanosilicate fibre Raman laser and a one-cascade phosphosilicate fibre laser producing the same frequency shift (with the optical loss level achieved in phosphosilicate fibres at present) has not been reported in the literature so far.

Therefore, to predict the output parameters of such lasers, it would be desirable to have the analytic solution of the problem. However, it seems that such a solution cannot be obtained in the general case. This attracts interest to the choice of a model problem that would be adequate to the real physical situation and simultaneously admit an analytic solution.

## 2. Choice of the model

Consider for definiteness the linear design of a cascade Raman laser. The three-cascade version of such a laser design in presented in Fig. 1.

Such a laser design has been studied in many papers (see, for example, [1 – 4]). A multicascade Raman laser is based on a fibre piece having the Raman gains $g_i$ ($i = 0, \ldots, N-1$,

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Received 12 May 2000
*Kvantovaya Elektronika* **30** (10) 873 – 877 (2000)
Translated by M N Sapozhnikov

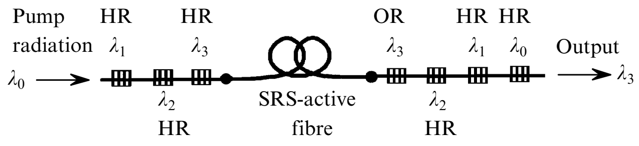


**Figure 1.** Scheme of a three-cascade Raman laser: HR (OR) are Bragg gratings with high (optimal) reflectivity at the wavelengths indicated; • are splicing points of fibres.

where $N$ is the number of cascades of the Raman laser) at wavelengths $\lambda_{i+1}$ upon pumping at the wavelength $\lambda_i$. The fibre is placed inside a system of $N$ enclosed optical cavities, which are formed by the same number of pairs of Bragg gratings of the refractive index recorded in the fibre core and serving as mirrors.

The first cascade of the Raman laser, to which a pair of Bragg gratings blazed at the wavelength $\lambda_1$ corresponds, is pumped by a single-mode laser (usually, a neodymium or an erbium fibre laser) at the wavelength $\lambda_0$. Each next cascade is pumped by the radiation from the previous cascade. The reflection coefficients of all the Bragg gratings are close to 100 %, except one of the gratings blazed at the wavelength $\lambda_N$ (this grating is denoted by OR in Fig. 1) whose transmission coefficient should be chosen optimal to obtain the maximum output power.

Besides, the laser contains an additional Bragg grating blazed at the wavelength $\lambda_0$, which returns the pump radiation back. The Bragg gratings can be recorded both directly in the active fibre and in pieces of a special fibre, which then are spliced with the active fibre. The optical loss at each of the wavelengths are characterised by their own distributed absorption coefficient $\alpha_i$.

If $z$ is the coordinate along the fibre axis ($0 \leqslant z \leqslant L$, where $L$ is the fibre length and pairs of Bragg mirrors in all cavities are located at points $z = 0$ and $z = L$), the generation of Stokes components in the fibre during cw lasing in a multicascade Raman laser is described by the well-known system of equations (see, for example, [5, 6])

$$\frac{\mathrm{d}P_0^{\pm}(z)}{\mathrm{d}z} = \mp\big[k_0 g_0 (P_1^+ + P_1^-)P_0^{\pm} + \alpha_0 P_0^{\pm}\big],$$

$$\frac{\mathrm{d}P_i^{\pm}(z)}{\mathrm{d}z} = \pm\big[g_{i-1}(P_{i-1}^+ + P_{i-1}^-)P_i^{\pm} - k_i g_i (P_{i+1}^+ + P_{i+1}^-)P_i^{\pm} - \alpha_i P_i^{\pm}\big], \quad 0 < i < N, \tag{1}$$

$$\frac{\mathrm{d}P_N^{\pm}(z)}{\mathrm{d}z} = \pm\big[g_{N-1}(P_{N-1}^+ + P_{N-1}^-)P_N^{\pm} - \alpha_N P_N^{\pm}\big].$$

Here, $P_i^{\pm}(z)$ is the power of radiation at the wavelength $\lambda_i$ propagating in the positive (+) and negative (−) directions along the $z$-axis, respectively ($0 < i < N$); $P_0^{\pm}$ is the pump radiation power; $k_i = \lambda_{i+1}/\lambda_i$ ($0 \leqslant i < N$).

The system of equations (1) is supplemented with boundary conditions, which specify the pump radiation coupling, reflection of radiation from Bragg gratings and outcoupling of the last Stokes radiation component from the cavity, as well as the radiation loss localised at points (for example, loss at the splicing points of fibres of different types, loss at Bragg gratings, etc.).

Note that the system of equations presented above takes into account only the stimulated generation of Stokes components and neglects any other nonlinear effects. Numerous experimental papers devoted to cw Raman lasers (see, for example, Ref. [7] and references therein) confirm the validity of such simplification.

The result of the numerical solution of the system of equations (1) for a three-cascade standard germanosilicate dispersion-shifted fibre laser is presented in Fig. 2. The laser was pumped by radiation at 1.06 μm and emitted at 1.234 μm. The solution of a similar problem for a six-cascade laser is presented in Ref. [4].

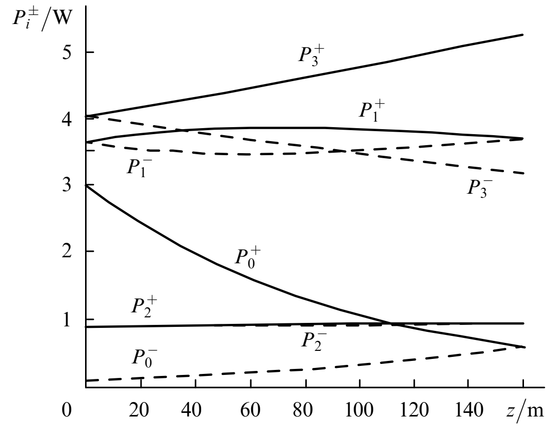


**Figure 2.** Pump power and powers of Stokes components propagating in the cavity of a three-cascade germanosilicate dispersion-shifted fibre Raman laser as functions of $z$. The pump power at the fibre input is 3 W, the fibge length is 160 m, the Raman gain of the fibre $g_0 = 5.2$ dB km$^{-1}$ W$^{-1}$, and the output power of the laser is $P_{\mathrm{out}} = P_3^+ - P_3^-$.

Note that the radiation powers of intermediate Stokes components counter-propagating in the cavity only slightly differ from each other and they can be approximately considered constant, independent of $z$. The distinct dependence on $z$ is observed only for the pump power $P_0^{\pm}$ and the power $P_N^{\pm}$ of the last Stokes component, which is related to the coupling of pump radiation into the cavity and the outcoupling of radiation from it. If we assume, as is usually done in the discussion of lasing in solid-state (not fibre) lasers, that the pump radiation is coupled uniformly over the cavity length and all loss in the cavities are distributed uniformly (see, for example, [8]), the dependence on $z$ in equations (1) will disappear and the approximation of powers of the waves propagating in a multicascade Raman laser by constants will become the exact solution of the approximate system of equations.

In this paper, we applied such an approximation to a multicascade fibre Raman laser. We assume, within the framework of this approximation, that the power of all the waves propagating in a fibre is independent of $z$ and all radiation loss in the fibre are uniformly distributed over its length and are described by the loss coefficients $A_i$.

Note that loss in the cavities of a Raman laser can be divided into three types. First, this is optical loss in the fibre (distributed loss), which are proportional to the fibre length and are characterised by coefficients $\alpha_i$. Second, this is optical loss caused by the elements of the laser design (lumped loss). They include, for example, loss at the splicing points of fibres in cavities and loss caused by the imperfections of Bragg mirrors. These loss are independent of the cavity length and are only determined by the method of fabrication of the cavity elements. Therefore, the loss coefficient averaged over a round trip in the cavity can be written as $\gamma/2L$.

The parameter $\gamma$ is defined as $\gamma = |\ln T|$, where $T = \prod_n T_n$ is the total transmission coefficient that takes into account only the lumped loss in the cavity; and $T_n$ is the 'transmission' coefficient at the $n$th point of the fibre (more exactly, of the $n$th short fibre piece) with lumped loss. Each quantity $T_n$ appears in the above product as many times as radiation passes the corresponding laser element during a round trip in the cavity. For example, the loss in each fibre splicing point (Fig. 1) will enter twice into the product. For simplicity, we will assume that $\gamma$ is independent of the emission wavelength.

Third, there are useful loss that are related to the outcoupling of the $N$th Stoke component from the laser cavity. It is known that the distributed loss are $\alpha_R = |\ln R(2L)^{-1}|$, where $R$ is the reflection coefficient of the output mirror. In simulations, the value of $\alpha_R$ should be chosen in such a way, depending on the other parameters of the problem, as to provide the maximum efficiency of the laser.

We also assume, within the framework of the approximation used, that the pump radiation is coupled uniformly over the cavity length rather than through the fibre end and that the highly reflecting mirrors (Bragg gratings) blazed at the pump wavelength are located at both ends of the fibre rather than at one end, as shown in Fig. 1.

Under such conditions, the pump radiation propagates in the fibre in the form of two counter-propagating waves with powers $P_0^+$ and $P_0^-$. It is absorbed in the cavity or is converted to the first Stokes component. For the total pump power coupled into the fibre equal to $P_{\rm p0}$, the power per the unit length is $P_{\rm p0}/2L$.

Then, because all the derivatives are zero and obvious relations $P_i^+ = P_i^- = P_i$ take place, the system of $2(N+1)$ differential equations (1) is reduced to the system of $N+1$ algebraic equations

$$\frac{P_{\rm p0}}{2L} = 2k_0 g_0 P_0 P_1 + A_0 P_0,$$

$$2g_{i-1}P_{i-1} - 2k_i g_{i+1} P_{i+1} - A_i = 0, \quad 0 < i < N, \tag{2}$$

$$2g_{N-1}P_{N-1} - A_N = 0,$$

where $A_i = \alpha_i + \gamma/2L\ (0 \leqslant i < N)$; $A_N = \alpha_N + \gamma/2L + \alpha_R$. For the problem formulated in this way, a certain power $P_i^{\pm}$ (which can be readily obtained analytically) and a certain laser efficiency correspond to each set of parameters $P_{\rm p0}$, $L$, and $\alpha_R$. By determining optimal values of $\alpha_R$ and $L$ that correspond to the maximum lasing efficiency for the given distributed and lumped loss in the cavity, we can obtain expressions for the maximum attainable efficiency. Note that all the coefficients in the system of equations (2) can be measured experimentally.

It follows from (2) that the maximum efficiency of a one-cascade Raman laser is

$$\eta_1 = \frac{1}{k_0}\left\{1 - \left[\frac{\gamma}{P_{\rm p0}}\left\langle\left[\left(\frac{\alpha_0}{g_0}\right)^{1/2} + \left(\frac{\alpha_1}{g_0}\right)^{1/2}\right]^2\right\rangle\right]^{1/2}\right\}^2. \tag{3}$$

The maximum efficiency is achieved for the optimal fibre length

$$L_{\rm opt1} = \frac{\gamma}{2(\alpha_0\alpha_1)^{1/2}}.$$

The analogous relations for a two-cascade Raman laser have the form

$$\eta_2 = \frac{1}{k_0 k_1}\left\{1 - \left[\frac{\gamma}{P_{\rm p0}}\left\langle\left[\left(\frac{\alpha_1(k_0 g_0 + g_1)}{g_0 g_1}\right)^{1/2} + \left(\frac{k_0\alpha_2 g_0 + \alpha_0 g_1}{g_0 g_1}\right)^{1/2}\right]^2\right\rangle\right]^{1/2}\right\}^2, \tag{4}$$

$$L_{\rm opt2} = \frac{\gamma}{2}\left(\frac{k_0 g_0 + g_1}{k_0 g_0 \alpha_1 \alpha_2 + \alpha_0 \alpha_1 g_1}\right)^{1/2}.$$

The corresponding relations for a three-cascade laser are more complicated:

$$\eta_3 = \frac{1}{k_0 k_1 k_2}\left\{1 - \left[\frac{\gamma}{P_{\rm p0}}\left\langle\frac{(k_0 g_0 \alpha_2 + g_1\alpha_0)(k_1 g_1 \alpha_3 + g_2\alpha_1)}{g_0 g_1 g_2}\right.\right.\right.$$
$$\left.\left.\left.\times\left[\left(\frac{k_0 g_0 + g_1}{k_0 g_0 \alpha_2 + g_1 \alpha_0}\right)^{1/2} + \left(\frac{k_1 g_1 + g_2}{k_1 g_1 \alpha_3 + g_2 \alpha_1}\right)^{1/2}\right]^2\right\rangle\right]^{1/2}\right\}^2, \tag{5}$$

$$L_{\rm opt3} = \frac{\gamma}{2}\left[\frac{(k_0 g_0 + g_1)(k_1 g_1 + g_2)}{(k_0 g_0 \alpha_2 + g_1\alpha_0)(k_1 g_1 \alpha_3 + g_2 \alpha_1)}\right]^{1/2}.$$

The corresponding relations for a Raman laser with a greater number of cascades are too cumbersome. However, they can be used if necessary.

## 3. Discussion

We can make the following conclusions based on the results obtained. The efficiency of lasers is always determined by the lumped loss $\gamma$ in cavities, which characterise the cavity design, by the distributed loss $\alpha_i$ in the fibre and Raman gains $g_i$ in the fibre, the quantities $\alpha_i$ and $g_i$ being dependent only on the fibre used.

The quantities $\alpha_i$ and $g_i$ appear in the expressions for the efficiency only as the ratio $\alpha/g$, which has the dimensionality of power. If the problem involves several Raman gains and several types of characteristic distributed loss, the final expression will contain some combination of the ratios $\alpha/g$. The expressions in angle brackets in formulas (3)–(5), which we denote as $\langle P\rangle$, contain all information on the fibre used and have the dimensionality of power. We have for a one-cascade laser

$$\langle P\rangle = \left[\left(\frac{\alpha_0}{g_0}\right)^{1/2} + \left(\frac{\alpha_1}{g_0}\right)^{1/2}\right]^2,$$

for a two-cascade laser

$$\langle P\rangle = \left\{\left[\frac{\alpha_1(k_0 g_0 + g_1)}{g_0 g_1}\right]^{1/2} + \left[\frac{k_0\alpha_2 g_0 + \alpha_0 g_1}{g_0 g_1}\right]^{1/2}\right\}^2,$$

and for a three-cascade laser

$$\langle P\rangle = \frac{(k_0 g_0 \alpha_2 + g_1\alpha_0)(k_1 g_1 \alpha_3 + g_2\alpha_1)}{g_0 g_1 g_2}$$

$$\times \left[ \left( \frac{k_0 g_0 + g_1}{k_0 g_0 \alpha_2 + g_1 \alpha_0} \right)^{1/2} + \left( \frac{k_1 g_1 + g_2}{k_1 g_1 \alpha_3 + g_2 \alpha_1} \right)^{1/2} \right]^2 .$$

The expression for the quantum efficiency of a multicascade laser can be written in the form

$$\eta_q = \left[ 1 - \left( \frac{\gamma}{P_{p0}} \langle P \rangle \right)^{1/2} \right]^2 . \quad (6)$$

The parameter $\langle P \rangle$ characterises only the fibre, which plays the role of a SRS-active medium. The efficiency of the cascade laser substantially depends on the optical loss in cavities, the role of the lumped loss in cavities being especially important. If the lumped loss are absent (more exactly, very small), the efficiency of the laser can be large (in the limit, the quantum efficiency tends to unity).

The question arises of the relation of this model to real laser systems. Let us compare the results of calculations based on this model with the real parameters of a one-cascade phosphosilicate fibre Raman laser (1.06 μm → 1.23 μm) studied in Ref. [9]. Fig. 3 shows the dependences of the output power of this laser on the fibre length calculated using our model (dashed line) and obtained by solving numerically the system of equations (1) (solid lines), as well as the experimental output power measured for a certain fibre length. The results of the numerical solution and calculations using the simple model well agree for long fibres, while their agreement is worse for short fibres. This is explained by the fact that the model neglects the pump loss after a round trip in the active fibre (the pump radiation that was not absorbed during a round trip in the fibre escapes from the system).

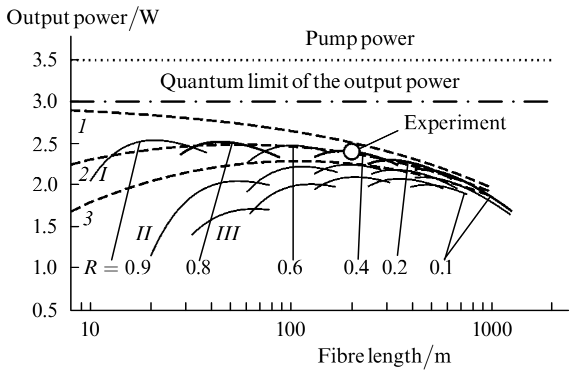


**Figure 3.** Dependences of the output power of a phosphosilicate fibre Raman laser ($\lambda_1 = 1.23$ μm, $\lambda_0 = 1.06$ μm, $P_{p0} = 3.5$ W) on the fibre length calculated using the proposed model (dashed curves) and by numerical solving the system of equations (1) (solid curves) for different reflectivities $R$ of the output Bragg grating and lumped loss in the cavity $\gamma = 0$ (*1* and group of curves *I*), 0.2 (*2* and *II*), and 0.4 dB (*3* and *III*).

The quantitative and, undoubtedly, qualitative agreement between a real laser and a simple model allows us to use analytic expressions of the model for estimates of real laser systems. For example, we can compare the efficiencies of various fibres as SRS-active media when they have different loss and gains. In particular, by using expression (3), we can compare the efficiency of one-cascade lasers based on different fibres, giving the preference to fibres with the maximum efficiency and assuming that such fibres will provide the highest efficiency in real laser systems as well.

As an example, Fig. 4 shows the dependences of the quantum efficiency $n_q$ on $\alpha_0/g_0$ and $\alpha_1/g_0$ calculated from expression (3) for the pump power $P_{p0} = 3.5$ W and the lumped loss in the cavity $\gamma = 0.4$ dB. Similarly, expressions (4) and (5) can be used to compare the efficiencies of fibres in two- and three-cascade lasers.

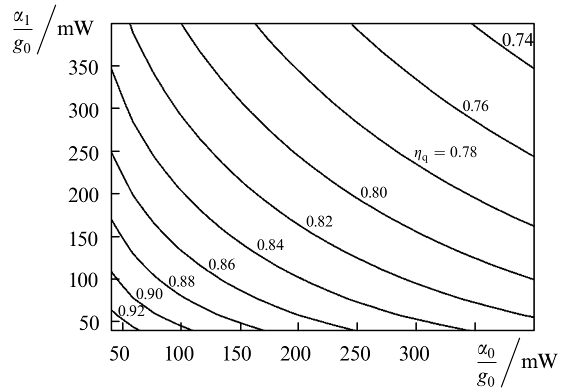


**Figure 4.** Dependences of the quantum efficiency $\eta_q$ of a model one-cascade Raman laser on parameters $\alpha_0/g_0$ and $\alpha_1/g_0$ for $P_{p0} = 3.5$ W and $\gamma = 0.4$ dB.

Finally, using the expressions for a one-cascade phosphosilicate fibre laser (3) and three-cascade germanosilicate fibre laser (5), we can compare the efficiencies of these lasers upon the 1.06 μm → 1.24 μm wavelength conversion. The parameter $\langle P \rangle$ for a three-cascade germanosilicate DSF laser equals 1.59 W, whereas for a one-cascade phosphosilicate fibre laser this parameter is only 0.88 W. In other words, despite the higher loss in phosphosilicate fibres manufactured at present ($g_0 = 5.3$ dB km$^{-1}$ W$^{-1}$, $\alpha_0 = 1.45$ dB km$^{-1}$, $\alpha_1 = 0.92$ dB km$^{-1}$), a reduction in the number of conversion cascades substantially affects the total efficiency of the laser. Thus, for $\gamma = 0.4$ dB and $P_{p0} = 3.5$ W, the maximum efficiency of the phosphosilicate fibre laser is 61 %, whereas that of the germanosilicate laser is only 46 %.

## 4. Conclusions

We proposed a simple model of the multicascade Raman laser and obtained analytic expressions describing its operation. We found the factors that determine the efficiency of multicascade Raman lasers. One of these factors is the lumped optical loss in the laser cavity. The results of model calculations allowed one to compare readily the efficiencies of different fibres as SRS-active media. Using fibres with different Stokes shifts, it is possible to compare Raman lasers with different numbers of cascades that provide the same total shift of the radiation frequency.

**Acknowledgements.** This work was partially supported by the Russian Foundation for Basic Research, Grant No. 99-02-17843.

## References

1. Grubb S G, Erdogan T, Mizrahi V, Strasser T, Cheung W Y, Reed W A, Lemmaire P J, Miller A E, Kosinski S G, Nykolak G, Beker P C, Peckman D W *Proc. Conf. Opt. Amp. Appl.* (Breckenridge, 1994), PD-3

2. Dainov E M, Grekov M V, Bufetov I A, Vasiliev S A, Medvedkov O I, Plotnichenko V G, Koltashev V V, Belov A V, Bubnov M M, Semjonov S L, Prokhorov A M *Electron. Lett.* **33** 1542 (1997)
3. Reed W A, Coughran W C, Grubb S G *Proc. Top Meet. OFC'95* (San Diego, 1995), p. 107
4. Vareille G, Audouin O, Desurvire E *Electron. Lett.* **34** 675 (1998)
5. Auyeung J, Yariv A *J. Opt. Soc Am.* **69** 803 (1979)
6. Stolen R H *Fib. Integr. Opt.* **3** 21 (1980)
7. Dianov E M, Bufetov I A, Karpov V I, Bubnov M M, Gurjanov A N, Khopin V F In: *Optical Networking* (London: Springer-Verlag), p. 165
8. Svelto O *Printsypy Lazerov (Principles of Lasers)* (Moscow: Mir, 1990)
9. Dianov E M, Bufetov I A, Bubnov M M, Grekov M V, Shubin A V, Vasiliev S A, Medvedkov O I, Semjonov S L, Guryanov A N, Khopin V F *Techn. Digest ECOC'99* (Nice, France, 1999), Mo B2.2; Dianov E M, Bufetov I A, Bubnov M M, Grekov M V, Shubin A V, Vasiliev S A, Medvedkov O I, Semenov S L, Egorova O N, Guryanov A N, Khopin V F, Yashkov M V, Varelas D, Iocco A, Constantini D, Limberger N G, Salathe R-P *Kvantovaya Elektron.* **29** 97 (1999) [*Quantum Electronics* **29** 935 (1999)]