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Fluctuations of the energy of Stokes pulses of resonance coherent SRS

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Abstract. An analytic solution is obtained for the equations of resonance coherent SRS by neglecting the population of the final level of the Raman transition for the systems with the active-medium length that is smaller than the wavelength of the incident light. For the extended systems, a numerical solution is obtained. The energy distribution of the Stokes pulses is found. The large-scale (about 100 %) fluctuations of the Stokes radiation energy were observed in the case of unsaturated amplified spontaneous emission.

1. Introduction

Raman scattering of laser radiation under the conditions of conservation of the phase memory of scattering centers, which is accompanied by the population of one of the intermediate levels, is called resonance transient (nonstationary) or coherent (cooperative) stimulated Raman scattering (SRS) [1 – 9]. A number of properties of this phenomenon have been established to date. In particular, upon excitation by a rectangular pulse, the Stokes radiation in resonance coherent SRS represents a single pulse for a transverse excitation (spatially homogeneous model) [5]. However, when the propagation effects (longitudinal excitation) are taken in account, the Stokes radiation is a set of decaying pulses [6, 7, 9].

Upon excitation by a high-power laser pulse, many molecules participate in the scattering, and it manifests cooperative properties [5, 7]. In addition, the excitation and Stokes waves exhibit a modulation [1, 2, 4 – 7] due to the oscillations of populations caused by the incident radiation [4, 6]. Upon weak coherent Raman scattering of 2π-pulses, stationary Stokes modes can be formed [3], whereas in nonlinear case the complete transformation of the excitation pulse into the Stokes pulse takes place [8].

In papers [1 – 9], radiation at the shifted frequency is formed due to amplification of the input Stokes pulse [1 – 4, 8] or spontaneous Stokes radiation [1 – 7, 9]. However, in the latter case, the stochastic nature of the SRS amplification onset is not taken in account. At the same time, since the place and time of creation of first Stokes photons and the direction of their propagation are random, one can expect that the Stokes pulse, which has been developed from a spontaneous noise, also will reveal random changes in its shape and energy. Indeed, such fluctuations were observed experimentally [10 – 12] and they were studied quite well theoretically [13 – 17] for nonresonance SRS. If the excitation radiation frequency is close to one of the intermediate transition frequencies, the phenomenon can reveal features that were not observed previously.

The aim of this paper is to study the statistical properties of the Stokes radiation energy in the resonance coherent SRS in the case of an exact resonance using the semiclassical approach and neglecting a change in the population of the final level of the Raman transition.

2. Basic equations

Let a laser wave

$$\boldsymbol{E}_{\mathrm{L}}(\boldsymbol{r},t)=\boldsymbol{e}_{\mathrm{L}}E_{\mathrm{L}}(\boldsymbol{r},t)\exp(-i\omega_{\mathrm{L}}t)+\mathrm{c.c.} \tag{1}$$

with frequency ω_{L} be incident on a Raman-active medium containing identical atoms. Here, $\boldsymbol{e}_{\mathrm{L}}$ is the unit polarisation vector, and E_{L} is the laser-wave amplitude.

We assume that the inequalities

$$|\omega_{\mathrm{L}}-\omega_{31}|\ll|\omega_{\mathrm{L}}-\omega_{i\alpha}|,\ \omega_{21},$$

$$|d_{i\alpha}E_{\mathrm{L}}|,|d_{i\alpha}E_{\mathrm{s}}|\ll\hbar|\omega_{\mathrm{L}}-\omega_{i\alpha}|,\ \hbar|\omega_{\mathrm{s}}-\omega_{i\alpha}|, \tag{2}$$

$$|d_{32}E_{\mathrm{L}}|,\ |d_{31}E_{\mathrm{s}}|\ll\hbar\omega_{21}. \tag{3}$$

are fulfilled. Here, $\omega_{\mathrm{s}}=\omega_{\mathrm{L}}-\omega_{21}$ is the central frequency of the scattered (Stokes) wave; $\omega_{i\alpha}$, $d_{i\alpha}$ and $d_{\beta\alpha}$ are the frequencies and dipole moments of the molecular transitions, where subscripts $\alpha,\beta\,(\alpha\neq\beta)=1,2,3$ correspond to the levels involved in the scattering, and the subscript $i=4,5,\ldots$ is related to all other levels (the $|1\rangle-|2\rangle$ transition is forbidden in the dipole approximation); and E_{s} is the Stokes wave amplitude. Inequalities (2) allow one to restrict the consideration to three levels (Fig. 1) and to neglect anti-Stokes scattering. Owing to the inequalities (3), one can neglect the $|2\rangle-|3\rangle$ transitions induced by the field E_{L} and the $|1\rangle-|3\rangle$ transitions induced by the field E_{s}.

In the one-dimensional approximation, the Stokes field is determined by the expression [18]

$$\boldsymbol{E}_{\mathrm{s}}(z,t)=-\frac{2\pi}{c}\int_0^L\dot{\boldsymbol{P}}_{\mathrm{s}}\left(t-\frac{|z-z'|}{c}\right)\mathrm{d}z', \tag{4}$$

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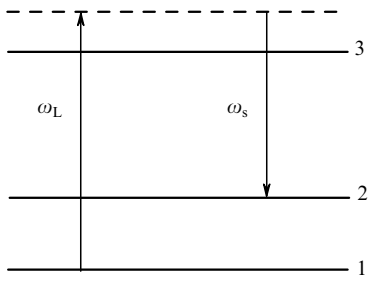

Figure 1. Energy level diagram.

where the z-axis is directed along a sample of length L.

The polarisation of the medium at frequency ω_s has the form [6, 7]

$$\boldsymbol{P}_s(z,t) = \boldsymbol{e}_s n d_{23} R_{32}(z,t)\exp(-i\omega_s t) + \text{c.c.} \tag{5}$$

Here, $R_{32}(z,t)$ is the amplitude of the nondiagonal element of the density matrix for the $|3\rangle - |2\rangle$ transition, i.e., $\rho_{32}(z,t) = R_{32}(z,t)\exp(-i\omega_s t)$; n is the concentration of atoms; and $\boldsymbol{e}_s$ is the unit polarisation vector of the Stokes wave.

Let us separate the fast dependence on time in the Stokes field:

$$\boldsymbol{E}_s(z,t) = \boldsymbol{e}_s E_s(z,t)\exp(-i\omega_s t) + \text{c.c.} \tag{6}$$

By substituting expressions (5) and (6) into (4) and neglecting the derivations of the slowly varying amplitudes, we find

$$E_s(z,t) = -\frac{2\pi\omega_s}{c} n d_{23} \int_0^L \dot{R}_{32}\left(z', t - \frac{|z-z'|}{c}\right) \times \exp\left(i\omega_s \frac{|z-z'|}{c}\right) dz'. \tag{7}$$

The elements of the density matrix for the coherent resonance SRS at an exact resonance satisfy the equations [6, 7]

$$\frac{\partial R_{21}}{\partial t} = \frac{i}{\hbar}(d_{23}E_s^* R_{31} - d_{31}E_L R_{32}^*), \tag{8}$$

$$\frac{\partial R_{32}}{\partial t} = \frac{i}{\hbar}(d_{32}E_s N_{23} + d_{31}E_L R_{21}^*), \tag{9}$$

$$\frac{\partial R_{31}}{\partial t} = \frac{i}{\hbar}(d_{31}E_L N_{13} + d_{32}E_s R_{21}), \tag{10}$$

$$\frac{\partial N_{13}}{\partial t} = \frac{2}{\hbar}\text{Re}[i(2d_{13}R_{31}E_L^* + d_{23}R_{32}E_s^*)], \tag{11}$$

$$\frac{\partial N_{23}}{\partial t} = \frac{2}{\hbar}\text{Re}[i(d_{13}R_{31}E_L^* + 2d_{23}R_{32}E_s^*)], \tag{12}$$

where $R_{\alpha 1}(z,t)$ is the amplitude of the nondiagonal element of the density matrix for the $|\alpha\rangle - |1\rangle$ $(\alpha = 2,3)$ transition, i.e., $\rho_{21}(z,t) = R_{21}(z,t)\exp(-i\omega_{21}t)$, $\rho_{31}(z,t) = R_{31}(z,t)\exp(-i\times\omega_L t)$; and $N_{\alpha 3} = \rho_{\alpha\alpha} - \rho_{33}(\alpha = 1,2)$ is the difference of the populations.

We assume that the boundary conditions for the resonance SRS are the same as those for the nonresonance SRS, which was studied earlier [19], i.e., atoms are not excited at the initial instant, and there are no fields in the medium. Raman scattering arises under the action of a rectangular pulse of height $E_L^{(0)}$ and duration t_L. Spontaneous Raman scattering from which the SRS amplification starts, is described by a given small initial Raman polarisation which is treated as a random quantity [19].

3. Solution of Maxwell–Bloch equations

3.1. Short sample

Consider first a medium small in size, i.e., a medium with $L \ll \lambda_{s,L}$ ($\lambda_{s,L}$ is the wavelength of Stokes (laser) radiation). In this case, one can neglect a retardation and dependence of the polarisation on z. Then,

$$E_s(t) = i\frac{2\pi n\omega_s L}{c} d_{23} R_{32}(t). \tag{13}$$

Taking into account that the pump pulse changes only slightly during its propagation in short systems [20], we use the approximation of a given pump field.

After substitution of expression (13) into (8) and (12), we obtain

$$\frac{dR_{21}}{d\tau} = R_{31}R_{32}^* - \mathscr{E}_L R_{32}^*, \tag{14}$$

$$\frac{dR_{32}}{d\tau} = -N_{23}R_{32} + \mathscr{E}_L R_{21}^*, \tag{15}$$

$$\frac{dR_{31}}{d\tau} = N_{13}\mathscr{E}_L - R_{21}R_{32}, \tag{16}$$

$$\frac{dN_{13}}{d\tau} = -2\mathscr{E}_L(R_{31} + R_{31}^*) + 2|R_{32}|^2, \tag{17}$$

$$\frac{dN_{23}}{d\tau} = -\mathscr{E}_L(R_{31} + R_{31}^*) + 4|R_{32}|^2, \tag{18}$$

where $\tau = t/\tau_s$; $\tau_s = \hbar c/2\pi n\omega_s|d_{23}|^2 L$ is the characteristic time scale of the resonance coherent SRS in the absence of propagation effects; and $\mathscr{E}_L = id_{31}E_L\tau_s/\hbar$.

Assume that the pulse intensity is such that the pulse produces oscillations of the populations of the resonance transition levels (i.e. of the levels $|1\rangle$ and $|3\rangle$) during the development of the process, but is not sufficient to populate the final level $|2\rangle$ of the Raman transition. It is possible if

$$T_R \leqslant t_L < t_0,$$

where $T_R = 2\pi/\omega_R$; $\omega_R = 2|d_{13}||E_L^{(0)}|/\hbar$ is the Rabi oscillation frequency for the $|1\rangle - |3\rangle$ transition and t_0 is the delay time of the Stokes pulse in a completely nonlinear problem [7]. In this case, one can neglect the effect of Stokes scattering on the pump. In addition, one can neglect terms in (16) and (17) that are proportional to R_{32}. Then, these equations take the form

$$\frac{dR_{31}}{d\tau} = \mathscr{E}_L Z_{13}, \tag{19}$$

$$\frac{dZ_{13}}{d\tau} = -2\mathscr{E}_L\text{Re}(R_{31}), \tag{20}$$

where $Z_{13} = 0.5N_{13}$. If $\mathscr{E}_L$ is real, then one can see from the latter equations that R_{13} can be also considered as real.

It follows from (19) and (20) that

$$\frac{d}{d\tau}\left(R_{31}^2 + Z_{13}^2\right) = 0,$$

which gives

$$R_{31}^2(\tau) + Z_{13}^2(\tau) = R_{31}^2(0) + Z_{13}^2(0) = 0.25. \quad (21)$$

One can conclude from (21) that

$$R_{31}(\tau) = 0.5\sin\theta(\tau), \qquad Z_{13}(\tau) = 0.5\cos\theta(\tau). \quad (22)$$

Substituting expression (22) into (19), we obtain

$$\frac{\mathrm{d}\theta}{\mathrm{d}\tau} = 2\mathscr{E}_{\mathrm{L}}. \quad (23)$$

In the case of a rectangular pulse, this equation has the solution:

$$\theta(\tau) = \begin{cases} \theta_{\mathrm{R}}\tau, & \tau \leqslant \tau_{\mathrm{L}}, \\ \theta_{\mathrm{R}}\tau_{\mathrm{L}}, & \tau \geqslant \tau_{\mathrm{L}}, \end{cases} \quad (24)$$

where $\theta_{\mathrm{R}} = 2\mathscr{E}_{\mathrm{L}}^{(0)} = \omega_{\mathrm{R}}\tau_{\mathrm{s}}$.

Because $\rho_{11} + \rho_{33} \approx 1$, then $\rho_{33} \approx 0.5 - Z_{13}$ and $N_{23} \approx -\rho_{33}$. Therefore,

$$N_{23}(\tau) = 0.5(\cos\theta - 1). \quad (25)$$

We will seek the solution of equations (14) and (15) for $R_{21}(\tau)$ and $R_{32}(\tau)$ in the form

$$R_{32}(\tau) = R(\tau)\sin\alpha(\tau), \; R_{21}^*(\tau) = R_{21}(\tau) = R(\tau)\cos\alpha(\tau). \quad (26)$$

Substituting (26) into (15), we find

$$\left[\frac{\mathrm{d}R}{\mathrm{d}\tau} + 0.5(\cos\theta - 1)R\right]\sin\alpha + \left(R\frac{\mathrm{d}\alpha}{\mathrm{d}\tau} - 0.5\theta R\right)\cos\alpha = 0.$$

Let us choose $R(\tau)$ and $\alpha(\tau)$ such that

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\tau} - 0.5\theta = 0, \quad (27)$$

$$\frac{\mathrm{d}R}{\mathrm{d}\tau} - 0.5(\cos\theta - 1)R = 0. \quad (28)$$

Equation (27) can be readily solved:

$$\alpha(\tau) = \begin{cases} 0.5\theta_{\mathrm{R}}\tau, & \tau \leqslant \tau_{\mathrm{L}}, \\ 0.5\theta_{\mathrm{R}}\tau_{\mathrm{L}}, & \tau \geqslant \tau_{\mathrm{L}}. \end{cases} \quad (29)$$

The solution of equation (28), which contains separable variables, has the form

$$R(\tau) = a\exp\left[0.5\left(\tau - \int_0^{\tau}\cos\theta\,\mathrm{d}\tau'\right)\right]$$

where

$$R(\tau) = \begin{cases} a\exp\left[0.5\left(\tau - \dfrac{1}{\theta_{\mathrm{R}}}\sin\theta_{\mathrm{R}}\tau\right)\right], \tau \leqslant \tau_{\mathrm{L}} \\ a\exp\left\{0.5\left[(1-\cos\theta_{\mathrm{R}}\tau_{\mathrm{L}})\tau + \tau_{\mathrm{L}}\cos\theta_{\mathrm{R}}\tau_{\mathrm{L}} - \dfrac{1}{\theta_{\mathrm{R}}}\sin\theta_{\mathrm{R}}\tau_{\mathrm{L}}\right]\right\}, \tau \geqslant \tau_{\mathrm{L}}. \end{cases} \quad (30)$$

Here, a is the integration constant, which can be easily found from expressions (26) and (30) as $a = R_{21}(0)$.

Let us define the intensity of Stokes radiation as the number of Stokes photons that leave the sample during the unit time per one atom:

$$I_{\mathrm{s}} = \frac{c|E_{\mathrm{s}}|^2}{2\pi\hbar\omega_{\mathrm{s}}nL} = R_{32}^2/\tau_{\mathrm{s}}. \quad (31)$$

The energy of the Stokes pulse for the time t_{L} is

$$\tilde{W}_{\mathrm{s}} = \int_0^{t_{\mathrm{L}}} I_{\mathrm{s}}(t)\mathrm{d}t = \int_0^{\tau_{\mathrm{L}}} R_{32}^2(\tau)\mathrm{d}\tau, \quad (32)$$

or, taking into account expression (30),

$$\tilde{W}_{\mathrm{s}} = a^2\int_0^{t_{\mathrm{L}}}\sin^2 0.5\tau_{\mathrm{R}}\exp\left(\tau - \frac{1}{\theta_{\mathrm{R}}}\sin\theta_{\mathrm{R}}\tau\right)\mathrm{d}\tau = a^2k^2, \quad (33)$$

where

$$k^2 = \frac{1}{2}\left[\exp\left(\tau_{\mathrm{L}} - \frac{1}{\theta_{\mathrm{R}}}\sin\theta_{\mathrm{R}}\tau\right) - 1\right]. \quad (34)$$

In the general case, the initial polarisation $R_{21}(0)$ is a complex random quantity described by a Gaussian distribution [19]. Taking into account that only long-wavelength fluctuations of the initial polarisation are mainly developed in the equations (14)–(18) [13], we will assume that the random quantity $R_{21}(0)$ is homogeneous over the sample for short systems. Then, $a = R_{21}(0)$ is a random quantity with the distribution

$$P_0(a) = \frac{2a}{\gamma^2}\exp\left(-\frac{a^2}{\gamma^2}\right), \quad (35)$$

where $\gamma = 1/\sqrt{N}$ is the width of the distribution [19] (N is a total number of atoms in the sample).

We see that the Stokes pulse energy fluctuates. The corresponding distribution function can be found using the expression

$$P(W_{\mathrm{s}}) = \left\langle\delta(\tilde{W}_{\mathrm{s}} - W_{\mathrm{s}})\right\rangle = \int\delta(\tilde{W}_{\mathrm{s}}(a) - W_{\mathrm{s}})P_0(a)\mathrm{d}a \simeq \frac{P_0(b)}{|\tilde{W}'_{\mathrm{s}}(b)|}, \quad (36)$$

where $\delta(x - x_0)$ is the delta function, and b is determined from the equation

$$\tilde{W}_{\mathrm{s}}(b) = W_{\mathrm{s}}, \quad \tilde{W}'_{\mathrm{s}}(b) = \left.\frac{\mathrm{d}\tilde{W}_{\mathrm{s}}}{\mathrm{d}a}\right|_{a=b},$$

which can be easily solved to give:

$$b = \frac{1}{k}\sqrt{W_{\mathrm{s}}}. \quad (37)$$

Calculating the derivative $\tilde{W}'_{\mathrm{s}}(b)$ and taking into account expression (19), we obtain the final expression for the distribution function:

$$P(W_{\mathrm{s}}) = \frac{1}{\gamma^2k^2}\exp\left(-\frac{W_{\mathrm{s}}}{\gamma^2k^2}\right). \quad (38)$$

The average energy of the Stokes pulse is

$$\bar{W}_{\mathrm{s}} = \int_0^{\infty} W_{\mathrm{s}}P(W_{\mathrm{s}})\mathrm{d}W_{\mathrm{s}} = \gamma^2k^2, \quad (39)$$

and its standard deviation is

$$\delta = \left[\int_0^1 (W_s - \bar{W}_s)^2 P(W_s) \mathrm{d}W_s\right]^{1/2} \bar{W}_s^{-1} = 1. \quad (40)$$

Thus, the distribution of Stokes pulses over their energies at the linear stage ($\rho_{22} \approx 1$) of the development of the resonance SRS has the exponential form

$$P(W_s) = \frac{1}{\bar{W}_s} \exp\left(-\frac{W_s}{\bar{W}_s}\right), \quad (41)$$

and the fluctuations of pulse energies about their average value reach 100 %. The average energy $\bar{W}_s$ of the Stokes pulse depends on the duration of the pump pulse and its field (see expressions (34) and (39)). The increment $\alpha_s = \tau_L - \theta_R^{-1} \sin\theta_R \tau_L$ is mainly determined by the excitation pulse width (the first term), and to a minor extent by its intensity (the second term). If τ_L and θ_R are chosen so that their product is equal to πm (m is an integer), i.e., the pump-pulse duration is multiple of the half period of Rabi oscillations, then $\alpha_s = \alpha_{s,0} = \tau_L$.

In the general case, the deviation of α_s from $\alpha_{s,0}$, given by the second term, is insignificant. This deviation and its sign are determined by the velocity $\dot{Z}_{13}$ of the transition of atoms from the level $|1\rangle$ to the level $|3\rangle$ at the moment $\tau = \tau_L$. However, note that a change in the parameters τ_L and $\mathscr{E}_L^{(0)}$ does not cause qualitative changes in the distribution $P(W_s)$.

3.2. Extended system

During propagation in extended systems, a pump pulse changes substantially its shape due to the resonance interaction with the medium [20]. Therefore, we have to discard the approximation of a given pump field. In addition, a consideration of the effects of propagation and retardation of the Stokes radiation becomes important. In this case, it is convenient to use Maxwell equations rather than the integral relations for fields.

Consider an excitation wave propagating along an extended sample. By neglecting back scattering and by separating the fast dependence $E_f \to E_f \exp(\mathrm{i}k_f z)$, ($k_f$ is the wave number, $f = L, s$) on the z coordinate in slowly-varying amplitudes introduced above, we represent the field in the sample as a superposition of the laser and forward Stokes waves. Similarly, we have for nondiagonal elements of the density matrix, $R_{21} \to R_{21} \exp[-\mathrm{i}(k_L - k_s)z]$, $R_{31} \to R_{31} \exp(-\mathrm{i}k_L z)$, $R_{32} \to R_{32} \exp(-\mathrm{i}k_s z)$. Then, for the three-level model of an atom, the truncated Maxwell equations for the amplitudes of the fields have the form [6, 7]

$$\frac{\partial \varepsilon_L}{\partial \xi} = -b_L R_{31}, \quad (42)$$

$$\frac{\partial \varepsilon_s}{\partial \xi} = -b_s q_s R_{32}. \quad (43)$$

Eqs (8) – (12) for the medium can be transformed to

$$\frac{\partial R_{21}}{\partial \tau} = -b_s \varepsilon_s^* R_{31} - b_L^* \varepsilon_L R_{32}^*, \quad (44)$$

$$\frac{\partial R_{31}}{\partial \tau} = b_L^* \varepsilon_L N_{13} + b_s^* \varepsilon_s R_{21}, \quad (45)$$

$$\frac{\partial R_{32}}{\partial \tau} = b_s^* \varepsilon_s N_{23} + b_L^* \varepsilon_L R_{21}^*, \quad (46)$$

$$\frac{\partial N_{13}}{\partial \tau} = -2\mathrm{Re}(2b_L R_{31} \varepsilon_L^* + b_s R_{32} \varepsilon_s^*), \quad (47)$$

$$\frac{\partial N_{23}}{\partial \tau} = -2\mathrm{Re}(b_L R_{31} \varepsilon_L^* + 2b_s R_{32} \varepsilon_s^*). \quad (48)$$

Here, we use the dimensionless quantities $\xi = z\Omega/c$, $\tau = \Omega(t - z/c)$, and $\varepsilon_f = \mathrm{i}d_{31}E_f/\hbar\Omega$ ($f = \mathrm{L}, \mathrm{s}$), where Ω^{-1} is the time scale and $c\Omega^{-1}$ is the spatial scale, taking into account the propagation effects in SRS; $\Omega = |d_{13}|(\pi n \omega_L/\hbar)^{1/2}$; $b_L = d_{13}/|d_{13}|$; $b_s = d_{23}/|d_{13}|$; and $q_f = \omega_f/\omega_L (f = \mathrm{L}, \mathrm{s})$.

Consider SRS in extended systems of length $l \leqslant 1$ at which an attenuation of the pump is insignificant, where $l = L\Omega/c$ [7]. Assume also that $b_L \approx b_s \approx 1$ and $q_s \approx 1$. The pump pulse duration $\tau_L = t_L\Omega$ is chosen such that the population of the level $|2\rangle$ during the exposure does not exceed 0.001.

To study the statistical properties of SRS in an extended medium, we accomplished a series of calculations of the kinetics of scattering radiation. As in the case of short samples, the Stokes pulse energy (the average number of Stokes photons emitted by a single atom)

$$\bar{W}_s = \frac{1}{l} \int_0^{\tau_L} |\varepsilon_s(L, \tau)|^2 \mathrm{d}\tau \quad (49)$$

fluctuates from one realisation of random polarisation to another. To provide a sufficient smoothness of the distribution, we considered an ensemble of the comparatively large number m of trajectories (about 1500). The range of the energy values $[0, 4\bar{W}_s]$ was divided into intervals of length $\Delta W_s = h\bar{W}_s$ ($h = 0, 2$). A number of trajectories with the values of $\bar{W}_s$ lying in the interval $[W_s - 0.5\Delta W_s, W_s + 0.5\Delta W_s]$, was calculated, where $W_s = \mathrm{i}h\bar{W}_s$. The function

$$P(W_s) = \frac{m_i}{hm\bar{W}_s}$$

is the required distribution.

We assumed in calculations, that $l = 1$, and a total number of atoms in the sample is $N = 5 \times 10^{13}$. The number n of transverse layers, into which the sample was divided upon specifying a random initial polarisation, was varied from 1 to 100 [19]. For a homogeneous random polarisation ($n = 1$, $\gamma = \sqrt{2} \times 10^{-7}$), the distribution $P(W_s)$ is exponential, and the deviation of the Stokes pulse energies from their average value is 98 % (Fig. 2). By comparing this result with expressions (40) and (41), we can see that the shape of the distribution $P(W_s)$ did not change in passing from a short sample to the extended one. An increase in the number n of layers did not virtually affect the distribution and the magnitude of the variation ($\delta = 100$, 102 and 97 % for $n = 10, 50$ and 100, respectively).

As in the case of a short sample, the average Stokes energy increases with the duration of the pump pulse (Table 1), but much weaker than it follows from expressions (34) and (39). This result is explained by the behaviour of the excitation pulse in the resonance medium. The pump pulse propagates in a such medium slowly at the group velocity that depends on the pulse intensity [20], and therefore the excitation of atoms occurs gradually as the field ε_L penetrates inside the sample.

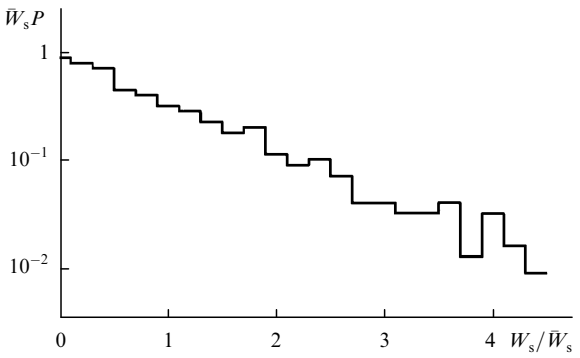

Figure 2. Probability density distribution over the energies of Stokes pulses for $\varepsilon_L^{(0)} = 0.25, l = 1$, and $\tau_L = 24\pi$.

Moreover, the field is inhomogeneous over the sample, and hence, Rabi frequencies $\theta_R = 2\varepsilon_L$ are different in different sites of the sample. It also explains the dependence of $\bar{W}_s$ on the input field $\varepsilon_L^{(0)}$ even when the half-period of Rabi oscillations related to $\varepsilon_L^{(0)}$ is a multiple of the pump pulse duration of (this case is considered in Table 1). The energy $\bar{W}_s$ increases with the pump intensity.

Table 1. Average energies and standard deviations of the Stokes pulse energy

τ_L	$\varepsilon_L^{(0)}$	$\bar{W}_s$	$\delta(\%)$
18π	0.25	8.7×10^{-7}	100.0
24π	0.25	8.6×10^{-5}	96.5
30π	0.25	3.8×10^{-3}	94.7
24π	0.125	1.1×10^{-6}	104.2
24π	0.50	9.2×10^{-4}	98.5

4. Conclusions

Thus, the resonance coherent SRS, which is not accompanied by the population of the final level of the Raman transition, exhibits large-scale fluctuations (of the order of 100 %) of the Stokes energy. The energy distribution of Stokes pulses is close to the exponential one. The average Stokes energy increases with the pump pulse duration and intensity. The dependence of $\bar{W}_s$ on τ_L and $\varepsilon_L^{(0)}$ in the extended system is more complicated due to the features of the coherent interaction of the pump radiation with the resonance medium.

The resonance SRS was observed at the electronic sublevels of alkali metal vapours [21 – 23] and for a number of other elements with relatively simple atomic energy level diagrams [24, 25]. Under typical experimental conditions, for instance, for the electronic SRS in In vapours upon excitation with a dye laser [24], $d_{13} = d_{23} = d \simeq 0.1$ D, $p \simeq 1$ kPa, $T \simeq 1$ kK, $T_{21} \simeq 10^{-8}$ s, $G_L \simeq 10^7$ W cm^{-2}, $\omega_L \simeq 4.6\times 10^{15}$ c^{-1}, $t_L \simeq 3\times10^{-9}$ s, $L \simeq 1$ cm.

Under such conditions, $\omega_R = dE_L/\hbar \simeq 1.5\times10^{10}$ s^{-1}, $\Omega = d(\pi n\omega_L/\hbar)^{1/2} \simeq 3.2\times10^{10}$ s^{-1}, $c\Omega^{-1} \simeq 1$ cm, so that $\varepsilon_L = dE_L/\hbar\Omega = \omega_R/\Omega \simeq 0.5\tau_L = t_L/\Omega^{-1} \simeq 100, l = L/c\Omega^{-1} \simeq 1$, which is close to the values of the parameters used in the calculation. Moreover, the condition $t_L < T_{21}$ is fulfilled. However, a comparison of the above theory with experiment seems to be impossible. To date, no experimental data on the statistical properties of the resonance coherent SRS are available.

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