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Nonparaxiality of dissipative optical solitons

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Abstract. A version of the perturbation theory is developed for determining the field distribution of spatial solitons with a 2D transverse profile in a medium with saturable absorption and gain in the case of small deviations from paraxial conditions. Starting from the unperturbed paraxial soliton with the linear polarisation of radiation, an approximate master equation is derived for transverse components of the electric field in the case of wide solitons. It is shown that its solution represents a stable weakly nonparaxial dissipative optical vector soliton with an axially asymmetric field distribution.

1. Introduction

Conservative spatial optical solitons represent stable self-channelling structures (pseudoparticles) in a transparent medium with the self-focusing nonlinearity of the refractive index, for which the diffraction spread of a beam is compensated by its nonlinear compression. They served as one of the first objects of nonlinear optics and have been actively studied since the early 1960s. [1]. Conservative solitons represent a family with a continuous spectrum of their basic characteristics, for instance, the maximum radiation intensity. Solitons of a qualitatively different kind, namely, dissipative optical solitons (DOSs) or autosolitons with a discrete spectrum of characteristics, are formed in dissipative systems, such as passive nonlinear interferometers excited by external radiation [2, 3], lasers with saturable absorption [4, 5], etc. (see also [6 – 8]).

The discrete nature of the spectrum of DOSs and a rigid (threshold) character of their excitation lead to an increased stability and an efficient suppression of noise, which is of interest for applications in optical data processing. For the majority of applications, generally speaking, of both conservative and dissipative solitons, it is desirable to minimise their size. When a soliton becomes comparable in size with the optical wavelength, its nonparaxiality becomes of substantial importance. Note that this feature is not taken into account within the framework of the standard quasi-optical approximation (the approximation of slowly varying variables or envelopes). The nonparaxiality may also have a strong effect on the polarisation structure of the soliton field, even in the case of rather wide solitons.

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As far as we know, the nonparaxiality has been previously considered in the literature only for conservative solitons. For weakly paraxial solitons, the method of the perturbation theory was proposed [9 – 11] in which the ratio of optical wavelength to the soliton width was used as a small parameter. In the region of strong nonparaxiality, optical needles, i.e., spatial solitons with the width smaller than the wavelength of light, were found by semianalytical and numerical methods [12]. In this paper, the emission of weakly nonparaxial DOSs is studied. To reveal the nonparaxiality, we consider monochromatic optical beams with a 2D transverse profile in a scheme without a cavity, which represents a continuous medium with saturable gain and absorption (see, e.g., [7]). We will be predominantly interested in polarisation effects, which are absent in the paraxial approximation.

2. Master equation

We start from the Maxwell equations for monochromatic radiation with frequency ω in a nonmagnetic medium (the magnetic permeability is equal to unity)

$$\mathrm{rot}\boldsymbol{E} = \mathrm{i}\frac{\omega}{c}\boldsymbol{H},\ \ \mathrm{rot}\boldsymbol{H} = -\mathrm{i}\frac{\omega}{c}\boldsymbol{D},\ \ \mathrm{div}\boldsymbol{H} = 0,\ \ \mathrm{div}\boldsymbol{D} = 0. \quad (1)$$

Here, $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic field strengths [in the complex notation, the factor $\exp(-\mathrm{i}\omega t)$ is omitted] and c is the speed of light in vacuum. The third- and higher-harmonic generation is assumed to be inefficient (the corresponding phase-matching conditions are not fulfilled). To separate the nonparaxiality, we use the simplified equation for the electric induction $\boldsymbol{D}$, which corresponds to the striction nonlinearity in the case of self-focusing

$$\boldsymbol{D} = \left[\varepsilon_0 + \delta\varepsilon\left(|\boldsymbol{E}|^2\right)\right]\boldsymbol{E}, \quad (2)$$

where ε_0 and $\delta\varepsilon$ are the linear (real) and nonlinear permittivites of a medium.

Using the method, which was previously employed in Ref. [11] for conservative solitons, we will derive an approximate closed equation for the transverse field components $\boldsymbol{E}_\perp = \{E_x, E_y\}$ of a stationary dissipative spatial soliton. For this soliton, the longitudinal (along the coordinate z) field variation is described by the factor $\exp(\mathrm{i}\Gamma z)$ with a real propagation constant Γ. Excluding from (1) the magnetic field strength, we obtain the generalised nonlinear Helmholtz equation

$$\Delta_\perp \boldsymbol{E}_\perp + (k^2 - \Gamma^2)\boldsymbol{E}_\perp + \frac{k^2}{\varepsilon_0}\delta\varepsilon\boldsymbol{E}_\perp - (\mathrm{grad}\,\mathrm{div}\boldsymbol{E})_\perp = 0, \quad (3)$$

where

$$\Delta_\perp = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

is the transverse Laplacian and $k = (\omega/c)\sqrt{\varepsilon_0}$ is the wave number of light in the linear medium. Taking into account the smallness of $|\delta\varepsilon|$, we obtain from the last of the Maxwell equations (1) the following equation for the longitudinal field component:

$$E_z = \frac{\mathrm{i}}{\Gamma}\left[\mathrm{div}_\perp \boldsymbol{E}_\perp + \frac{1}{\varepsilon_0 + \delta\varepsilon}\big(\boldsymbol{E}_\perp, \mathrm{grad}_\perp \delta\varepsilon(|\boldsymbol{E}_\perp|^2)\big)\right]$$

$$\approx \frac{\mathrm{i}}{k}\,\mathrm{div}_\perp \boldsymbol{E}_\perp. \quad (4)$$

As a result, we obtain a closed approximate equation for the transverse field components of wide stationary spatial solitons

$$\Delta_\perp \boldsymbol{E}_\perp + \left(k^2 - \Gamma^2\right)\boldsymbol{E}_\perp + \frac{k^2}{\varepsilon_0}\delta\varepsilon\big(|\boldsymbol{E}_\perp|^2\big)\boldsymbol{E}_\perp = \boldsymbol{Q}_\perp(\boldsymbol{E}_\perp), \quad (5)$$

where

$$\boldsymbol{Q}_\perp(\boldsymbol{E}_\perp) = -\frac{1}{\varepsilon_0}\Big[\delta\varepsilon'\big(|\boldsymbol{E}_\perp|^2\big)|\mathrm{div}_\perp \boldsymbol{E}_\perp|^2 \boldsymbol{E}_\perp$$

$$+ \mathrm{grad}_\perp\big(\boldsymbol{E}_\perp, \mathrm{grad}_\perp \delta\varepsilon\big(|\boldsymbol{E}_\perp|^2\big)\big)\Big]; \quad (6)$$

$$\delta\varepsilon'(I) = \frac{\mathrm{d}\delta\varepsilon}{\mathrm{d}I}; \quad I = |\boldsymbol{E}|^2. \quad (7)$$

For the Kerr nonlinearity ($\delta\varepsilon = \varepsilon_2|\boldsymbol{E}|^2, \varepsilon_2 > 0$), these relations are transformed into the corresponding equations of Ref. [11].

To emphasise the role of energy dissipation, we consider a medium with nonlinearity of the absorption and gain only. In this case,

$$\delta\varepsilon\big(|\boldsymbol{E}|^2\big) = -\mathrm{i}\varepsilon_0\mu f\big(|\boldsymbol{E}|^2\big),$$
$$f\big(|\boldsymbol{E}|^2\big) = -1 - \frac{a_0}{1 + |\boldsymbol{E}|^2/I_a} + \frac{g_0}{1 + |\boldsymbol{E}|^2/I_g}, \quad (8)$$

where $k\mu$ is the coefficient of constant (nonresonance) absorption (in intensity); $\mu \ll 1$; a_0 and g_0 are the linear coefficients of resonant absorption and gain, which are normalised to the nonresonance absorption coefficient; and I_a and I_g are the saturation intensities for amplification and absorption. For the characteristic shift of the propagation constant, we introduce the dimensionless quantity

$$\alpha = \frac{k^2 - \Gamma^2}{k^2\mu}, \quad (9)$$

and the dimensionless transverse coordinates that are obtained by multiplying the dimensional coordinates by $k\sqrt{\mu}$. As a result, Eqn (5) takes the form

$$\Delta_\perp \boldsymbol{E}_\perp + \alpha\boldsymbol{E}_\perp - \mathrm{i}f\big(|\boldsymbol{E}_\perp|^2\big)\boldsymbol{E}_\perp = \mathrm{i}\mu\boldsymbol{q}_\perp(\boldsymbol{E}_\perp), \quad (10)$$

where

$$\boldsymbol{q}_\perp = \boldsymbol{E}_\perp|\mathrm{div}_\perp \boldsymbol{E}_\perp|^2 f'\big(|\boldsymbol{E}_\perp|^2\big)$$

$$+ \mathrm{grad}_\perp\big(\boldsymbol{E}_\perp, \mathrm{grad}_\perp f\big(|\boldsymbol{E}_\perp|^2\big)\big), \quad f'(I) = \frac{\mathrm{d}f}{\mathrm{d}I}. \quad (11)$$

In Cartesian coordinates,

$$q_x = \left|\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}\right|^2 E_x f' + \frac{\partial}{\partial x}\left(E_x\frac{\partial f}{\partial x} + E_y\frac{\partial f}{\partial y}\right),$$
$$q_y = \left|\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}\right|^2 E_y f' + \frac{\partial}{\partial y}\left(E_x\frac{\partial f}{\partial x} + E_y\frac{\partial f}{\partial y}\right). \quad (12)$$

The parameter α, which plays the role of an eigenvalue in Eqn (10), should be determined. Recall that the spectrum of α for DOSs being studied here is discrete, whereas the spectrum for conservative solitons is continuous. Moreover, the field envelope of DOSs is described by complex functions, i.e., the wave front of DOSs is necessarily curved, whereas the class of conservative solitons contains solitons with a plane front.

3. Solution of the master equation in the perturbation theory

When deriving Eqn (10), we restricted our consideration to the lowest corrections to the standard quasi-optical (nonparaxial) approximation, assuming that the nonlinearity is weak and the soliton width is sufficiently large (compared to the wavelength λ). Therefore it is reasonable to solve this equation using the perturbation theory, with $\mu \sim (\lambda/w)^2$ considered as a small parameter, where w is the characteristic soliton width. Note that expression (11) enables one to determine the desired quantities only in the zero and first orders of the perturbation theory. To find them in higher orders of the perturbation theory, one should include in (11) additional terms.

The solution of Eqn (10) is sought in the form

$$\boldsymbol{E}_\perp = \boldsymbol{E}_{\perp 0} + \mu\delta\boldsymbol{E}_\perp + ..., \quad \alpha = \alpha_0 + \mu\delta\alpha + ... \quad (13)$$

The zero approximation ($\boldsymbol{E}_\perp = \boldsymbol{E}_{\perp 0}$ and $\alpha = \alpha_0$) corresponds to the standard paraxial description in which $\boldsymbol{q}_\perp = 0$ and (10) is transformed into the vector quasi-optical equation

$$\Delta_\perp \boldsymbol{E}_\perp + \alpha_0\boldsymbol{E}_\perp - \mathrm{i}f\big(|\boldsymbol{E}_\perp|^2\big)\boldsymbol{E}_\perp = 0. \quad (14)$$

In the first order of the perturbation theory, we obtain for $\delta\boldsymbol{E}_\perp$ the inhomogeneous linear equation

$$\Delta_\perp \delta\boldsymbol{E}_\perp + \alpha_0\delta\boldsymbol{E}_\perp - \mathrm{i}f\big(|\boldsymbol{E}_{\perp 0}|^2\big)\delta\boldsymbol{E}_\perp - \mathrm{i}\boldsymbol{E}_{\perp 0}f'[(\boldsymbol{E}^*_{\perp 0}, \delta\boldsymbol{E}_\perp)$$

$$+ (\boldsymbol{E}_{\perp 0}, \delta\boldsymbol{E}^*_\perp)] = -\delta\alpha\boldsymbol{E}_{\perp 0} + \mathrm{i}\boldsymbol{q}(\boldsymbol{E}_{\perp 0}). \quad (15)$$

Let us write Eqn (14) in Cartesian coordinates

$$\Delta_\perp E_{x0} + \alpha_0 E_{x0} - \mathrm{i}f\big(|E_{x0}|^2 + |E_{y0}|^2\big)E_{x0} = 0,$$
$$\Delta_\perp E_{y0} + \alpha_0 E_{y0} - \mathrm{i}f\big(|E_{x0}|^2 + |E_{y0}|^2\big)E_{y0} = 0. \quad (16)$$

The solutions of this system are most completely studied in the literature for the linear polarisation. We shall restrict our further analysis to this case (note that more general relations, which are presented in Ref. [11] for conservative solitons, are valid for DOS as well). For definiteness, we consider the lowest (fundamental) solitons with linear polarisation and an axially symmetric (in the zero approximation) field distribution:

$$E_{x0} = E_{x0}(\rho), \quad E_{y0} = 0, \quad \rho = \left(x^2 + y^2\right)^{1/2}. \qquad (17)$$

In the first order of the perturbation theory, the equations linearised with respect to a small perturbation $\delta \boldsymbol{E}_\perp$ can be written in the form

$$L_x(\delta E_x) = -\delta\alpha E_{x0} + \mathrm{i} q_{x0}, \quad L_y(\delta E_y) = \mathrm{i} q_{y0}, \qquad (18)$$

where

$$L_x(\delta E_x) = \left[\Delta_\perp + \alpha_0 - \mathrm{i}\left(f + f'|E_{x0}|^2\right)\right]\delta E_x - \mathrm{i} f' E_{x0}^2 \delta E_x^*;$$

$$L_y(\delta E_y) = \left(\Delta_\perp + \alpha_0 - \mathrm{i} f\right)\delta E_y; \qquad (19)$$

$$q_{x0} = f' E_{x0}\left|\frac{\partial E_{x0}}{\partial x}\right|^2 + \frac{\partial}{\partial x}\left(E_{x0}\frac{\partial f}{\partial x}\right); \quad q_{y0} = \frac{\partial}{\partial y}\left(E_{x0}\frac{\partial f}{\partial x}\right).$$

The solutions of the homogeneous system

$$L_x(\delta E_x) = 0, \quad L_y(\delta E_y) = 0, \qquad (20)$$

which corresponds to (18), for finite boundary conditions and a sufficiently fast decrease in $\delta E_\perp$ with increasing transverse coordinates have the same form as for conservative solitons [11].

To solve linear inhomogeneous equation (18), we represent $\boldsymbol{q}_{\perp 0}$ in the form

$$q_{x0} = q_0(\rho) + q_2(\rho)\cos 2\varphi, \quad q_{y0} = q_2(\rho)\sin 2\varphi, \qquad (21)$$

where

$$q_0(\rho) = \frac{1}{2}\left(\frac{\mathrm{d}}{\mathrm{d}\rho} + \frac{1}{\rho}\right)G; \quad q_2(\rho) = \frac{1}{2}\left(\frac{\mathrm{d}}{\mathrm{d}\rho} - \frac{1}{\rho}\right)G;$$
$$G = E_{x0} f' \frac{\mathrm{d}|E_{x0}|^2}{\mathrm{d}\rho}. \qquad (22)$$

The second of the equations (18) takes the form

$$(\Delta_\perp + \alpha_0 - \mathrm{i} f)\delta E_y = \mathrm{i} q_2(\rho)\sin 2\varphi. \qquad (23)$$

The corresponding homogeneous equation has the unique nontrivial axially symmetric solution $\delta E_y = E_{x0}(\rho)$. Because this solution (as well as the solution of the corresponding orthogonal equation) is orthogonal to the right-hand side of Eqn (23), this equation can be solved. Without loss of generality one may equate to zero the solution of the homogeneous equation that corresponds to the rotation of a DOS as a whole in the plane xy. Then, we can assume that Eqn (23) with appropriate boundary conditions has a unique solution of the form

$$\delta E_y = \delta E_{y2}(\rho)\sin 2\varphi. \qquad (24)$$

The function $\delta E_{y2}(\rho)$ is determined as a solution, finite on the interval $0 < \rho < \infty$, of the ordinary differential equation

$$\frac{\mathrm{d}^2\delta E_{y2}}{\mathrm{d}\rho^2} + \frac{1}{\rho}\frac{\mathrm{d}\delta E_{y2}}{\mathrm{d}\rho} + \left(\alpha_0 - \frac{4}{\rho^2} - \mathrm{i} f\right)\delta E_{y2} = \mathrm{i} q_2(\rho). \qquad (25)$$

The linearised equation for δE_x has the form

$$\left[\Delta_\perp + \alpha_0 - \mathrm{i}\left(f + f'|E_{x0}|^2\right)\right]\delta E_x - \mathrm{i} f' E_{x0}^2 \delta E_x^*$$
$$= -\delta\alpha E_{x0} + \mathrm{i} q_0(\rho) + \mathrm{i} q_2(\rho)\cos 2\varphi. \qquad (26)$$

One may claim that Eqn (26) also has a unique solution of the form

$$\delta E_x = \delta E_{x0}(\rho) + \delta E_{x2}(\rho)\cos 2\varphi. \qquad (27)$$

The determination of the radial functions $\delta E_{x0}(\rho)$ and $\delta E_{x2}(\rho)$, taking into account the value found for $\delta\alpha$, is reduced to finding the solution of two linear homogeneous ordinary differential equations

$$\frac{\mathrm{d}^2\delta E_{x0}}{\mathrm{d}\rho^2} + \frac{1}{\rho}\frac{\mathrm{d}\delta E_{x0}}{\mathrm{d}\rho} + \left(\alpha_0 - \mathrm{i} f - \mathrm{i} f'|E_{x0}|^2\right)\delta E_{x0}$$
$$- \mathrm{i} f' E_{x0}^2 \delta E_{x0}^* = \mathrm{i} q_0(\rho) - \delta\alpha E_{x0}(\rho),$$
$$\frac{\mathrm{d}^2\delta E_{x2}}{\mathrm{d}\rho^2} + \frac{1}{\rho}\frac{\mathrm{d}\delta E_{x2}}{\mathrm{d}\rho} + \left(\alpha_0 - \frac{4}{\rho^2} - \mathrm{i} f - \mathrm{i} f'|E_{x0}|^2\right)\delta E_{x2} \qquad (28)$$
$$- \mathrm{i} f' E_{x0}^2 \delta E_{x2}^* = \mathrm{i} q_2(\rho),$$

which is finite on the interval $0 < \rho < \infty$. In accordance with (24), the longitudinal field component has the form

$$E_z \approx \frac{\mathrm{i}}{k}\frac{\partial E_{x0}}{\partial x} = \frac{\mathrm{i}}{k}\frac{\mathrm{d}E_{x0}}{\mathrm{d}\rho}\cos\varphi. \qquad (29)$$

In our opinion, the most striking experimental manifestation of the nonparaxiality of DOSs is the change in the polarisation of radiation. To be specific, the polarisation becomes elliptic, and the state of polarisation changes over the cross section. The lowest nonparaxial correction is given by the longitudinal field component E_z (29). The end of the electric field vector rotates in time along a strongly elongated ellipse, which lies in the plane xz. In the next order, a weaker field component δE_y appears. If this component is taken into account, the ellipse leaves the plane xz, and the orientation of its plane varies over the cross section.

The radial profile of the function $\delta E_{y2}(\rho)$, obtained by numerical solution of linear Eqn (25), is shown in Fig. 1. The angular dependence of δE_y has the simple form (24). The radial profile of the y component of the radiation intensity $I_y = |\delta E_y|^2$ has a well-pronounced maximum near the inflection point of the radial dependence of $|E_{x0}|$. Because of this, the intensity I_y in the plane xy is represented by four symmetrically positioned peaks, whereas the intensity of the z component is represented by two peaks [see (29)]. Note that this field structure is also typical of optical needles, which represent strongly nonparaxial conservative solitons [12].

The stability of weakly nonparaxial DOSs does not require an additional analysis. Indeed, we analysed the initial paraxial solitons only inside the region of their stability (the latter is presented in Fig. 3 of Ref. [8]). In this case, the eigenvalues of the corresponding linearised equations for weak perturbations either vanish (for neutral modes corresponding to the symmetry of the problem) or have a negative real part, which corresponds to the soliton stability. The nonparaxial corrections retain the symmetry of the problem and, there-

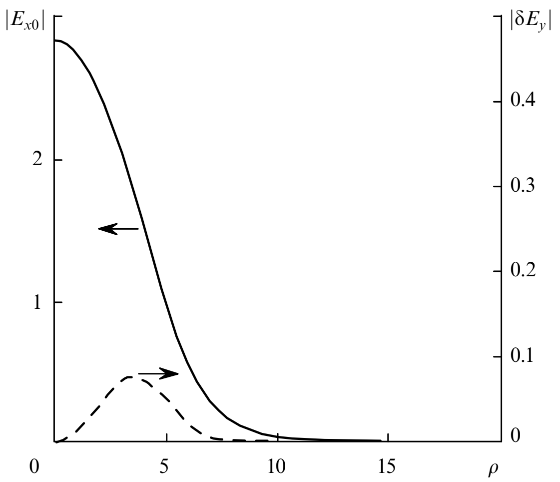

Figure 1. Radial profiles of the magnitudes of amplitudes of the initial paraxial soliton $|E_{x0}|$ (solid curve) and the y component of the normalised nonparaxial correction $|\delta E_y|$ (dashed curve) for the parameters of a medium $a_0 = 2$, $g_0 = 2.1$, and $I_g/I_a = 10$.

fore, the zero eigenvalues. As for the nonzero eigenvalues, the sign of their real part cannot be changed, at least because of the smallness of nonparaxial corrections. In reality, it is reasonable to expect that the nonparaxiality additionally stabilises solitons. In this respect, DOSs once again manifest a particular stability, for instance, compared to conservative solitons in a transparent medium with the Kerr nonlinearity. In the latter case, nonparaxial solitons have purely imaginary eigenvalues, so that even a slightest perturbation can qualitatively change the character of stability. This problem will be analysed elsewhere.

4. Conclusions

We have developed here the version of the consistent perturbation theory, which enables one to determine the lowest nonparaxial corrections to the field structure of dissipative optical vector solitons with a 2D transverse profile. Their field has a simple form of polarisation and azimuthal structure. The deviation from nonparaxiality leads to the appearance of a noticeable longitudinal field component, and because of this it is inappropriate to analyse the nonparaxiality of the electromagnetic field within the framework of the scalar nonlinear Helmholtz equation. It seems that the nonparaxiality is best evident in the experiment in the form of a typical azimuthal structure of the field component with linear polarisation that is orthogonal to the major one.

The nonparaxial soliton produced by a paraxial soliton with linear polarisation is axially symmetric. The approach developed here can also be used for the analysis of three-dimensional conservative and dissipative solitons, i.e., the so-called optical and laser bullets [7, 13, 14]. Moreover, it may be of interest to study strongly nonparaxial supernarrow spatial solitons, i.e., optical needles [12], which were not considered here. Such solitons are most naturally formed in media having nonlinearity of the refractive index (Kerr nonlinearity) in addition to the nonlinearity of gain.

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