

Propagation of ultrashort electromagnetic pulses in a Kerr medium with impurity atoms under quasi-resonance conditions

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Abstract. The study of the propagation of an electromagnetic ultrashort pulse in a Kerr medium with impurity atoms under quasi-resonance conditions is reported, which demonstrates the efficiency of the unitary transformation method in nonlinear optics. An equation is derived, which differs from the known equations describing the evolution of a pulse envelope and takes into account the dispersion of the nonlinear response and the dispersion of group velocities.

1. Introduction

One of the objects studied in nonlinear fibre optics is an optical fibre containing resonance impurity atoms [1–18]. If the population of resonance levels is inverted by an additional pumping, a fibre doped with Er^{3+} ions serves as an active medium of a fibre amplifier [1, 2]. A fibre directional coupler, whose channels are doped with resonance atoms, may be used as an all-optical switch [3, 4]. Under certain conditions, the self-induced transparency is possible in a doped optical fibre [5–10]. The experimental observation of this phenomenon [12] in an optical fibre doped with Er^{3+} ions stimulated a further study of the coherent propagation of optical USP in such media. In Refs [12–18], the amplification of USP was considered, and in Refs [19–21], new regimes of their propagation were studied. Models of resonance and nonresonance subsystems were further generalised in [22–24]. In Ref. [11], the nonlinear absorption caused by the two-photon resonance was studied.

When the self-induced transparency is not realised, impurities can introduce additional losses. As noted in Ref. [12], the energy of an optical soliton travelling in a nonlinear fibre without resonantly absorbing impurities is several hundred times lower than the energy required for the total inverse population of resonance levels. Thus, the effect of resonance impurities on optical solitons in a fibre is reduced only to absorption (provided special means, as in Ref. [12], are not used). The effect of losses can be strongly weakened by increasing detuning Δ from the resonance. In this case, the role of impurity atoms is reduced mainly to

the modification of the refractive index of a fibre. It is evident that the frequency detuning Δ should not be too large. Otherwise, it is meaningless to speak about resonance impurities. This situation, which will be called quasi-resonance, is characterised by the inequalities $|\Delta| \ll \omega$ and $|\Delta| \ll |\omega - \omega_{\alpha\beta}|$, where ω is the carrier-wave frequency and $\omega_{\alpha\beta}$ are the frequencies of atomic transitions in an impurity (except the resonance transition).

The theory describing the propagation of optical USP under quasi-resonance conditions has been developed rather long ago [25–27] and is based on the adiabatic following approximation (AFA). The time of changes in the envelope of an optical pulse was assumed to be much greater than Δ . This enables one to replace the initial system of equations with a single nonlinear wave equation for the slowly varying complex pulse envelope. However, the approximation of slowly varying envelope and phase of an optical pulse (SVEPA) was already used in the initial system of Maxwell-Bloch equations. Some effects, for instance, the level shift due to the quadratic Stark effect, are not taken into account in this approximation. The resonance (or quasi-resonance) conditions separate a pair of levels from the total spectrum of an impurity atom, thereby introducing the concept of a two-level atom as a model of a resonance medium. It would be more correct to develop the AFA on the basis of equations for the density matrix that are obtained for a two-level atom without any other assumptions.

In this paper, the adiabatic following approximation will be obtained starting from the model of two-level atoms without the use of Bloch equations in the rotating wave approximation, which makes possible a more exact representation of the expression for the polarisation of the resonance subsystem. In this case, one can take into account the resonant contributions through the phenomenological introduction of nonlinear susceptibilities. As a result, corrections to the basic equations [25–27] will be obtained.

An efficient means for solving nonlinear optical problems is the unitary transformation method [28–30]. It enables one to obtain a more exact, in comparison with Ref. [25], solution of Bloch equations, which describe the evolution of the state of a two-level atom, and find an expression for the polarisation in the form of a series in powers of the ratio of the instantaneous Rabi frequency to the frequency detuning from the exact resonance. The resulting equations differ from the known equations [25–27] by the correction terms, which take into account the dispersion of the nonlinear response of the resonance system. To illustrate the application of the general equations, we consider the initial stage of formation of a shock wave in an optical fibre.

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2. Formulation of the problem

Let electromagnetic radiation with the electric field strength

$$\mathbf{E} = \vec{\mathcal{E}}e^{-i\Phi} + \vec{\mathcal{E}}^*e^{i\Phi}, \quad \Phi = kz - \omega t - \varphi_0 \quad (1)$$

and the carrier frequency $\omega = kv_{\text{ph}}$ (v_{ph} is the phase velocity) be travelling in the z -direction in a medium containing resonance impurity atoms. The resonance impurities are represented by two-level atoms, whose Hamiltonian H_0 is assumed to be the same for all the atoms and independent of the impurity position in the medium. Thus, we neglect the inhomogeneous broadening of the spectral line of impurity atoms and other effects associated with the position of an impurity atom in the medium.

The basic equations describing the propagation of wave (1) are the wave equation

$$\frac{\partial^2}{\partial z^2} \mathbf{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{E} = \frac{4\pi}{c^2} \frac{\partial^2 \mathbf{P}}{\partial t^2} \quad (2)$$

and the equations determining the polarisation \mathbf{P} of the medium. Because the polarisation of the medium is the dipole moment of its unit volume, we represent it in the form $\mathbf{P} = \mathbf{P}_m + \mathbf{P}_{\text{im}}$, where the term \mathbf{P}_m describes the medium without impurity atoms and the second term \mathbf{P}_{im} is caused by the interaction of the electromagnetic wave with impurities. The term \mathbf{P}_m is related to the electric field through optical susceptibilities, which will be taken into account up to third-order terms in the field inclusive, and \mathbf{P}_{im} is determined by the density matrix of impurity atoms ρ_{im} :

$$\mathbf{P}_{\text{im}} = \text{Sp}(\rho_{\text{im}} \mathbf{d}_{\text{im}}), \quad (3)$$

where \mathbf{d}_{im} is the dipole moment operator of an impurity atom. The density matrix ρ_{im} is assumed to be normalised to the density of impurity atoms: $\text{Sp}(\rho_{\text{im}}) = N_{\text{im}}$.

The density matrix of impurity atoms is found from the equation

$$i\hbar \left(\frac{\partial}{\partial t} + \hat{\Gamma} \right) \rho_{\text{im}} = (H_0 - \mathbf{E} \mathbf{d}_{\text{im}}) \rho_{\text{im}} - \rho_{\text{im}} (H_0 - \mathbf{E} \mathbf{d}_{\text{im}}), \quad (4)$$

where $\hat{\Gamma}$ is the relaxation operator and the Hamiltonian H_0 has eigenvectors $|a\rangle$ and $|b\rangle$: $H_0|a\rangle = E_a|a\rangle$, $H_0|b\rangle = E_b|b\rangle$, $E_b - E_a = \hbar\omega_{ba}$.

We neglect polarisation effects by assuming that all the quantities determining them ($\vec{\mathcal{E}} = \mathcal{E} \mathbf{e}_x$, $\mathbf{P}_{\text{im}} = P_{\text{im}} \mathbf{e}_x$, $\mathbf{d}_{\text{im}} = d \mathbf{e}_x$) are parallel to the x -axis. For the slowly varying amplitude $\mathcal{E} = \mathcal{E}(z, t)$ of field (1), we have the wave equation

$$\left\{ -2ik \left(\frac{\partial}{\partial z} + \frac{1}{v_g} \frac{\partial}{\partial t} \right) + \frac{1 - kv'_g}{v_g^2} \frac{\partial^2}{\partial t^2} \right\} \mathcal{E}(z, t) = \frac{12\pi k^2}{\varepsilon(\omega)} \chi^{(3)}(-\omega; \omega, -\omega, \omega) |\mathcal{E}(z, t)|^2 \mathcal{E}(z, t) + 4\pi \frac{k^2}{\varepsilon(\omega)} \mathcal{P}_{\text{im}}, \quad (5)$$

where v_g is the group velocity; $v'_g = dv_g/d\omega$; and $\varepsilon(\omega)$ and $\chi^{(3)}(-\omega; \omega, -\omega, \omega)$ are the dielectric permittivity and the third-order susceptibility of the medium without impurity atoms. \mathcal{P}_{im} is the slowly varying polarisation amplitude:

$$P_{\text{im}} = \mathcal{P}_{\text{im}} e^{-i\Phi} + \mathcal{P}_{\text{im}}^* e^{i\Phi}. \quad (6)$$

The harmonic generation is ignored in (5).

Equations (3)–(5) describe the propagation of the electromagnetic field in a nonlinear medium with impurity atoms.

3. Unitary transformation method for the adiabatic following

Consider the case when the frequency detuning from the resonance $\Delta = \omega - \omega_{ba}$ is much greater than the Rabi frequency $\Lambda = 2\mathcal{E}d_{ba}/\hbar$ and the spectral line width $1/\gamma$, but is much smaller than the carrier frequency:

$$|\Delta| \gg |\Lambda|, \quad |\Lambda| \gg 1/\gamma, \quad |\Delta| \ll \omega. \quad (7)$$

Under conditions (7), the polarisation of impurity atoms adiabatically follows field (1). It is common to analyse this case by expanding the solution of the equation for the density matrix in power series. However, here, we will demonstrate a more elegant unitary transformation, which showed its efficiency in various problems of nonlinear optics where resonance conditions are present in one form or another.

Instead of solving equation (4) we transform the density matrix and the Hamiltonian of impurity atoms in field (1) so that the Hamiltonian be diagonal. Because of condition (7), one may neglect the relaxation operator and obtain

$$\tilde{\rho} = e^{-iS} \rho_{\text{im}} e^{iS}, \quad i\hbar \frac{\partial}{\partial t} \tilde{\rho} = [\tilde{H}, \tilde{\rho}], \quad (8)$$

$$\tilde{H} = e^{-iS} H_0 e^{iS} - e^{-iS} \mathbf{E} d e^{iS} - i\hbar e^{-iS} \frac{\partial}{\partial t} e^{iS}.$$

Hereafter, the square brackets denote the commutator of operators. It is evident that the operator e^{-iS} should be unitary and the operator S should be Hermitian. Because this transform cannot be obtained exactly, we expand the operator S and the effective Hamiltonian \tilde{H} in a power series in the electric field:

$$S = S^{(1)} + S^{(2)} + \dots, \quad \tilde{H} = \tilde{H}^{(0)} + \tilde{H}^{(1)} + \tilde{H}^{(2)} + \dots, \quad (9a)$$

$$\tilde{H}^{(0)} = H_0, \quad \tilde{H}^{(1)} = -\mathbf{E} d - i[S^{(1)}, H_0] + \hbar \frac{\partial}{\partial t} S^{(1)}, \quad (9b)$$

$$\begin{aligned} \tilde{H}^{(2)} = & \frac{i}{2} [S^{(1)}, \mathbf{E} d] - \frac{i}{2} [S^{(1)}, \tilde{H}^{(1)}] \\ & - i[S^{(2)}, H_0] + \hbar \frac{\partial}{\partial t} S^{(2)}, \end{aligned} \quad (9c)$$

$$\begin{aligned} \tilde{H}^{(3)} = & \frac{i}{2} [S^{(2)}, \mathbf{E} d] - \frac{i}{2} [S^{(1)}, \tilde{H}^{(2)}] - \frac{i}{2} [S^{(2)}, \tilde{H}^{(1)}] \\ & + \frac{1}{12} [S^{(1)}, [S^{(1)}, \mathbf{E} d]] + \frac{1}{12} [S^{(1)}, [S^{(1)}, \tilde{H}^{(1)}]] \\ & - i[S^{(3)}, H_0] + \hbar \frac{\partial}{\partial t} S^{(3)}. \end{aligned} \quad (9d)$$

We require the fulfilment of the conditions

$$\tilde{H}^{(1)} = 0, \quad \tilde{H}_{ab}^{(2)} = E_a^{(2)} \delta_{ab}, \quad \tilde{H}_{ab}^{(3)} = E_a^{(3)} \delta_{ab}, \quad (10)$$

where $E_a^{(2)}$ and $E_a^{(3)}$ contain no oscillating exponentials $e^{im\Phi}$, $m = \pm 1, \pm 2, \dots$ (a and b label the energy levels of the two-level system). As a result, the transformed density matrix has the diagonal form: $\tilde{\rho}_{ab} = \tilde{\rho}_a^{(0)} \delta_{ab}$.

Polarisation (3) of a two-level atom with frequency detuning Δ from the resonance is determined by the formula

$$P_{\text{im}}(\Delta) = \text{Sp}(\tilde{\rho} e^{-iS} d e^{iS}) = \text{Sp} \left\{ \tilde{\rho} \left(d - i[S, d] - \frac{1}{2}[S, [S, d]] + \frac{i}{6}[S, [S, [S, d]]] + \dots \right) \right\}. \quad (11)$$

Let us find $P_{\text{im}}(\Delta)$, accurate to within third-order terms in the field inclusive. For this purpose, one should derive expressions for $S^{(1)}$, $S^{(2)}$, and $S^{(3)}$.

Using integration by parts, we obtain from Eq.(9b) and condition (10)

$$\begin{aligned} S_{ab}^{(1)} &= \frac{d_{ab}}{\hbar} \left\{ \frac{\mathcal{E} e^{-i\Phi}}{i(\omega_{ab} - \omega)} + \frac{\mathcal{E}^* e^{i\Phi}}{i(\omega_{ab} + \omega)} \right\} \\ &+ \frac{d_{ab}}{\hbar} \left\{ \frac{\partial \mathcal{E}}{\partial t} e^{-i\Phi} \frac{1}{(\omega_{ab} - \omega)^2} + \frac{\partial \mathcal{E}^*}{\partial t} e^{i\Phi} \frac{1}{(\omega_{ab} + \omega)^2} \right\} \\ &- \frac{d_{ab}}{\hbar} \left\{ \frac{\partial^2 \mathcal{E}}{\partial t^2} e^{-i\Phi} \frac{1}{i(\omega_{ab} - \omega)^3} + \frac{\partial^2 \mathcal{E}^*}{\partial t^2} e^{i\Phi} \frac{1}{i(\omega_{ab} + \omega)^3} \right\} \\ &- \frac{d_{ab}}{\hbar} \left\{ \frac{\partial^3 \mathcal{E}}{\partial t^3} e^{-i\Phi} \frac{1}{(\omega_{ab} - \omega)^4} + \frac{\partial^3 \mathcal{E}^*}{\partial t^3} e^{i\Phi} \frac{1}{(\omega_{ab} + \omega)^4} \right\} + \dots \end{aligned} \quad (12)$$

for the adiabatic field switching. In addition to condition (7) we require that the field change in the characteristic time τ_p be sufficiently rapid:

$$\frac{\Delta(\Delta\tau_p)}{\omega} \ll 1. \quad (13)$$

In this case, one may retain in (12) only the terms

$$\begin{aligned} S_{ba}^{(1)} &= \frac{d_{ba}}{\hbar} \left(-i\Delta^{-1} \mathcal{E} e^{-i\Phi} + \Delta^{-2} \frac{\partial \mathcal{E}}{\partial t} e^{-i\Phi} + i\Delta^{-3} \frac{\partial^2 \mathcal{E}}{\partial t^2} e^{-i\Phi} \right. \\ &\left. - \Delta^{-4} \frac{\partial^3 \mathcal{E}}{\partial t^3} e^{-i\Phi} \right) = S_{ab}^{(1)*}, \quad S_{bb}^{(1)} = S_{aa}^{(1)} = 0. \end{aligned}$$

From Eq. (9c) follows

$$\begin{aligned} E_a^{(2)} &= \frac{|d_{ba}|^2}{\hbar} \left\{ \Delta^{-1} |\mathcal{E}|^2 + \frac{i}{2} \Delta^{-2} \left(\mathcal{E} \frac{\partial \mathcal{E}^*}{\partial t} - \mathcal{E}^* \frac{\partial \mathcal{E}}{\partial t} \right) \right. \\ &\left. - \frac{1}{2} \Delta^{-3} \left(\mathcal{E}^* \frac{\partial^2 \mathcal{E}}{\partial t^2} + \mathcal{E} \frac{\partial^2 \mathcal{E}^*}{\partial t^2} \right) \right\} = -E_b^{(2)}, \quad S_{ab}^{(2)} = 0. \end{aligned}$$

Finally, we obtain from (9d) the expression

$$\frac{\partial}{\partial t} S_{ba}^{(3)} + i\omega_{ba} S_{ba}^{(1)} = \frac{i}{\hbar} S_{ba}^{(1)} E_a^{(2)}$$

$$- \frac{1}{6\hbar} \left(|S_{ba}^{(1)}|^2 E d_{ba} - S_{ba}^{(1)2} E d_{ab} \right). \quad (14)$$

Retaining in this expression only the terms proportional to $e^{\pm i\Phi}$, we have, in view of (13),

$$\begin{aligned} S_{ba}^{(3)} &= -i \frac{4}{3} \frac{|d_{ba}|^2 d_{ba}}{\hbar^3} \left\{ \Delta^{-3} \mathcal{E} |\mathcal{E}|^2 \right. \\ &\left. + i \frac{1}{2} \Delta^{-4} \left(\mathcal{E}^2 \frac{\partial \mathcal{E}^*}{\partial t} - 3 |\mathcal{E}|^2 \frac{\partial \mathcal{E}}{\partial t} \right) \right\} e^{-i\Phi}. \end{aligned} \quad (15)$$

The results obtained enable one to calculate polarisation (11) of two-level impurity atoms within an accuracy of third-order terms inclusive. However, we additionally assume that $\tilde{\rho}_a^{(0)} = N_{\text{im}}$. Then, retaining the above accuracy, we obtain

$$\begin{aligned} P_{\text{im}}(\Delta) &= N_{\text{im}} \left\{ -i \left(S_{ab}^{(1)} d_{ba} - d_{ab} S_{ba}^{(1)} \right) \right. \\ &\left. \times \left(1 - \frac{2}{3} |S_{ba}^{(1)}|^2 \right) - i \left(S_{ab}^{(3)} d_{ba} - d_{ab} S_{ba}^{(3)} \right) \right\}, \end{aligned}$$

from which follows the expression

$$\begin{aligned} \mathcal{P}_{\text{im}}(\Delta) &= N_{\text{im}} \frac{|d_{ab}|^2}{\hbar} \left\{ -\frac{1}{\Delta} \mathcal{E} + \frac{i}{\Delta^2} \frac{\partial \mathcal{E}}{\partial t} + \frac{1}{\Delta^3} \frac{\partial^2 \mathcal{E}}{\partial t^2} - \frac{i}{\Delta^4} \frac{\partial^3 \mathcal{E}}{\partial t^3} \right. \\ &\left. + \frac{|d_{ab}|^2}{\hbar^2 \Delta^3} \left(2 \mathcal{E} |\mathcal{E}|^2 - 6i \frac{|\mathcal{E}|^2}{\Delta} \frac{\partial \mathcal{E}}{\partial t} + \frac{4i}{3\Delta} \frac{\partial (\mathcal{E} |\mathcal{E}|^2)}{\partial t} \right) \right\}. \end{aligned} \quad (16)$$

which is accurate to within terms of the order of $|\Delta|^{-4}$.

Note that this expression takes into account dispersion up to the third order (the term with the third time derivative) and the dispersion of the nonlinear response of quasi-resonance atoms (the last two terms). It is these terms that differ the results obtained here from the results found in the adiabatic following approximation [25–27].

4. Nonlinear equation for the optical-pulse envelope

Let us represent equation (5) in the form

$$\begin{aligned} i \left(\frac{\partial}{\partial z} + \frac{1}{v_g} \frac{\partial}{\partial t} \right) \mathcal{E} - \frac{1}{2} D \frac{\partial^2 \mathcal{E}}{\partial t^2} \\ + \frac{6\pi\omega\chi^{(3)}}{cn(\omega)} |\mathcal{E}(z, t)|^2 \mathcal{E}(z, t) = -\frac{2\pi\omega}{cn(\omega)} \mathcal{P}_{\text{im}}, \end{aligned} \quad (17)$$

where $n(\omega)$ is the refractive index of the fibre core at the carrier frequency and

$$D = (1 - kv_g') k^{-1} v_g^{-2} = \frac{1}{2} \frac{d^2 k^2}{d\omega^2}$$

is the dispersion parameter, which characterises the second-order group velocity dispersion that is caused only by the fibre itself.

We have two characteristic lengths, namely, the resonance absorption length L_{abs} for the initial pulse with duration t_p and the dispersion length L_D :

$$L_{\text{abs}} = cn(\omega) \hbar (2\pi\omega N_{\text{im}} |d_{ba}|^2 t_p)^{-1}, \quad L_D = t_p^2 / 2|D|.$$

Here, we chose the pulse duration t_p as a characteristic time of the problem. We introduce the new normalised variables $\tau = (t - z/v_g)t_p^{-1}$, $\zeta = z/L_{\text{abs}}$, and $q = A_0^{-1}\mathcal{E}$. Taking into account (16) and rewriting Eq. (17) in terms of these variables, we obtain

$$i\frac{\partial q}{\partial \zeta} - \frac{\sigma}{2}\varepsilon_D\frac{\partial^2 q}{\partial \tau^2} + \mu|q|^2q = J_1q - iJ_2\frac{\partial q}{\partial \tau} - J_3\frac{\partial^2 q}{\partial \tau^2} + iJ_4\frac{\partial^3 q}{\partial \tau^3} - \frac{\varepsilon_R^2}{2}J_3|q|^2q + i\frac{\varepsilon_R^2}{2}J_4\left\{3|q|^2\frac{\partial q}{\partial \tau} - \frac{2}{3}\frac{\partial(|q|^2q)}{\partial \tau}\right\}, \quad (18)$$

where $J_1 = (\Delta t_p)^{-1}$; $J_2 = (\Delta t_p)^{-2}$; $J_3 = 2(\Delta t_p)^{-3}$; $J_4 = 2(\Delta t_p)^{-4}$; $\sigma = \text{sign}D$; $\mu = 6\pi\omega\chi^{(3)}A_0^2L_{\text{abs}}/cn(\omega)$; and A_0 is the peak amplitude of the initial pulse. We also use the parameters $\varepsilon_D = L_{\text{abs}}/L_D$ and $\varepsilon_R = (2|d_{ba}|A_0/\hbar)t_p = \Omega_R t_p$. Using the nonlinear refractive index n_2 , which is related to the cubic susceptibility $\chi^{(3)}$ by the expression $n_2 = 3\pi\chi^{(3)}/n(\omega)$, one can rewrite the parameter μ in the form $\mu = 2(\omega/c)n_2A_0^2L$. Because of the high-frequency Kerr effect, the wave acquires the phase shift $\Delta\varphi = (\omega/c)n_2A_0^2L$ after passing the path L . Thus, the parameter μ is interpreted as the phase shift produced at a distance equal to two absorption lengths L_{abs} .

Making a change to the new variables $\eta = \tau - (\Delta t_p)^{-2}\zeta$ and $\tilde{q}(\zeta, \eta) = q \exp[i\zeta/\Delta t_p]$, one can write Eq. (18) in the form

$$i\frac{\partial \tilde{q}}{\partial \zeta} + \tilde{\varepsilon}_D\frac{\partial^2 \tilde{q}}{\partial \eta^2} + \tilde{\mu}|\tilde{q}|^2\tilde{q} = iJ_4\frac{\partial^3 \tilde{q}}{\partial \eta^3} + i\frac{\varepsilon_R^2}{2} \times J_4\left\{3|\tilde{q}|^2\frac{\partial \tilde{q}}{\partial \eta} - \frac{2}{3}\frac{\partial(|\tilde{q}|^2\tilde{q})}{\partial \eta}\right\}, \quad (19)$$

where $\tilde{\varepsilon}_D = -\sigma\varepsilon_D/2 + (\Delta t_p)^{-3}$ and $\tilde{\mu} = \mu + 1/2\varepsilon_R^2(\Delta t_p)^{-3}$. It has the form of the generalised nonlinear Schrödinger equation. Here, the third-order group velocity dispersion and the dispersion of nonlinear responses manifest themselves due to the influence of impurity atoms.

5. An example of the influence of quasi-resonance impurities

In the range of picosecond durations of optical pulses, the dispersion length is of the order of 100 m, which may be many-fold greater than the resonance absorption length. Assuming that $L_{\text{abs}} \ll L_D$ and omitting the terms proportional to $(\Delta t_p)^{-3}$ and $(\Delta t_p)^{-4}$, Eq. (19) can be written in the form

$$i\frac{\partial \tilde{q}}{\partial \zeta} + \tilde{\mu}|\tilde{q}|^2\tilde{q} = i\beta\left(3|\tilde{q}|^2\frac{\partial \tilde{q}}{\partial \eta} - \frac{2}{3}\frac{\partial(|\tilde{q}|^2\tilde{q})}{\partial \eta}\right), \quad (20)$$

where

$$\beta = \frac{\varepsilon_R^2 J_4}{2} \approx \left(\frac{\Omega_R}{A}\right)^2 (\Delta t_p)^{-2}.$$

Making a change to real variables (defining them by the relation $\tilde{q} = a \exp(i\phi)$), we represent Eq. (20) by the system of real equations

$$\frac{\partial a}{\partial \zeta} - \beta a^2 \frac{\partial a}{\partial \eta} = 0, \quad \frac{\partial \phi}{\partial \zeta} - \frac{7}{3} \beta a^2 \frac{\partial \phi}{\partial \eta} = \tilde{\mu} a^2.$$

The first equation of this system describes the wave that

travels with velocity depending on its amplitude. The solution of this equation can be written in the implicit form as

$$a(\eta, \zeta) = f(\eta + \beta a^2(\eta, \zeta)\zeta),$$

where the function $f(\eta) = a(\eta, \zeta = 0)$ is determined by the initial profile of an optical pulse. If $\beta > 0$ ($\beta < 0$), the peak of a pulse travels with velocity greater (smaller) than the group velocity. This is accompanied by the self-steepening of the leading (trailing) edge of a pulse, and finally a shock wave is formed. One can estimate the maximum values of β from the requirement that the quasi-resonance conditions $\Omega_R/A \leq 10^{-1}$ and $1/t_p A \leq 10^{-1}$ be necessarily fulfilled. This gives $\beta \leq 10^{-4}$. Note that the terms omitted in Eq. (19) become substantial in this limiting case and their influence not only hinders the formation of a discontinuity at the leading (trailing) edge, but, on the contrary, provides the suppression of oscillations of the pulse envelope and subsequent disintegration of the initial pulse into a set of weak pulses.

In many cases [22, 24], the factors at the terms in Eq. (19) are chosen in an arbitrary way, and sometimes they are chosen so that the resulting equations be integrable by the inverse scattering method. In our case, the correct approach leads to clearly defined relations between the coefficients of the evolution equation and gives no way of using completely integrable versions of this equation as a zero approximation.

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