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On the feasibility of raising the density and the extension of atomic ensembles in laser cooling

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Abstract. The ways of raising the limiting density of cooled atoms and the extension of the volume occupied by them are considered. These include raising the laser beam intensity above the saturation intensity of the working transition, the use of a noncollinear geometry of the laser beam and the atomic medium, and coherent cooling by sequences of counterpropagating π-pulses.

1. Introduction

The light pressure of the photon flux of an optical laser has become an efficient tool of a physics experiment, in particular, in the problems on cooling of neutral atoms and manipulation of their motion (see, e. g., Ref. [1]). To solve some of these problems [2, 3], it is necessary that the density of cool atoms should be significantly increased for a relatively long extension of the volume they occupy. The characteristic desirable parameter magnitudes are as follows: a density of atoms occupying a filamentary volume of length $L = 100 - 300$ cm and a lateral diameter $D = 10^{-3} - 10^{-2}$ cm should be $n = 10^{12} - 10^{13}$ cm^{-3}.

Among the known direct obstacles to increasing the density and volume are the limitations in the optical depth of the atomic medium [1]. A usual elementary estimate of the laser photon penetration depth

$$\delta_0 = (\sigma_0 n)^{-1} \tag{1}$$

indicates that, for instance, for an atomic density $n = 10^{12}$ cm^{-3} and the absorption cross section $\sigma_0 = 2 \times 10^{-9}$ cm^2, the penetration depth δ_0 is negligible (it does not exceed 5×10^{-4} cm). Nevertheless, this elementary picture is not exhaustive.

2. Effect of saturation on the penetration depth

When estimating the penetration depth, account must be taken of the effect of saturation of the working atomic transition. Indeed, the excited (n_2) and unexcited (n_1) atom densities ($n_1 + n_2 = n$) are given by the stationary rate equation

$$R(n_2 - n_1) + n_2/\tau = 0, \tag{2}$$

where [1]

$$R = \frac{I/\tau I_0}{1 + 2I/I_0 + (2\tau\Delta)^2}; \tag{3}$$

τ is the spontaneous relaxation time; $\Delta = \omega - \omega_0 < 0$ is the laser frequency detuning relative to the central transition frequency ω_0; I is the laser beam intensity; I_0 is the saturation intensity at which $R = h(3\tau)^{-1}$, if $\Delta = 0$; hereafter, we will disregard the statistical weights of the states.

From expression (2), we obtain

$$n_1 - n_2 = \frac{n}{1 + 2R\tau}, \tag{4}$$

and, hence, the penetration depth is

$$\delta = \frac{I/R}{n_1 - n_2} = I_0 \frac{\tau}{n}\left[1 + 4\frac{I}{I_0} + (2\tau\Delta)^2\right]. \tag{5}$$

For $I/I_0 \to 0$,

$$\delta = I_0 \frac{\tau}{n}\left[1 + (2\tau\Delta)^2\right], \tag{6}$$

which should correspond by the order of magnitude to the estimate (1) for $\Delta = 0$:

$$I_0 \frac{\tau}{n} = \delta_0 = (\sigma_0 n)^{-1}. \tag{7}$$

Hence, $I_0 \approx (\sigma_0 \tau)^{-1} = 5 \times 10^{16}$ cm^{-2} s^{-1} (~ 10 mW cm^{-2}) for $\tau = 10^{-8}$ s and the numerical data assumed in the estimate (1).

Comparison of expressions (5), (6), and (7) gives

$$\delta = \delta_0 \left[1 + 4\frac{I}{I_0} + (2\tau\Delta)^2\right]. \tag{8}$$

For instance, $\delta/\delta_0 = 22$ for $I/I_0 = 5$ and $2\tau\Delta = 1$, increasing by more than an order of magnitude compared to the case when $I/I_0 \to 0$.

3. Minimal temperature and cooling rate

The equilibrium temperature T of the atoms being cooled is proportional to the ratio between the coefficient of momentum diffusion and the friction coefficient [1]:

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$$kT = -\frac{1+z}{8\tau}\hbar\left[2\tau\Delta + \frac{1}{2\tau\Delta}\left(1 + 2\frac{I}{I_0}\right)\right], \quad (9)$$

where k is the Boltzmann constant; $z = 0.4$ and 0.3 for linearly and circularly polarised light, respectively. For a given detuning $\Delta_0 < 0$, the temperature reaches a minimum

$$kT_{\min} = -\frac{1+z}{2}\hbar\Delta_0 = \frac{1+z}{4\tau}\hbar\left(1 + \frac{2I'}{I_0}\right)^{1/2} \quad (10)$$

for the photon beam intensity

$$I' = \frac{I_0}{2}\left[(2\tau\Delta_0)^2 - 1\right]. \quad (11)$$

As the light is absorbed upon penetrating the atomic medium, the intensity I falls off compared to the initial intensity I' at the entrance to the medium. This results in a departure of the temperature T from the initial minimal temperature $T_{\min}$ for an invariable detuning $\Delta_0 = \text{const}$:

$$T = T_{\min}\left(1 - \beta\frac{I'/I_0}{1 + 2I'/I_0}\right), \quad (12)$$

where $\beta = (I' - I)/I'$.

An effort to accomplish cooling down to this minimal temperature requires a significant detuning $|\Delta_0| > (2\tau)^{-1}$. This results in a lowering of the cooling rate

$$R = \frac{R_0}{2} \quad (13)$$

for the detuning $\Delta = \Delta_0$, compared to the rate R_0 corresponding to the zero detuning $\Delta \to 0$.

Eventually one can see that the saturation of the working transition with increasing intensity I results in a significant increase in the penetration depth δ at the expense of a reduction in the rate R. Nevertheless, the penetration depth δ is still not large enough. In the above example, it amounts to $\sim 10^{-2}$ cm.

4. Cooling in the case of noncollinearity of the photon and atomic beams

The noncollinearity is supposedly capable of resolving the conflict between the increased, though still not large enough, penetration depth and the required large extension of the volume occupied by the atoms, if this volume is filamentary in shape and has a lateral diameter D comparable with the penetration depth estimated by formula (8). If the photons are incident at an angle α relative to the axis of the atomic beam, then the cooling of a beam of arbitrary length L requires a relatively small penetration depth

$$\delta = \frac{D}{\sin\alpha}, \quad (14)$$

which can be attained at an increased intensity I.

This is gained at the expense of a reduction in the efficiency of cooling in the longitudinal atomic degree of freedom, because only the axial component of the photon momentum $p_z = (\hbar\omega/c)\cos\alpha$ acts in this direction. In addition, the total output power of the laser source should be substantially increased (by several orders of magnitude) up to

$$\mathscr{P} = \hbar\omega I\pi DL\sin\alpha, \quad (15)$$

and the relatively complex optics should be used to form the corresponding 'band-like' beam. For instance, to irradiate an atomic beam of length $L = 100$ cm at a grazing angle of $\alpha = 10°$, a light beam with the cross section 20×0.1 cm with a total power of about $50 - 100$ mW is required, which can be provided by present-day lasers of numerous types in different spectral regions.

The role of the optical Stark effect in a high-intensity light field with $I > I_0$ calls for a special consideration.

5. Coherent atom cooling by sequences of counterpropagating π pulses

This technique involves the coherent interaction of laser radiation with a two-level atom, specifically, the interaction of atoms with the sequences of counterpropagating π pulses. Earlier investigations [4–10] were concerned with the application of this technique for the acceleration and deflection of neutral atoms, and also for other manipulations with them. It seems that this technique can also be used to cool atomic ensembles.

As is known, the probability of the ground-to-excited state atomic transition induced by the electromagnetic field with frequency ω and electric vector $\boldsymbol{E}$ is

$$w = \frac{1/2}{1 + (\Delta/2\Omega_R)^2}\left\{1 - \cos 2\Omega_R\left[1 + \left(\frac{\Delta}{2\Omega_R}\right)^2\right]^{1/2} t\right\}, \quad (16)$$

where

$$\Omega_R = \langle\mu\rangle\frac{E}{\hbar} = \frac{E}{8\pi}\left(\frac{3\lambda^3}{2\pi\hbar\tau}\right)^{1/2} \quad (17)$$

is the Rabi frequency; $\langle\mu\rangle$ is the transition matrix element;

$$\langle\mu\rangle^2 = \frac{3\hbar\lambda^2}{(4\pi)^3\tau}; \quad (18)$$

λ is the radiation wavelength; and t is the time.

For a strong field, when

$$E^2 \gg \frac{4}{3}(2\pi)^5 c^2\hbar\left(\frac{\Delta}{\omega}\right)^2\frac{\tau}{\lambda^5} \quad (19)$$

(c is the velocity of light), expression (16) is simplified, since

$$\left(\frac{\Delta}{2\Omega_R}\right)^2 \ll 1, \quad (20)$$

and describes the Rabi oscillations:

$$w \approx \frac{1}{2}(1 - \cos 2\Omega_R t). \quad (21)$$

During the time interval

$$T_\pi = \frac{\pi}{2\Omega_R} = \frac{18\ \text{ps}}{E}\left(\frac{\tau}{\lambda^3}\right)^{1/2}, \quad (22)$$

which is referred to as the π-pulse duration, the probability w varies from the initial probability ($w = 0$) at $t = 0$ to unity (hereafter, we express, in numerical formulas, E in kV cm^{-1}, τ in nanoseconds, and λ in micrometres). This means that the atom absorbed a photon with the energy $\hbar\omega$ and acquired the momentum

$$\Delta \boldsymbol{p}_1 = \hbar \boldsymbol{k}_1, \quad \Delta p_1 = \frac{\hbar\omega}{c}. \tag{23}$$

which coincides in direction with the wave vector $\boldsymbol{k}$.

At the moment $t = T_\pi$, the wave train is interrupted and the atom becomes subjected to the action of a wave with the same ω and E but of the opposite direction, so that

$$\boldsymbol{k}_2 = -\boldsymbol{k}_1. \tag{24}$$

According to (21), by the moment $t = 2T_\pi$ this wave reverts the atom to the initial state with $w = 0$. In this case, a photon with a wave vector $\boldsymbol{k}_2$ (24) is emitted and the atom gains the momentum

$$\Delta \boldsymbol{p}_2 = -\hbar\boldsymbol{k}_2 = \hbar\boldsymbol{k}_1 = \Delta\boldsymbol{p}_1. \tag{25}$$

Eventually, over the absorption – emission cycle with a duration of $\Delta t_\mathrm{R} = 2T_\pi$, the atom gains the momentum

$$\Delta \boldsymbol{p}_\mathrm{R} = \Delta\boldsymbol{p}_1 + \Delta\boldsymbol{p}_2 = 2\hbar\boldsymbol{k}_1, \quad \Delta p_\mathrm{R} = 2\hbar\omega/c. \tag{26}$$

Note that the given directionality of the momentum acquired by the atom occurs, unlike the usual process involving spontaneous emission, in every cycle rather than as a result of averaging over many cycles.

A multiple repetition of the above cycles of interrupted Rabi oscillations in counterpropagating alternating π pulses is accompanied by the appearance of a light pressure force

$$\boldsymbol{F}_\mathrm{R} = \frac{\Delta\boldsymbol{p}_\mathrm{R}}{\Delta t_\mathrm{R}}, \ F_\mathrm{R} = \frac{2\hbar\omega}{2cT_\pi} = \frac{E}{2\pi}\left(\frac{3\hbar\lambda}{2\pi\tau}\right)^{1/2}$$
$$= (3.7\times 10^{-12}\,\mathrm{dyn})E\left(\frac{\lambda}{\tau}\right)^{1/2}. \tag{27}$$

To be more precise, two counterpropagating sequences of similar π pulses alternately act on the atom to exert a stationary force $\boldsymbol{F}_\mathrm{R}$ aligned with the wave vectors of the photons being absorbed, i. e., in essence with the first pulse in the sequence. In this case, the shift of the laser frequency to the wings of the Doppler profile of the atomic line opens up the way to selective acceleration or deceleration of the groups of atoms with a given thermal velocity, i. e., to modification of their velocity distribution, which can involve, in particular, cooling of the atomic ensemble.

It should be emphasised that the F_R force (27), which arises in coherent Rabi oscillations, is far greater that the force of light pressure F_s in the sequence of resonance absorption – spontaneous emission cycles employed in conventional cooling schemes. The ratio of these forces is:

$$\frac{F_\mathrm{R}}{F_\mathrm{s}} = \frac{E}{2\pi^2}\left(\frac{3\lambda^3\tau}{2\pi\hbar}\right)^{1/2} \approx 115E(\lambda^3\tau)^{1/2}. \tag{28}$$

A numerical example: for $\lambda = 1$ μm, $\tau = 10$ ns, and $E = 10$ kV cm^{-1} (135 kW cm^{-2}), we have $T_\pi = 5.6$ ps, $F_\mathrm{R} = 1.2\times 10^{-11}$ dyn, and $F_\mathrm{R}/F_\mathrm{s} = 3600$.

Naturally, the time interval between the pulses of each of the sequences should exceed T_π to accommodate the counterpropagating π pulse but should be significantly shorter than the relaxation time τ to maintain the process coherence. The pulse repetition rate should therefore lie in the range

$$\frac{1}{\tau} \ll f \leqslant \frac{1}{2T_\pi} = \frac{\Omega_\mathrm{R}}{\pi}. \tag{29}$$

Recall that the spectrum of such a regular pulse sequence is by no means similar to a very broad continuous spectrum of a single ultrashort pulse, but constitutes a series of discrete lines spaced at intervals f enclosed by the envelope of the form $(\sin x)/x$ (for rectangular pulses), where $x = \pi\Delta/(2\Omega_\mathrm{R})$. In particular, for $f = (2T_\pi)^{-1}$, two satellites, apart from the central line, are present under the envelope with the effective width $4\Omega_\mathrm{R}$, the satellite amplitudes being $2/\pi$ times lower than that of the central line. The separation between these satellites is $2f$ and may significantly exceed the Doppler width of the atomic line.

Finally, addressing ourselves to time concepts, the atom may be said to interact with an ultrashort π pulse with a wavelength λ. For a spectral representation and the corresponding laser tuning, the interaction takes place only with the central line of an extended line spectrum. The linewidth does not depend on the π-pulse duration whatsoever. Therefore, the task of efficient cooling involves production of a regular π-pulse sequence stabilised so that the width of the central line corresponds to the desired minimal temperature of the atomic ensemble. A mode-locked laser is supposedly best suited for the solution of this problem. In this case, the atoms interact in fact with only the field of the central mode.

During the acceleration or deceleration of the atoms, a Doppler shift of the resonance transition frequency occurs and a spatial displacement of the atoms takes place. The shift of the transition frequency obeys inequality (20) if the velocity variation is not too large:

$$\left(\frac{\Delta v}{c}\right)^2 \ll \left(\frac{\lambda}{2cT_\pi}\right)^2, \tag{30}$$

which is knowingly fulfilled for thermal velocities (on the order of 10^4 cm s^{-1}). The spatial displacement of the atoms is far smaller than the spatial scale of the π pulse ($cT_\pi \approx 1$ cm) if the variation in the kinetic energy of an atom of mass M is small compared to the photon energy:

$$\frac{M}{2}(\Delta v)^2 \ll \hbar\omega. \tag{31}$$

The coherent cooling technique opens up the important opportunity for unlimited increase in the depth of penetration of counterpropagating π-pulse sequences into the atom medium being cooled. This possibility results from the fact that each of the sequences alternately loses energy in the absorption of the π pulse by the atoms being excited and acquires the lost energy in the next event of their coherent emission.

In this case, it is necessary that the energy of a π pulse absorbed by an atom in every single half-cycle should not noticeably lower its initial energy:

$$\hbar\omega nc\Delta t \ll \hbar\omega I T_\pi, \tag{32}$$

where Δt is the interval between the neighbouring counterpropagating π pulses, i. e., virtually the duration of the half-cycle, because $\Delta t \gg T_\pi$. A serious limitation on the atom density follows from inequality (32):

$$n \ll \frac{\pi}{c\lambda}\left(\frac{2\pi}{3}\frac{I}{\Delta t}\right)^{1/2} \approx \frac{0.05\ \mathrm{cm}^{-3}}{\lambda}\left(\frac{I}{\Delta t}\right)^{1/2}, \tag{33}$$

where I is expressed in cm^{-2} s^{-1} and Δt in nanoseconds. It follows from this estimate that using this technique to obtain cooled atomic ensembles with a density of over 10^{12} cm^{-3} will require π pulses with an intensity of the order of several GW cm^{-2} produced by lasers with an output power of the order of several tens of kilowatts.

Note that the above peak power is attained in mode-locked lasers of different types, although with a relatively low pulse repetition rate, which is far below the desired high repetition rates (up to hundreds of gigahertz). These high repetition rates are realised in short-cavity semiconductor lasers but for a significantly lower peak power (below 100 mW). It is possible that the optimal solution might involve amplification of high-repetition-rate sequence of low-power picosecond pulses by a broadband laser amplifier.

Consider now the results of experiments on passive mode locking in a multisection semiconductor laser with a 15-cm-long external cavity staged to verify the aforesaid [11]. Fig. 1 gives the peak power $\mathscr{P}_p$, the average power $\mathscr{P}$, and the pulse duration τ_p as functions of the injection current J for a repetition rate of 1 GHz. Minimal pulse lengths of less than 5 ps are attained simultaneously with a peak power of about 5 W as a maximum for moderate currents. Applying the MOPA system, i. e., a further amplification of such pulse sequences by a travelling-wave linear optical amplifier in the form of a wedge-shaped semiconductor laser diode with anti-reflection-coated faces, an amplification factor of 39 dB, and a response time of about 2 ps [12, 13] would show promise of obtaining the required pulses with a power of the order of several kilowatts and a repetition rate of 1 GHz and over.

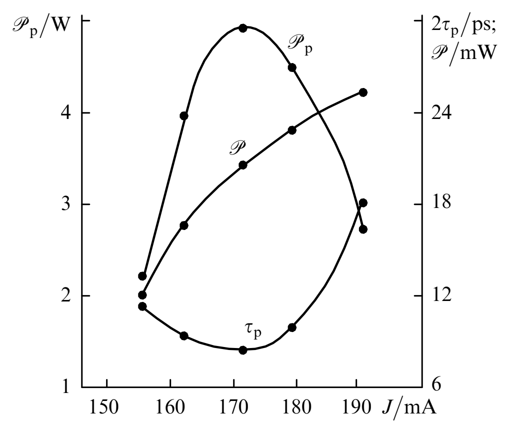

Figure 1.

Alternative sources of high-power picosecond pulses may be laser diode-pumped fibre amplifiers with a response time of less than 1 ps and a two-stage amplification factor up 50 dB. In particular, the authors of Ref. [14] amplified the low-power 1.8-ps pulses of a master semiconductor laser up to 500 kW in peak power. The required pulse repetition rates of the order of several hundred megahertz can also be obtained employing fibre amplifiers with an average output power into the hundreds of watts [15].

6. Conclusions

The above analysis suggests that the limitations imposed on the density of cooled atoms and the length of the volume they occupy may be overcome by raising the laser beam intensity above the saturation intensity of the working atomic transition, applying the noncollinear geometry of the laser beam and the atomic medium, and using coherent cooling by sequences of counterpropagating high-intensity π pulses, as well as by combining these techniques. All the above techniques call for a substantial rise of the laser beam intensity compared to the traditional Doppler cooling technique.

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