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The formation dynamics of a shock wave of the ultrashort pulse envelope in a medium with relaxing cubic nonlinearity

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Abstract. The effect of relaxation of nonlinearity on the formation of a shock wave of the intensity envelope of ultrashort laser pulses is studied on the basis of numerical analysis of the corresponding truncated wave equation. The relaxation of nonlinearity is shown to lead to the stabilisation of the envelope sharpening process, with the minimum width of the shock-wave front being limited by the relaxation time of nonlinearity. It is established that depending on the nonlinearity sign, the relaxation causes compression or spreading of the propagating pulse.

A feasibility of forming envelope shock waves of high-intensity ultrashort light pulses in nonlinear media was pointed out for the first time by Ostrovskii [1]. In his first publications [1, 2], he showed that the nonlinear distortion of the shape of a powerful laser pulse propagating in a nonlinear medium was caused by the dependence of the group velocity on the pulse intensity. A nonlinear addition to the group velocity of an ultrashort pulse (USP) in media with $n_2 > 0$ brings about a steepening of its trailing edge, whereas in media with $n_2 < 0$ the leading edge of the pulse becomes steeper. As the pulse propagates in a nonlinear medium, the accumulating changes in the USP shape may lead to the formation of a shock wave of the pulse envelope. This situation is quite similar to shock wave generation in acoustics [3].

The formation dynamics of the USP envelope shock wave in nonlinear media has been considered in many papers (see, e.g., [1 – 8]). However, the authors investigated primarily the spectral broadening of an USP caused by nonlinear distortion of its shape. The detailed analysis of the formation dynamics of the USP envelope shock wave was made mostly for media with inertialess cubic nonlinearity [4]. From general considerations it is evident that when one of the edges of an USP propagating in a nonlinear medium becomes steeper, the delay of the nonlinear response becomes more appreciable. In addition, the interaction length, over which the quasistatic (inertialess) approximation is valid, becomes substantially smaller.

The results of experimental investigations of the formation of the envelope shock wave of picosecond pulses in capillary fibre waveguides filled with organic liquids are reported in Ref. [9]. The comparison between the experimental data and well-known theoretical models revealed [9] that the delay of the nonlinear response of the fibre 'filler' caused by various mechanisms plays a decisive role in the envelope shock wave formation and spectral broadening of USPs. Therefore, the analysis of the formation dynamics of the USP envelope shock wave in nonlinear media is of practical interest.

In this paper, we performed the numerical modelling of formation of the USP envelope shock wave in a Kerr medium with relaxing nonlinearity. Manifestations of the wave nonstationary behaviour, namely, the dependence of the group velocity of a light pulse on the intensity, are considered using a truncated wave equation, in which the first derivative of the nonlinear polarisation amplitude of the medium is retained in the right-hand side [3]. The analysis of the formation dynamics of the USP envelope shock wave in nonli near media is based on the numerical solution of the system consisting of the truncated wave equation and dynamic equation for the nonlinear response. The analog of the system is a single integro-differential equation for the complex amplitude of a pulse.

For the high-frequency Kerr effect, the dynamic equation for the nonlinear addition $\Delta\chi$ to the linear susceptibility of a medium has the form [3]

$$t_0 \frac{\partial \Delta\chi}{\partial t} + \Delta\chi = \chi^{(3)} |E|^2, \qquad (1)$$

where $\chi^{(3)}$ is the cubic susceptibility of the medium ($\chi^{(3)} \sim n_2$), t_0 is the relaxation time of the nonlinear response, and E is the pulse amplitude. From Eqn (1) for the nonlinear polarisation $\mathscr{P}^{(3)}$ of the medium, we find the expression

$$\mathscr{P}^{(3)} = \frac{\chi^{(3)}}{t_0} \int_{-\infty}^{t} |E|^2 \exp\left(\frac{t' - t}{t_0}\right) \mathrm{d}t'$$

$$\times E \exp[-\mathrm{i}(\omega t - kz)] \equiv \chi_{\mathrm{n}} \mathscr{E}, \qquad (2)$$

where ω and $k = \omega n_0 / c$ are the frequency and wave number, n_0 is the linear refractive index, and $\mathscr{E}(z, t) = E(z, t) \times \exp[-\mathrm{i}(\omega t - kz)]$. Taking into account Eqn (2), from the wave equation

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$$\left(\Delta - \frac{n_0^2}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathscr{E} = \frac{4\pi}{c^2}\frac{\partial^2 \mathscr{P}^{(3)}}{\partial t^2} \quad (3)$$

one can easily obtain the following equation for the pulse amplitude:

$$\frac{\partial E}{\partial z} + \frac{1}{v_0}\frac{\partial E}{\partial t} = \mathrm{i}2\frac{k}{n_0^2}\frac{\chi^{(3)}}{t_0}\left(1 - \mathrm{i}\frac{2}{\omega}\frac{\partial}{\partial t}\right)$$
$$\times\left[E\int_{-\infty}^{t}|E|^2\exp\left(\frac{t'-t}{t_0}\right)\mathrm{d}t'\right], \quad (4)$$

where $v_0 = c/n_0$. The integro-differential equation (4) takes into account the first derivative of the polarisation amplitude (2) because it is responsible, as was mentioned above, for the distortion of the pulse shape in a nonlinear medium.

Using Eqn (4), we will consider the formation dynamics of the envelope shock wave for USPs having the Gaussian shape

$$E(0,t) = E_0\exp\left[-2\left(\frac{t}{t_\mathrm{p}}\right)^2\right], \quad (5)$$

where E_0 is the real maximum amplitude and t_p is the pulse duration. After the substitution $E = |E|\exp(\mathrm{i}\varphi)$ and simple transformations, Eqn (4) is reduced to the system of two equations in dimensionless variables $x = z/L$ and $\tau = t/t_\mathrm{p}$ for the real normalized intensity $I = |E|^2/|E_0|^2$ and the phase φ of the pulse:

$$\frac{\partial I}{\partial x} + \left[\frac{L}{v_0 t_\mathrm{p}} + \frac{a}{\alpha}\int_{-\infty}^{\tau} I\exp\left(-\frac{\tau-\tau'}{\alpha}\right)\mathrm{d}\tau'\right]\frac{\partial I}{\partial \tau}$$
$$= -\frac{2a}{\alpha}I\int_{-\infty}^{\tau}\frac{\partial I}{\partial \tau'}\exp\left(-\frac{\tau-\tau'}{\alpha}\right)\mathrm{d}\tau', \quad (6)$$

$$\frac{\partial \varphi}{\partial x} + \left[\frac{L}{v_0 t_p} + \frac{a}{\alpha}\int_{-\infty}^{\tau} I\exp\left(-\frac{\tau-\tau'}{\alpha}\right)\mathrm{d}\tau'\right]\frac{\partial \varphi}{\partial \tau}$$
$$= \frac{\omega t_\mathrm{p}}{2}\frac{a}{\alpha}\int_{-\infty}^{\tau} I\exp\left(-\frac{\tau-\tau'}{\alpha}\right)\mathrm{d}\tau', \quad (7)$$

where L is the length of a nonlinear medium, $\alpha = t_0/t_\mathrm{p}$, $a = 4\pi k\chi^{(3)}|E_0|^2 L/n_0^2\omega t_\mathrm{p}$, $0 \leqslant x \leqslant 1$. According to Eqn (5), the boundary conditions for Eqns (6) and (7) are

$$I(0,\tau) = \exp\left[-(2\tau)^2\right],\ \varphi(0,\tau) = 0. \quad (8)$$

It is evident that Eqn (6) describing the formation dynamics of the envelope shock wave does not depend on the pulse phase. Equations (6) and (7) are derived for arbitrary values of the parameter α. Specifically, for $\alpha \to 0$, which corresponds to the transition to the inertialess response of the medium, Eqn (6) takes the known form [3]:

$$\frac{\partial I}{\partial x} + \left(\frac{L}{v_0 t_\mathrm{p}} + 3aI\right)\frac{\partial I}{\partial \tau} = 0. \quad (9)$$

Equation (9), which has the analytic solution in the implicit form, was studied quite thoroughly in Ref. [4].

We will study the effect of the delay of the nonlinear response of a medium on the formation of the envelope shock wave using the numerical analysis of Eqn (6) with the boundary conditions (8). The numerical solution of Eqn (7), together with the obtained solution of Eqn (6), makes it possible to analyze how the process of the USP frequency deviation depends on the parameters of a nonlinear medium and radiation.

The results of the numerical solution of Eqns (6) and (7) are shown in Fig. 1. For comparison, Fig. 1a shows the results corresponding to the instantaneous response of a nonlinear medium ($\alpha = 0$) [4]. One can see from Fig. 1a that in this case, the shapes of the intensity envelope $I(\xi)$ and of the dependence of the frequency $\partial\varphi/\partial\xi$ on the time $\xi = \tau - xL/(v_0 t_\mathrm{p})$ corresponding to the opposite signs of the nonlinearity parameter $a \sim \chi^{(3)}$, are symmetric with respect to the initial position of the pulse maximum (the point $\xi = 0$).

As soon as we allow for the relaxation of nonlinear response of the medium, the symmetry becomes violated due to the shift of the amplitude maximum $\chi_\mathrm{n}(\xi)$ toward the trailing edge of the pulse (see Fig. 1b, c). In this case, the qualitative distinctions are observed in the character of the formation dynamics of intensity envelope shock waves according to the sign of the nonlinearity parameter a. A characteristic feature of the deformation of the intensity envelope in the case $a < 0$ is the emergence of a narrow peak with the duration $\sim t_0$ at the leading edge of the pulse. In the case $a > 0$, the steepening of the trailing edge of the pulse is accompanied by an increase in the pulse duration and a decrease in its maximum intensity.

The comparison of the results of numerical experiments corresponding to different relaxation times (Fig. 1b, c) reveals that the delay of nonlinearity hinders the shock wave formation. The sharpening of the pulse envelope is stabilised when the width of the pulse edge is of the same order of magnitude as the nonlinearity relaxation time.

One can see from Fig. 1 that the delay of the nonlinear response also brings about the violation of symmetry of the function $\partial\varphi/\partial\xi$ relative to the point $\xi = 0$ when the sign of the nonlinearity parameter $a \sim \chi^{(3)}$ changes. The maximum absolute value of the function $\partial\varphi/\partial\xi$ is reached for $a > 0$ at the trailing edge of the pulse due to the 'accumulating' nonlinear response, whereas for $a < 0$ it is achieved at the leading edge of the pulse due to its high peak intensity caused by the formation of a shock wave.

Thus, the delay of nonlinearity affects substantially the dynamics of the intensity envelope of a propagating USP. First, the relaxation of nonlinearity limits the sharping of USP envelope so that the width of the shock wave front has a characteristic scale of the order of the relaxation time t_0. Second, the delay of nonlinearity causes qualitative differences in the regimes of front steepening for USPs propagating in media with different signs of n_2. For positive values of n_2, the delay of nonlinearity leads to decompression of the pulse, whereas for negative n_2, the opposite effect is observed.

Note that the formation of a short intense peak at the leading edge of the envelope (the case $\alpha \neq 0$ and $a < 0$) is interesting for using this effect for shortening the duration of an USP propagating in media with negative relaxing nonlinearity.

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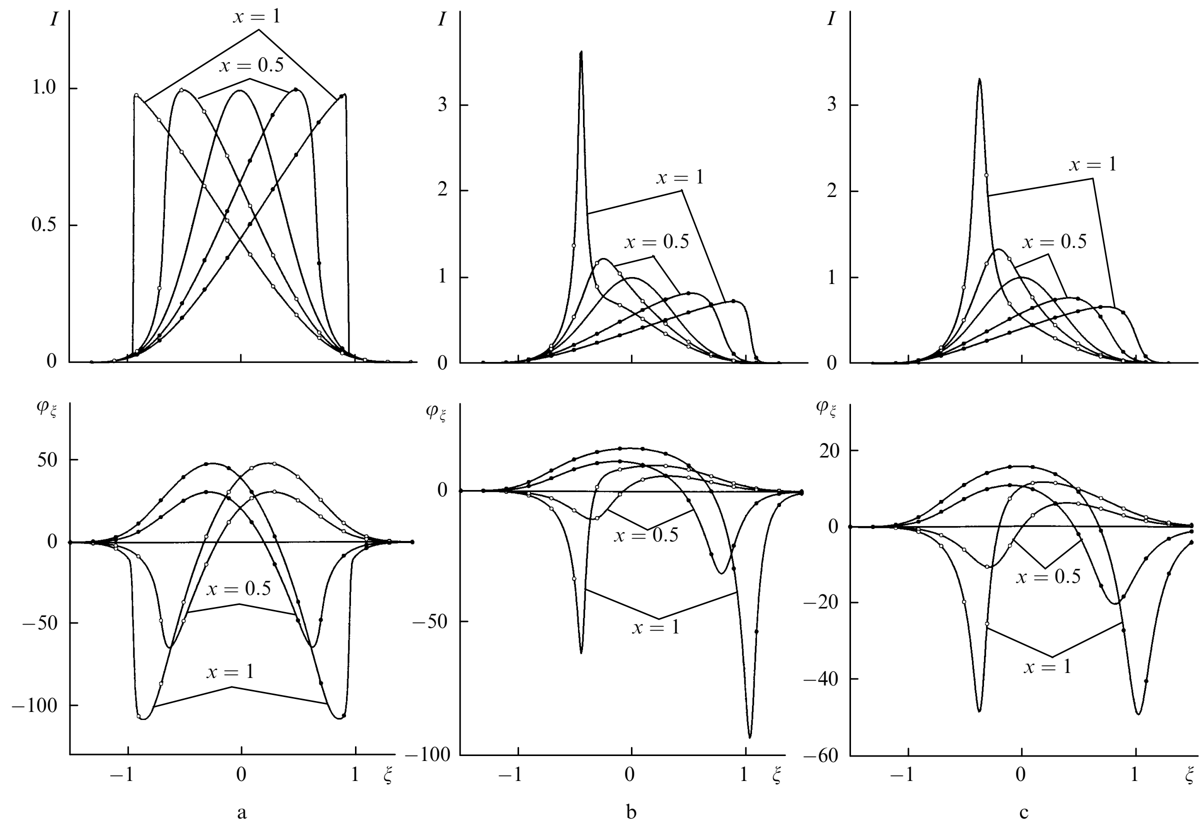

Figure 1. The evolution of the envelope I and frequency $\varphi_\xi = \partial\varphi/\partial\xi$ of a Gaussian pulse in a medium with cubic nonlinearity for various x and $\omega t_{\mathrm{p}} = 10^2$ and for $\alpha = 0$, $a = \pm 1/3$,(a), $\alpha = 0.1$, $a = 0.5$, $a = -0.175$ (b), and $\alpha = 0.2$, $a = 0.5$, $a = -0.2$ (c). The curves identified with dark (light) marker correspond to $a > 0$ ($a < 0$). The unmarked curves show the envelopes of an input pulse ($x = 0$).

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