Quantum Electronics **31** (1) 50 – 54 (2001)

NONLINEAR OPTICAL PHENOMENA

PACS 42.81.Dp; 42.65.Sf
DOI: 10.1070/QE2001v031n01ABEH001890

Modulation instability of the wave packet in a periodically inhomogeneous nonlinear fibre

I O Zolotovskii, D I Sementsov

Abstract. The modulation instability conditions are studied for a wave packet consisting of two strongly coupled modes co-propagating in a two-mode fibre. The two modes are linearly coupled by the phase matching conditions, which take the longitudinal periodicity of the fibre into account. The nonlinear coupling of the modes is provided by the cross-modulation interaction. The influence of the initial conditions of the fibre excitation on the subsequent development of the modulation instability is analysed. It is shown that the modulation instability can exist in a region of normal material dispersion of the fibre.

Keywords: *nonlinear fibre, optical pulse, modulation instability*.

The instability of quasi-continuous radiation with respect to a temporal modulation was first considered in Ref. [1]. It was shown that the modulation instability (MI) of the wave in a nonlinear single-mode fibre arises due to the self-action only in the region of anomalous dispersion. The MI in fibres with Kerr nonlinearity was observed in Ref. [2]. Several authors [3 – 5] have shown that MI is possible in media with other kinds of nonlinearity as well. The authors [4, 5] studied the influence of the higher order dispersion on the MI in the absence of the second-order dispersion.

In the frequency region of normal dispersion, the MI can be caused by cross-modulation [6, 7]. For example, the authors of Refs [8, 9] have demonstrated experimentally a modulation instability arising due to the phase cross-modulation in a two-mode nonlinear fibre, in particular, in the region of normal dispersion. It is quite interesting to consider effects related to the wave packet instability caused by relatively long interaction between short pulses propagating in nonlinear fibres. In particular, such instability can arise in systems consisting of two fibres with a distributed coupling or in two-mode fibres with a strong linear and nonlinear coupling between the modes.

A strong coupling between co-propagating modes can be achieved in long-period fibre gratings, which were recently produced by laterally irradiating germanosilicate fibre with the UV light [10, 11]. Unlike the conventional Bragg gratings, which couple the fundamental fibre mode with the counter-propagating mode [12] and have a spatial period Λ in the medium on the order of a wavelength, the proposed photoinduced gratings have a period of 100 – 500 μm and can efficiently couple co-propagating eigenmodes of the fibre. The analysis of linear and nonlinear regimes of the optical mode conversion in such fibres shows that they have unique dispersion properties [13], which allow the efficient compression of propagating pulses [14, 15].

I O Zolotovskii, D I Sementsov Ul'yanovsk State University, ul. L. Tolstogo 42, 432700 Ul'yanovsk, Russia

Received 20 June 2000; revised version received 11 October 2000
Kvantovaya Elektronika **31** (1) 50 – 54 (2001)
Translated by I V Bargatin

In this work, we study the emergence of the modulation instability of a wave packet propagating in a two-mode periodic fibre. We consider various types of fibre excitation and take the first- and second-order dispersion effects, as well as nonlinear self- and cross-modulation effects, into account.

Suppose that a light pulse of duration τ_0 and peak input amplitude A_0 is injected into a two-mode longitudinally periodic fibre. The modes propagating in the fibre are efficiently coupled if the phase matching condition is satisfied. Upon pulsed excitation of the periodic fibre, the phase matching condition should be satisfied at the central frequency:

$$\beta_1(\omega_0) - \beta_2(\omega_0) - \frac{2\pi}{\Lambda} = 0. \tag{1}$$

In a longitudinally homogeneous fibre, the phase matching condition $\beta_1(\omega) - \beta_2(\omega) = 0$ cannot be satisfied because the propagation constants are different in the working frequency range (for example, for LP_{01} and LP_{02} modes). Employing the periodicity, one can easily satisfy the phase matching condition. If the parameters satisfy this condition closely, we can write the equations for the temporal envelopes of the coupled pulse modes in the current-time coordinates, $\tau = t - z/u$, where u is the group velocity of the wave packet. Taking into account the group velocity mismatch between the modes, the material dispersion, and the nonlinear effects (phase self-modulation and cross-modulation), these equations can be written in the form [13]

$$\begin{aligned}
&\frac{\partial A_1}{\partial z} - \frac{1}{v}\frac{\partial A_1}{\partial \tau} - \mathrm{i}\frac{d}{2}\frac{\partial^2 A_1}{\partial \tau^2} \\
&\qquad = -\mathrm{i}\sigma_{12}A_2 - \mathrm{i}R(\gamma_{11}|A_1|^2 + 2\gamma_{12}|A_2|^2)A_1, \\
&\frac{\partial A_2}{\partial z} + \frac{1}{v}\frac{\partial A_2}{\partial \tau} - \mathrm{i}\frac{d}{2}\frac{\partial^2 A_2}{\partial \tau^2} \\
&\qquad = -\mathrm{i}\sigma_{21}^{*}A_1 - \mathrm{i}R(2\gamma_{21}|A_1|^2 + \gamma_{22}|A_2|^2)A_2 .
\end{aligned} \tag{2}$$

Here, $v^{-1} = (u_1 - u_2)/2u^2$; $u_j = (\partial\beta_j/\partial\omega)_0^{-1}$; $u_j = (\partial\beta_j/\partial\omega)_0^{-1}$ is the group velocity of the jth mode; $2u = u_1 + u_2$; d is the material dispersion of the fibre; R is the nonlinearity parameter of the fibre;

$$\sigma_{ij}=\left(k^2\varepsilon_0 m\int f(r)U_i^*U_j r\mathrm{d}r\right)\left(2\beta_i\int U_i^*U_i r\mathrm{d}r\right)^{-1},$$
$$\gamma_{ij}=\left(\int U_i^2U_j^2 r\mathrm{d}r\right)\left(\int U_i^2 r\mathrm{d}r\right)^{-1} \tag{3}$$

are intermodal coupling constants (σ_{ij}), phase self-modulation (γ_{ii}) and cross-modulation (γ_{ij}) parameters determined by the corresponding overlap integrales for the profile functions $U(r)$ of the fibre modes; and ε_0 is the dielectric constant at the fibre axis. The function $f(r)$ describes the distribution of the dielectric constant over the fibre cross section; parameter $m \ll 1$ is the degree of modulation of the longitudinal optical inhomogeneity; and $k=\omega/c$ is the wave number.

Equations (2) should be solved together with the initial conditions for the temporal mode envelopes A_j, which are determined by the fibre excitation conditions. These initial conditions can be quite generally described by the relation $A_2(\tau,0)=\xi A_1(\tau,0)$, where parameter ξ determines the fibre excitation regime. The case of $\xi=\pm 1$ corresponds to the symmetric or antisymmetric fibre excitation, whereas $\xi=0$ or $\xi^{-1}=0$ correspond to the single-mode excitation.

Let us introduce the characteristic lengths: the dispersion length, $L_{\mathrm{d}}=\tau_0^2|d|^{-1}$, the group velocity mismatch length, $L_{\mathrm{g}}=\tau_0 v$, and the nonlinear and intermodal coupling lengths, $L_{\mathrm{n}}=|RA_0^2|^{-1}$ and $L_\sigma=|\sigma|^{-1}$, where $2\sigma=\sigma_{12}+\sigma_{21}^*$.

In our case, we have $L_\sigma \ll L_{\mathrm{d}}, L_{\mathrm{g}}, L_n$; therefore, in the absence of losses, the variation in the pulse intensity caused by the material dispersion of the fibre, the group velocity dispersion of the modes, and the cubic nonlinearity of the fibre is negligibly small over the length L_σ. Consequently, we can regard the condition $|A_1|^2+|A_2|^2=\mathrm{const}$ as reasonably well satisfied over the length L_σ. It follows from this condition that $\sigma_{12}\simeq\sigma_{21}^*=\sigma$, $\gamma_{11}\simeq\gamma_{22}=\gamma_{\mathrm{sm}}$, and $2\gamma_{21}\simeq 2\gamma_{12}=\gamma_{\mathrm{cm}}$. We will solve equation (2) in the case of the phase matching between the modes and within the approximation of slowly varying amplitudes. The temporal envelope of the corresponding mode will be represented as a sum of two partial pulses:

$$A_j=(-1)^{j+1}a_1(\tau,z)\exp(\mathrm{i}\sigma z)+a_2(\tau,z)\exp(-\mathrm{i}\sigma z), \tag{4}$$

where a_f are amplitudes that vary slowly with the coordinate z. Inserting expression (4) into Eqn (2), we obtain the following equations for the amplitudes of the partial pulses ($f=1,2$):

$$\frac{\partial a_f}{\partial z}-\frac{\mathrm{i}D_f}{2}\frac{\partial^2 a_f}{\partial\tau^2}+\mathrm{i}\left(\chi|a_f|^2+2R\gamma_{\mathrm{sm}}|a_{3-f}|^2\right)a_f=0, \tag{5}$$

where $\chi=R(\gamma_{\mathrm{cm}}+\gamma_{\mathrm{sm}})$. It follows from Eqns (4) and (5) that the original pulse can be represented as a combination of partial pulses a_1 and a_2, for which the effective fibre dispersion is given by

$$D_f=(-1)^f d_{\mathrm{m}}+d, \tag{6}$$

where $d_{\mathrm{m}}=1/v^2\sigma$ is the intermodal dispersion of the group velocities of the modes. The sign of the material dispersion d is determined by the actual type of the fibre material dispersion (normal or anomalous) at the central frequency of the pulse. If parameters v, σ, and d are chosen appropriately, the fibre can become a waveguide having zero effective dispersion for one of the pulses. The condition $D_f=0$ then holds at the working frequency.

Consider the solution of equations (5) in the case when small temporal perturbations appear against the background of a sufficiently powerful quasi-continuous pump. If the duration of the wave packet coupled to the fibre is sufficiently long and the quasi-monochromatic approximation is applicable to the pump wave (i. e., the dispersion terms are negligibly small for it, which should hold for $\tau_0 \gg 10^{-10}$ s), we can write the solution of equations (5) for the amplitudes of the partial pulses as

$$a_f(z,\tau)=[a_{f0}+\Delta a_f(z,\tau)]\exp\left[-\mathrm{i}(\chi a_{f0}^2+2R\gamma_{\mathrm{sm}}a_{3-f0}^2)\right], \tag{7}$$

where Δa_f is the complex amplitude of the perturbation, satisfying the inequality $a_{f0}\gg|\Delta a_f|$.

Inserting expression (7) into equations (5) and linearising them with respect to small perturbations Δa_f, we derive a system of equations for the perturbations:

$$\frac{\partial}{\partial z}\Delta a_f-\frac{\mathrm{i}D_f}{2}\frac{\partial^2}{\partial\tau^2}\Delta a_f=-\mathrm{i}\chi a_{f0}^2\left(\Delta a_f+a_f^*\right)$$
$$-2\mathrm{i}R\gamma_{\mathrm{sm}}a_{10}a_{20}\left(\Delta a_{3-f}+\Delta a_{3-f}^*\right). \tag{8}$$

Taking into account the conditions of the pulse injection into the fibre, we have for the initial amplitudes of the partial pulses

$$a_{f0}^2=\frac{I_0[1+(-1)^f\xi]^2}{4(1+\xi^2)}, \tag{9}$$

where I_0 is the intensity of the injected pulse. For the following discussion, the symmetric ($\xi=1, f=2$) and antisymmetric ($\xi=-1, f=1$) fibre excitation regimes, when the propagating pulse consists of only one partial pulse, will be particularly interesting. In these cases, the product $a_{10}a_{20}=0$ in accordance with Eqn (9), and system of equations (8) reduces to a system of two identical equations for perturbations:

$$\frac{\partial}{\partial z}\Delta a_f-\frac{\mathrm{i}D_f}{2}\frac{\partial^2}{\partial\tau^2}\Delta a_f=-\mathrm{i}\chi a_{f0}^2\left(\Delta a_f+\Delta a_f^*\right),\ f=1,2. \tag{10}$$

We will seek the solution of this system in the case of harmonic perturbations in the form

$$\Delta a_f=b_f\cos(hz-\Omega\tau)+\mathrm{i}l_f\sin(hz-\Omega\tau), \tag{11}$$

where h and Ω are the wave number and perturbation frequency, respectively. Inserting (11) into (10), we obtain a system of two homogeneous equations for amplitudes b_f and l_f. By solving this equation, we derive the dispersion relationship

$$h_f=\pm\chi I_0\frac{\Omega}{\Omega_f}\left(\frac{\Omega^2}{\Omega_f^2}+\operatorname{sign}D_f\right)^{1/2}, \tag{12}$$

where $\Omega_f^2=2\chi I_0/|D_f|$. It follows from Eqn (12) that the stability of the stationary state depends on the sign of the effective dispersion describing the propagating partial pulse.

For $D_f > 0$, the wave number h_f is real at any Ω, and the stationary state of the pulses is stable with respect to small perturbations. For $D_f < 0$ and $|\Omega| < \Omega_f$, the wave number h_f becomes imaginary, and a perturbation $\Delta a_f(z,\tau)$ grows exponentially with z. The perturbation gain is determined by the relation

$$G_f = 2\mathrm{Im}(h_f) = 2\chi I_0 \frac{\Omega}{\Omega_f}\left[1 - \left(\frac{\Omega}{\Omega_f}\right)^2\right]^{1/2}. \quad (13)$$

The maximum gain $G_f = \chi I_0$ is reached at the frequency $|\Omega_{\mathrm{m}}| = \Omega_f/\sqrt{2}$. An important feature of the case considered is that the MI can be observed in the frequency region corresponding to normal material dispersion d of the fibre.

Fig. 1 shows the dependence of the perturbation gain G_f on the modulus of the perturbation frequency $|\Omega|$ obtained for the effective dispersion $D_f = -10^{-26}$ s^2 m^{-1} and parameters $\chi I_0 = 0.5$, 1, 1.5, and 2 m^{-1} in the case of symmetric (antisymmetric) fibre excitation. One can see that the width $\Delta\Omega$ of the MI region and the maximum value of G_f increase with increasing intensity of the injected radiation or increasing nonlinearity parameter. At the same time, the maximum of the gain shifts towards higher perturbation frequencies.

Because the effective dispersion D_f of the partial pulse is different for the symmetric and antisymmetric ($f = 1$, 2) fibre excitation regimes at the same values of the material dispersion d, the width of the MI region and the maximum gain will be different for these two types of excitation. Furthermore, when $|d| < d_{\mathrm{m}}$, the MI is not observed upon symmetric fibre excitation, whereas it can take place upon antisymmetric excitation.

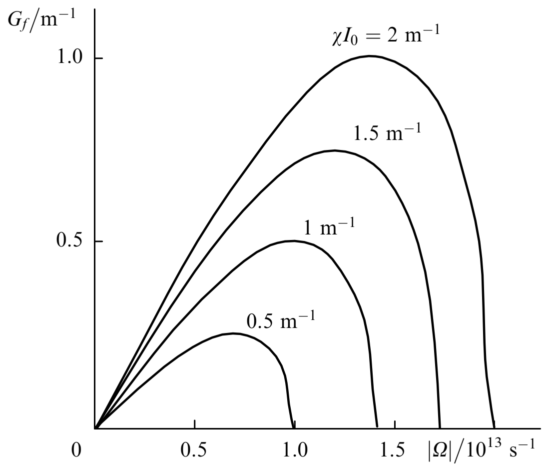

Рис.1. Dependence of the gain G_f on the modulus of the perturbation frequency $|\Omega|$ upon symmetric or antisymmetric fibre excitation for $D_f = -10^{-26}$ s^2 m^{-1} and various values of χI_0.

Fig. 2 shows the dependence of G_f on the parameter χI_0 obtained for $D_f = -10^{-26}$ s^2 m^{-1} and $|\Omega| = (0.5 - 2) \times 10^{13}$ s^{-1}. One can see that the threshold intensity (nonlinearity) sufficient to induce the MI increases with increasing perturbation frequency.

In the general case of $\xi \neq \pm 1$, the propagating pulse cannot be described by a single partial pulse; therefore, one has to solve the complete system of coupled equations (8). Taking solutions in the form (11), we obtain the general dispersion equation

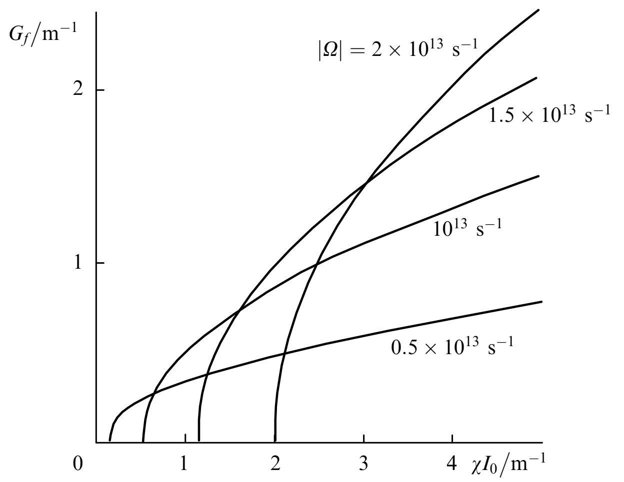

Рис.2. Dependence of the gain G_f on the parameter χI_0 upon symmetric or antisymmetric fibre excitation for $D_f = -10^{-26}$ s^2 m^{-1} and various values of $|\Omega|$.

$$h^4 - (K_1 + K_2)h^2 + K_1K_2 - F = 0,$$

$$K_f = 0.25 D_f \Omega^2 (D_f \Omega^2 + 4\chi a_{f0}^2), \quad (14)$$

$$F = 4D_1D_2R^2\gamma_{\mathrm{sm}}^2\Omega^4 a_{10}^2 a_{20}^2.$$

Equation (14) has the following solution:

$$h_\pm^2 = 0.5\Big\{K_1 + K_2 \pm [(K_1 - K_2)^2 + 4F]^{1/2}\Big\}. \quad (15)$$

The modulation instability of the pulse corresponds to the regions of parameter values were one or both of the wave numbers $h_\pm$ are imaginary. Consider the case of the single-mode fibre excitation, when either $\xi = 0$ or $\xi^{-1} = 0$ and $a_{10}^2 = a_{20}^2 = I_0/4$. In this case, we have

$$h_\pm^2 = \frac{\Omega^2}{4}\Big\{\Omega^2(d^2 + d_{\mathrm{m}}^2) + \chi I_0 d \pm 2\Big[d_{\mathrm{m}}^2(\Omega^2 d + \chi I_0/2)^2 + \gamma_{\mathrm{sm}}^2 R^2 I_0^2 (d^2 - d_{\mathrm{m}}^2)\Big]^{1/2}\Big\}. \quad (16)$$

We will consider some special cases of material and intermodal dispersion. When the material dispersion almost vanishes ($d \simeq 0$), we have

$$h_\pm^2 = \frac{\Omega^2}{4}\Big[\Omega^2 d_{\mathrm{m}}^2 \pm I_0 d_{\mathrm{m}}(\chi^2 - 4\gamma_{\mathrm{sm}}^2 R^2)^{1/2}\Big]. \quad (17)$$

It follows from (17) that, if the intensity of the injected radiation is

$$I_0 > \Omega^2 d_{\mathrm{m}}(\chi^2 - 4\gamma_{\mathrm{sm}}^2 R^2)^{-1/2} \quad (18)$$

the wave number h_- becomes imaginary and the corresponding perturbation leads to the MI. At $d_{\mathrm{m}} \to 0$, which is possible in the case of a strong intermodal coupling, $h \approx 0.5\Omega^2 d_{\mathrm{m}}$ and the MI is not realised.

The case of $|d| = d_m$, which corresponds to the zero effective dispersion D_f for one of the partial pulses, is quite interesting from the practical viewpoint. In this case, we obtain from (16) for $d > 0$,

$$h_+^2 = \Omega^2 d(\Omega^2 d + 0.5\chi I_0), \quad h_- = 0; \tag{19}$$

whereas for $d < 0$,

$$h_-^2 = \Omega^2 |d|(\Omega^2 |d| - 0.5\chi I_0), \quad h_+ = 0. \tag{20}$$

It follows from the derived equations that the MI takes place in the case of anomalous material dispersion ($d < 0$). The wave number h_- becomes imaginary in the frequency interval $|\Omega| < (\chi I_0/2|d|)^{1/2}$. The maximum perturbation gain $G_{\max} = \chi I_0/2$, observed at the frequency $|\Omega_m| = (\chi I_0/2|d|)^{1/2}$ is two times lower than $G_{\max}$ of the two-mode excitation.

Consider now the conditions for the appearance of the MI when the carrier frequency is strongly shifted to the blue, where the material dispersion is normal ($d > 0$). If $d > d_m$, the effective dispersion $D_{1,2} > 0$ for both partial pulses. According to Eqn (15), the appearance of the MI in this case is determined by the inequality $F > K_1 K_2$, which yields the condition $\gamma_{sm} > \gamma_{cm}$, taking into account the expressions for F and K_f. The MI takes place for perturbations within the frequency interval

$$|\Omega| < \Omega_s = \left\{[(g_1 + g_2)^2 + \Delta]^{1/2} - g_1 - g_2\right\}^{1/2}, \tag{21}$$

where

$$g_f = \frac{\chi I_0}{2D_f}\frac{[1 + (-1)^f \xi]^2}{1 + \xi^2};$$

$$\Delta = (\gamma_{sm} - \gamma_{cm})(3\gamma_{sm} + \gamma_{cm})\left(\frac{RI_0}{D_f}\frac{1 - \xi^2}{1 + \xi^2}\right)^2.$$

Assuming that $d \gg d_m$, we have $D_1 \simeq D_2 \simeq d$. In this case, the gain is given by

$$G = d|\Omega|(\Omega_s^2 - \Omega^2)^{1/2}.$$

The maximum perturbation gain $G_{\max} = d\Omega_s^2/2$ is reached at the perturbation frequency $\Omega_m = \Omega_s/\sqrt{2}$.

Fig. 3 shows the dependence of $G_{\max}$ on the parameter ξ, which characterises the type of the fibre excitation. The dependence has been calculated for $d = 10^{-26}$ s^2 m^{-1}, $d_m = 10^{-27}$ s^2 m^{-1}, $\gamma_{sm} RI_0 = 1$ m^{-1}, and the ratios $\gamma_{cm}/\gamma_{sm} = 0$, 1/3, and 2/3. One can see that in this case ($D_1 > 0$ and $D_2 > 0$), the gain reaches its maximum value upon single-mode fibre excitation (ξ or $\xi^{-1} = 0$), whereas $G = 0$ for all values of γ_{cm}/γ_{sm} in the case of symmetric excitation ($\xi = 1$). The gain increases with increasing strength of the cross-modulation nonlinear effects.

Our analysis has shown that the MI can appear in longitudinally periodic fibres and similar systems with distributed coupling regardless of the sign of the material dispersion. Although the development of the MI in systems with a linear intermodal coupling (in particular, periodic waveguides) is similar to the development of the MI in nonlinear birefringent waveguides, the parameters characterising the dynamics of the MI in such systems (the effective dispersion, the nonlinearity parameter, and the amplitudes of partial pulses) differ from their conventional form.

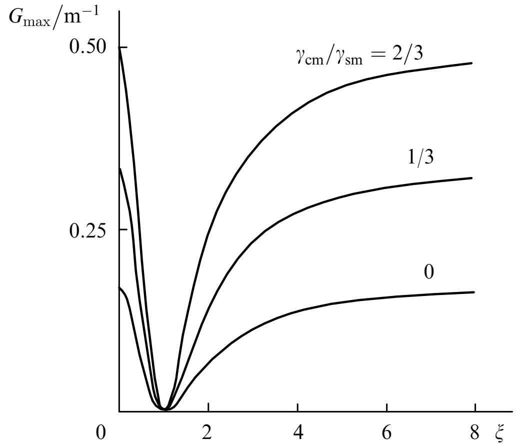

Рис.3. Dependence of the gain $G_{\max}$ on the parameter ξ, which describes the fibre excitation regime, for $d = 10^{-26}$ s^2 m^{-1}, $d_m = 10^{-27}$ s^2 m^{-1}, and various ratios γ_{cm}/γ_{sm}.

Thus, the effective dispersion of a system with an intermodal coupling can differ significantly from the material dispersion of the fibre itself [15]. In particular, the effective dispersion becomes negative in the frequency range where the material dispersion is normal. In our opinion, the most important property of the effective parameters that describe systems with an intermodal coupling is their strong dependence on the conditions of the radiation injection into the fibre, which are determined by the parameter ξ. This property makes it possible to control efficiently the MI dynamics in such fibres and appears to be the most useful property for practical applications. Note also that the results obtained are valid only for the initial stage of the MI. The developed stage of the MI, when the perturbation $|\Delta a_f|$ becomes comparable to a_{f0}, can be analysed only by numerical methods.

References

1. Hasegawa A *Opt. Lett.* **9** 288 (1984).
2. Tai A, Hasegawa A, Tomita A *Phys. Rev. Lett.* **56** 135 (1986)
3. Lyra M L, Gouveia-Neto A S *Opt. Commun.* **108** 117 (1994)
4. Hickmann J M, Cavalcanti S B, Borges M N, Gouveia E A, Gouveia-Neto A S *Opt. Lett.* **18** 182 (1993)
5. Feng Lu, Liu Xiumin, Feng Qiyuan *Kvantovaya Elektron.* **27** 269 (1999) [*Quantum Electron.* **29** 546 (1999)]
6. Agrawal G P *Phys. Rev. Lett.* **59** 880 (1987)
7. Agrawal G P *Nonlinear Fiber Optics* (Boston: Academic Press, 1989
8. Dinda P T, Millot G, Steve E, Haelterman M *Opt. Lett.* **21** 1640 (1996)
9. Millot G, Seve E, Dinda P T, Haelterman M *Opt. Lett.* **22** 1686 (1997)
10. Vasil'ev S A, Dianov E M, Kurkov A S, Medvedkov O I, Protopopov V N *Kvantovaya Elektron.* **24** 151 (1997) [*Quantum Electron.* **27** 146 (1997)]
11. Vasil'ev S A, Dianov E M, Starodubov D S, Frolov A A, Medvedkov O I *Kvantovaya Elektron.* **24** 160 (1997) [*Quantum Electron.* **27** 155 (1997)]

12. Galvanauskas A, Fermann M E, Roman J E *Appl. Phys. Lett.* **66** 1053 (1995)

13. Zolotovskii I O, Semetsov D I *Kvantovaya Elektron.* **27** 273 (1999) [*Quantum Electron.* **29** 550 (1999)]

14. Vysloukh V A, Gevorkyan L P *Izv. Akad. Nauk SSSR* **55** 323 (1991)

15. Zolotovskii I O, Semetsov D I *Opt. Spectrosk.* **85** 304 (1998) [*Opt. Spectrosc.* **85** 281 (1998)]