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Transformation of the order of Bessel beams in uniaxial crystals

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***Abstract*. The optical transformation of a zeroth-order Bessel beam to a second-order Bessel beam is studied theoretically and experimentally in the case when the beam propagates along the optical axis of a uniaxial crystal. It is shown that, if the crystal length or the cone angle of the incident beam are chosen properly, the energy of the input field can be almost completely converted into a second-order Bessel beam.**

***Keywords*:** *Bessel beams, uniaxial crystas, conoscopic images.*

1. Introduction

Bessel light beams (BLBs) are a separate object of optical research for 15 years already. This field has been initiated by papers [1, 2], in which it has been shown that BLBs are described by exact solutions of the Helmholtz equation and have the remarkable property of continuous self-reproduction of their profile, which is called the ray-like property. So far, most studies have been concentrated on zeroth-order BLBs (see, e.g., Refs [3–6]), which is explained by their importance for practical applications. Zeroth-order BLBs have a deep focus — a narrow, relatively intense central maximum that is diffraction-free at large distances. It is important that fields with a deep focus can be produced using a simple scheme with an axicon, a fact known long before the publication of papers [1, 2] (see Refs [7–11]). Apart from the above-mentioned feature of the spatial structure, the Fourier spectrum structure of BLBs is also quite important. The cone of the wave vectors of Bessel beams makes it possible to realise various vector interactions in nonlinear optics (see, e.g., Ref. [12]).

Recently, a new application employing diffraction-free BLBs has emerged: BLBs can be used to confine cold atoms and control their motion [13–16]. Of some advantages is the variant when atoms are localised in a zero-intensity region. One can use either higher order BLBs or circular BLBs, formed in the far-field zone of a limited BLB of an arbitrary order. In the former case, one can create an optical trap with transverse dimensions of a few wavelengths provided that the used BLBs have a sufficiently large cone angle.

Thus, the problem of generating higher order BLBs is of current interest. At present, BLBs are mostly generated with the help of holograms [14, 17, 18]. The main advantage of the holographic method is its universality: it can create BLBs of arbitrary orders or their superpositions. Its main drawback is a relatively low efficiency, amounting to $\sim 40\,\%$. The second method for the generation of higher order BLBs employs the conversion of higher order Laguerre–Gauss modes in an axicon. This process is similar to the conversion of a fundamental mode (Gaussian beam) to a zeroth-order BLB [19, 20].

The authors of Ref. [16] have recently performed a comprehensive study of this method and confirmed its high efficiency. However, this conclusion concerns the last stage of the conversion, when an axicon is used, whereas the main restriction of this method currently comes from the limited efficiency of the generation of higher-order Laguerre–Gauss modes. In Refs [16,19], Laguerre–Gauss modes have been formed by holograms; in Ref. [20], a biaxial crystal has been used for this purpose, allowing a conversion efficiency of approximately 60 %.

The stage of generating higher order Laguerre–Gauss modes is excluded in a two-stage method employing an axicon and a biaxial crystal, earlier proposed in Ref. [21]. In this scheme, the fundamental laser mode can be almost completely converted into a first-order BLB. A drawback of this method is the necessity to implement the n-fold cascading of the optical scheme to generate a nth-order BLB.

In this paper, we continue the study of transformation of the Bessel beam order by anisotropic crystals, in particular, the possibility of using uniaxial crystals for this purpose.

2. Theoretical model

Let us determine the refracted field induced by a BLB that is incident on a uniaxial crystal from an isotropic medium along its optic axis c (Fig. 1). For a uniaxial crystal, the Maxwell equations have two solutions corresponding to ordinary (o) and extraordinary (e) plane waves. The electric field vectors corresponding to these solutions can be written as

$$\hat{\boldsymbol{E}}_{\rm o,e}(R) = \boldsymbol{E}_{\rm o,e}(\rho)\exp({\rm i}k_{\rm o,e}z + {\rm i}m\varphi), \qquad (1)$$

where $R = (\rho, \varphi, z)$ are the cylindrical coordinates and m is

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an integer. The components of the vector amplitudes $\boldsymbol{E}_{\mathrm{o,e}}$ can be expressed in terms of mth-order Bessel functions of the first kind $J_m(q\rho)$ and their derivatives $J'_m(q\rho) = \partial J_m(q\rho)/\partial(q\rho)$ as

$$E_{\mathrm{o}\rho} = \frac{\mathrm{i}m}{q\rho} J_m(q\rho), \quad E_{\mathrm{o}\varphi} = -J'_m(q\rho), \quad E_{\mathrm{o}z} = 0 \tag{2}$$

for an ordinary beam and

$$E_{\mathrm{e}\rho} = \mathrm{i}(\cos\gamma_{\mathrm{e}}) J'_m(q\rho), \quad E_{\mathrm{e}\varphi} = -(\cos\gamma_{\mathrm{e}}) \frac{m}{q\rho} J_m(q\rho),$$

$$E_{\mathrm{e}z} = (\sin\gamma_{\mathrm{e}}) \frac{\varepsilon_{\mathrm{o}}}{\varepsilon_{\mathrm{e}}} J_m(q\rho) \tag{3}$$

for an extraordinary beam. The cone parameter q, equal to the radial component of the BLB wave vectors, is continuous at the interface due to the boundary conditions. For the both beams inside the crystal, it coincides with the cone parameter $q = k_1 \sin\gamma_1$ of the incident beam, where $k_1 = n_1\omega/c$; n_1 is the refractive index of the isotropic medium (see Fig. 1). The longitudinal components of wave vectors $k_{\mathrm{o},z} = k_0 n_{\mathrm{o}} \cos\gamma_{\mathrm{o}}$ and $k_{\mathrm{e}z} = k_0 n_e(\gamma_{\mathrm{e}}) \cos\gamma_{\mathrm{e}}$ are related to the radial component q by expressions

$$q^2 + k_{\mathrm{o}z}^2 = k_0^2 n_o^2, \quad q^2 + k_{\mathrm{e}z}^2 = k_0^2 n_{\mathrm{e}}^2(\gamma_{\mathrm{e}}),$$

where $n_{\mathrm{o}}^2 = \varepsilon_{\mathrm{o}}$; $n_{\mathrm{e}}^2(\gamma) = \varepsilon_{\mathrm{o}}\varepsilon_{\mathrm{e}}/(\varepsilon_o \sin^2\gamma + \varepsilon_{\mathrm{e}}\cos^2\gamma)$; ε_{o} and $\varepsilon_{\mathrm{e}} = n_{\mathrm{e}}^2$ are the principal values of the dielectric permittivity tensor of the crystal.

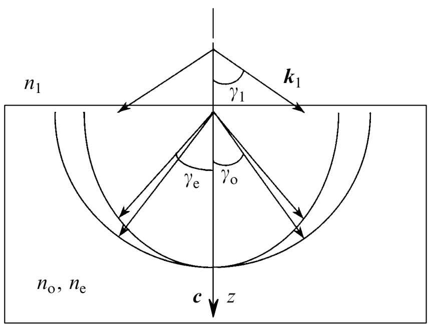


Figure 1. Orientations of the wave vectors of the incident and two refracted BLBs at the boundary of the uniaxial crystal.

One can see from equations (2) and (3) that, similarly to the case of ordinary and extraordinary plane waves, the polarisation vector of the BLB has nonzero φ- and ρ-components only if $m = 0$ (Fig. 2). Higher-order BLBs additionally have components that are orthogonal to the indicated ones.

To induce fields (2) and (3), which have homogeneous azimuthal intensity distribution, the incident beam must be circularly polarised [21]. In the general case of vector BLBs, this condition is satisfied by beams whose electric field vector has a transverse component of the form

$$\hat{\boldsymbol{E}}^{\pm}_{1\perp}(R) = \mathrm{i}A_1 \left[\frac{(\boldsymbol{e}_\rho \pm \mathrm{i}\tau \boldsymbol{e}_\varphi)}{(1+\tau^2)^{1/2}} \frac{m}{q\rho} J_m(q\rho) \right.$$

$$\left. \pm \frac{(\tau\boldsymbol{e}_\rho \pm \mathrm{i}\boldsymbol{e}_\varphi)}{(1+\tau^2)^{1/2}} J'_m(q\rho) \right] \exp[\mathrm{i}(k_z z + m\varphi)], \tag{4}$$

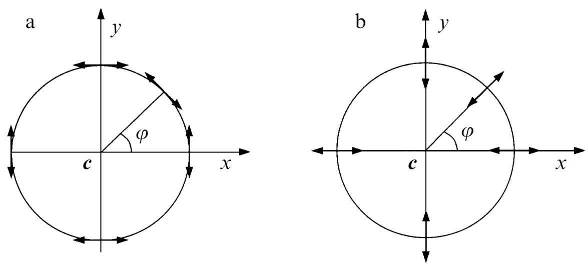


Figure 2. Polarisation vectors of a zeroth-order BLB in a uniaxial crystal as a function of the azimuthal coordinate φ, corresponding to the rotation about the optical axis c, in the case of ordinary (a) and extraordinary (b) waves.

where $\boldsymbol{e}_{\rho,\varphi}$ are the unit vectors of the cylindrical system of coordinates; $\tau = \cos\gamma_1$; A_1 is a constant amplitude factor. The longitudinal field components of beams (4) are given by

$$E_z^{\pm} = \pm A(\sin\gamma) J_m(q\rho).$$

One can see from Eqn (4) that, at small cone angles ($\gamma_1 \to 0$), the field polarisation is close to the right-handed (+) or left-handed (−) circular polarisation (quasi-circular polarisation). A numerical estimation shows that the ellipticity τ of field (4) is close to unity for angles γ_1 as high as $\sim 20°$.

For definitiness, we will assume that the crystal is irradiated by a right-polarised BLB (4). In order to find the amplitudes $A_{\mathrm{o,e}}$ of the two refracted beams, we need to solve the boundary problem, in which the incident, reflected, and refracted waves satisfy the continuity condition for the tangential components of the electric and magnetic fields. Its solution yields the following expressions for the amplitude refraction coefficients $t_{\mathrm{o,e}} = A_{\mathrm{o,e}}/A_1$:

$$t_{\mathrm{o}} = \frac{2n_1}{n_1\cos\gamma_1 + n_{\mathrm{o}}\cos\gamma_{\mathrm{o}}} \frac{\cos\gamma_1}{(1+\cos^2\gamma_1)^{1/2}}, \tag{5}$$

$$t_{\mathrm{e}} = \frac{2n_1}{n_1\cos\gamma_{\mathrm{e}} + n_{\mathrm{o}}^2\cos\gamma_1/n_{\mathrm{e}}(\gamma_{\mathrm{e}})\cos\gamma_{\mathrm{e}}} \frac{\cos\gamma_1}{(1+\cos^2\gamma_1)^{1/2}}. \tag{6}$$

Thus, a superposition of o- and e-polarised BLBs is produced inside the crystal. The electric field of this superposition has the transverse component

$$\hat{\boldsymbol{E}}_\perp(R) = t_{\mathrm{o}}\boldsymbol{E}_{\mathrm{o}\perp}(\rho)\exp(\mathrm{i}k_{\mathrm{o}z}z + \mathrm{i}m\varphi)$$

$$+ t_{\mathrm{e}}\boldsymbol{E}_{\mathrm{e}\perp}(\rho)\exp(\mathrm{i}k_{\mathrm{e}z}z + \mathrm{i}m\varphi), \tag{7}$$

where, in accordance with Eqns (2) and (3),

$$\boldsymbol{E}_{\mathrm{o}\perp}(\rho) = \mathrm{i}\boldsymbol{e}_\rho \frac{m}{q\rho} J_m(q\rho) - \boldsymbol{e}_\varphi J'_m(q\rho), \tag{8}$$

$$\boldsymbol{E}_{\mathrm{e}\perp}(\rho) = \mathrm{i}(\cos\gamma_{\mathrm{e}})\boldsymbol{e}_\rho J'_m(q\rho) - (\cos\gamma_{\mathrm{e}})\boldsymbol{e}_\varphi \frac{m}{q\rho} J_m(q\rho). \tag{9}$$

A numerical estimation of coefficients $t_{\mathrm{o,e}}$, given by formulas (5) and (6), shows that the ratio $t_{\mathrm{o}}/t_{\mathrm{e}}$ differs from unity by less than 3 % inside the cone angle of the incident beam, where the polarisation can be treated as circular. Then, assuming $t_{\mathrm{o}} = t_{\mathrm{e}}$ in Eqn (7) and introducing the notation $k_{z\mathrm{o,e}} = k_z \pm \Delta k/2$, we have

$$\hat{\boldsymbol{E}}_\perp(R) = t_{\mathrm{e}} \left[E_{\mathrm{o}\perp}(\rho) \exp\left(\mathrm{i}\,\frac{\Delta kz}{2}\right) + \right.$$

$$+\boldsymbol{E}_{e\perp}(\rho)\exp\left(-\mathrm{i}\frac{\Delta kz}{2}\right)\Bigg]\exp(\mathrm{i}k_z z+\mathrm{i}m\varphi). \quad (10)$$

Inserting the vector amplitudes (8) and (9) into Eqn (10) and omitting the insignificant phase factor $\exp(\mathrm{i}k_z z)$, we obtain

$$\hat{\boldsymbol{E}}_\perp(R)=t\left[\left(\cos\frac{\Delta kz}{2}\right)J_{m-1}(q\rho)\boldsymbol{e}_+\exp(-\mathrm{i}\varphi)\right.$$
$$\left.+\mathrm{i}\left(\sin\frac{\Delta kz}{2}\right)J_{m+1}(q\rho)\boldsymbol{e}_-\exp(\mathrm{i}\varphi)\right]\exp(\mathrm{i}m\varphi). \quad (11)$$

Here, $t=2n_1/(n_1+n_o)$ is the Fresnel amplitude transmittivity and $\boldsymbol{e}_\pm=(\boldsymbol{e}_1\pm\boldsymbol{e}_2)/\sqrt{2}$ are the basis vectors of the right- and left-circular polarisations.

In the case when a left-polarised beam is incident on a uniaxial crystal, the transverse component of the refracted field can be found in a similar fashion:

$$\hat{\boldsymbol{E}}_\perp(R)=t\left[\left(\cos\frac{\Delta kz}{2}\right)J_{m+1}(q\rho)\boldsymbol{e}_-\exp(\mathrm{i}\varphi)\right.$$
$$\left.+\mathrm{i}\left(\sin\frac{\Delta kz}{2}\right)J_{m-1}(q\rho)\boldsymbol{e}_+\exp(-\mathrm{i}\varphi)\right]\exp(\mathrm{i}m\varphi). \quad (12)$$

Using formulas (11) and (12), one can study the fields induced inside the crystal by circularly polarised BLBs and their arbitrary superpositions.

3. Analysis of the theoretical results

To analyse the expressions obtained, we will write the electric field of the incident beam (4) in the paraxial approximation:

$$\hat{\boldsymbol{E}}_1^\pm(R)=\mathrm{i}\boldsymbol{e}_\pm J_{m\mp1}(q\rho)\exp[\mathrm{i}(k_z z+(m\mp1)\varphi)]. \quad (13)$$

Comparison of expressions (11) and (12) with expression (13), shows that the first terms in Eqns (11) and (12) describe the incident field. Thus, in the both cases, the refracted field is a superposition of the incident BLB and an orthogonally polarised BLB. As the beams propagate in the crystal, their amplitudes oscillate as $\cos(\Delta kz/2)$ and $\sin(\Delta kz/2)$. The most important property of Eqns (11) and (12) is that the order of the orthogonally polarised BLB differs from that of the incident BLB by two units.

In the special case of $m=1$, when the incident right-polarised field (13) is a zeroth-order BLB, equation (11) yields

$$\hat{\boldsymbol{E}}_\perp(R)=t\left[\left(\cos\frac{\Delta kz}{2}\right)J_0(q\rho)\boldsymbol{e}_+\right.$$
$$\left.+\mathrm{i}\left(\sin\frac{\Delta kz}{2}\right)J_2(q\rho)\boldsymbol{e}_-\exp(2\mathrm{i}\varphi)\right]. \quad (14)$$

The intensity of the field (13) is given by

$$I(\rho,z)=|\hat{\boldsymbol{E}}_\perp\rho,z)|^2$$
$$=t^2\left[\left(\cos^2\frac{\Delta kz}{2}\right)J_0^2(q\rho)+\left(\sin^2\frac{\Delta kz}{2}\right)J_2^2(q\rho)\right]. \quad (15)$$

For the incident left-polarised BLB of the zeroth order ($m=-1$), we derive from Eqn (12)

$$\hat{\boldsymbol{E}}_\perp(R)=t\left[\left(\cos\frac{\Delta kz}{2}\right)J_0(q\rho)\boldsymbol{e}_-\right.$$
$$\left.+\mathrm{i}\left(\sin\frac{\Delta kz}{2}\right)J_2(q\rho)\boldsymbol{e}_+\exp(-2\mathrm{i}\varphi)\right]. \quad (16)$$

The intensity of the field (16) is also described by formula (15).

The refracted fields (14) and (16) are superpositions of circularly polarised beams of the zeroth and second order. For $\Delta kz_n/2=\pi/2+\pi n$, the radiation transmitted by the crystal will be BLBs of the second and minus second orders, respectively.

Thus, if we neglect the reflected radiation, the zeroth-order BLB is fully converted into a second-order BLB. Fig. 3 shows the intensity oscillation half-period, $z_0=\pi/\Delta k(\gamma_1)$, as a function of the cone angle of the KDP crystal. One can see that the half-period z_0 falls off drastically with increasing angle γ_1. For example, the energy is fully converted in a 2.5-cm-thick crystal if $\gamma_1\approx1.7°$.

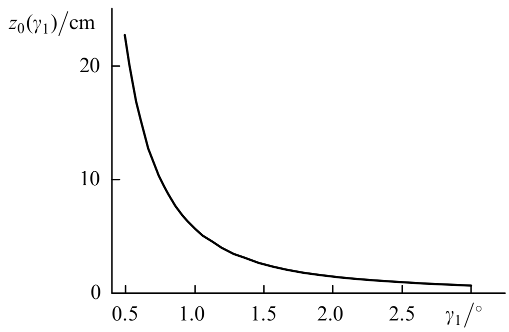


Figure 3. Dependence of the conversion half-period of the Bessel beam order as a function of the cone angle inside the KDP crystal for $n_o=1.51$, $n_e=1.47$, and $n_1=1$.

The obtained results allow us to calculate the structural transformation of a linearly polarised incident beam as well. This beam has the form $\hat{\boldsymbol{E}}_1=[\hat{\boldsymbol{E}}_{1\perp}^+(m=1)+\hat{\boldsymbol{E}}_{1\perp}^-(m=-1)]/\sqrt{2}$ and is a superposition of left- and right-polarised BLBs (4). Inserting the vector amplitudes given by (4), at $\tau=1$ we obtain

$$\hat{\boldsymbol{E}}_1(\rho,z)=\boldsymbol{e}_1J_0(q\rho)\exp(\mathrm{i}k_z z), \quad (17)$$

i.e., a zeroth-order BLB linearly polarised along the x-axis. The field inside the crystal can be found by adding Eqns (14) and (16):

$$\hat{\boldsymbol{E}}_\perp(R)=t\left[\left(\cos\frac{\Delta kz}{2}\right)J_0(q\rho)\boldsymbol{e}_1\right.$$
$$\left.+\mathrm{i}\left(\sin\frac{\Delta kz}{2}\right)J_2(q\rho)(\boldsymbol{e}_1\cos2\varphi+\boldsymbol{e}_2\sin2\varphi)\right]. \quad (18)$$

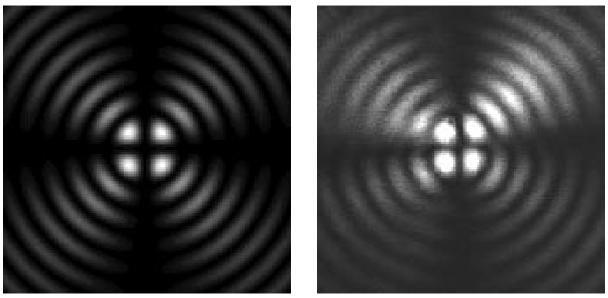
In an optical scheme employing a crossed configuration of the polariser and the analyser, the transmitted field has a component polarised along $\boldsymbol{e}_2$, whose intensity is given by

$$I(R) = t^2 J_2^2(q\rho) \sin^2\left(\frac{\Delta kz}{2}\right) \sin^2 2\varphi. \qquad (19)$$

In addition, it follows from Eqn (18) that, in the case of $\sin(\Delta kz/2) = 1$, the field component polarised along $\boldsymbol{e}_1$ is $\mathrm{i}tJ_2(q\rho)\cos 2\varphi$, that is, an azimuthally modulated BLB of the second order.

4. Comparison with the experiment

Both in the cases of circularly and linearly polarised inputs, the uniaxial crystal transforms a zeroth-order BLB into a second-order BLB. In the case of a circularly polarised input field, the output beam contains a second-order screw dislocation of the wavefront. The sign of the screw dislocation changes with a change in the direction of the circular polarisation. If the input beam is linearly polarised, the amplitude of the output second-order BLB is modulated along the azimuthal coordinate. The corresponding intensity distribution is shown in Fig. 4a, numerically calculated with the help of formula (19). One can see that the number of lopes contained in the output intensity distribution is an evident criterion of the order of the Bessel function.



a b

Figure 4. Intensity distribution of an azimuthally modulated second-order BLB calculated according to formula (19) (a) and measured experimentally (b).

We have tested this theoretical result in an experiment. Fig. 5 shows the scheme of our experimental setup. A collimated beam produced by a He – Ne laser, 4 mm in diameter, was converted with the help of an axicon *4* into a zeroth-order BLB with a cone angle of $\sim 0.65°$ and then entered a 26-mm-long KDP crystal *6*. An analyser *3* separated the transmitted radiation component whose polarisation was orthogonal to that of the incident beam. Axicon *5* transformed the circular field into a second-order BLB with a

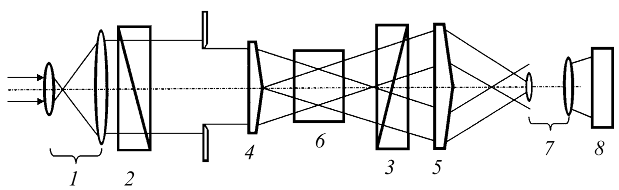


Figure 5. Optical scheme of the experiment: (*1*) 20-fold telescope; (*2, 3*) polariser and analyser in a crossed configuration; (*4, 5*) axicons subtending angles of ~ 1.3 and 2.5°, respectively; (*6*) uniaxial KDP crystal; (*7*) microscope; (*8*) CCD array.

cone angle of $\sim 0.6°$, whose transverse structure was detected by a CCD array *8*. Fig. 4b shows the detected intensity distribution, which confirms the theoretical results predicted by formula (19). The conversion efficiency was relatively small ($\sim 10\,\%$); it was determined by the parameters of the available axicons and crystal.

It is important to compare the calculated fields with the well-known conoscopic images, produced by a linearly polarised input Gaussian beam in a scheme with a crossed configuration of the polariser and analyser [22]. Fig. 6a shows the conoscopic image produced by a Gaussian input beam in the scheme shown in Fig. 5 with the axicons replaced by spherical lenses. One can see that the main distinction between the conoscopic images produced by Bessel and Gaussian beams lies in the radial intensity distribution.

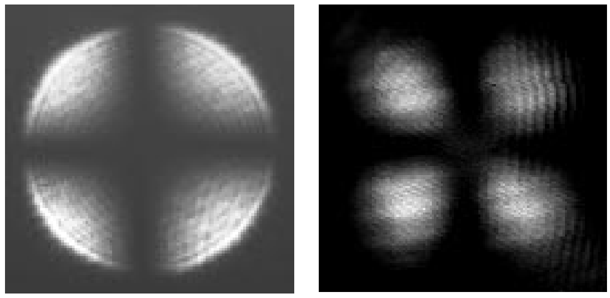


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Figure 6. Conoscopic image of a Gaussian beam (a) and the intensity distribution of a single-circle second-order Laguerre – Gauss beam (b).

Qualitatively, this distinction can be explained by representing the Gaussian beam as a superposition of zeroth-order BLBs of different cone angles. Each of these partial BLBs forms its own conoscopic image, as described above. The observed pattern will be a superposition of many conoscopic images, generally resulting in a complex radial distribution of the intensity.

Note, however, that by appropriately focusing a Gaussian beam on the crystal, one can produce fields close to optical modes. Fig. 6b shows an example of such a field, produced by focusing a collimated Gaussian beam on the crystal by a lens with the focal distance of 7 cm. In a good approximation, this field is a one-ring second-order Laguerre – Gauss mode. Upon passing through the axicon, it is converted into a second-order BLB, which almost coincides with the one shown in Fig. 4b. Unlike the previous situation, however, one cannot realise in this way a complete conversion of the input beam into a Laguerre – Gauss or Bessel beam.

5. Conclusions

The theoretical and experimental investigation of the transformation of the order of Bessel beams in uniaxial crystals has shown that an input zeroth-order circularly polarised BLB transforms into a second-order BLB as it propagates along the optic axis of the crystal. If the input beam is linearly polarised, it is converted into an azimuthally modulated second-order BLB. The conversion efficiency can be made close to 100 % by choosing an appropriate crystal thickness or cone angle. By cascading the optical scheme, one can obtain BLBs of higher even orders.

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