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# Laser-induced self-organisation of coupled thermodeformation fields on surfaces of solids

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***Abstract*. A theory of self-organisation of periodic surface thermodeformation (TD) fields in crystals irradiated by laser pulses is developed. The period of the TD structure is obtained as a function of the pulse duration and the temperature of the medium. The theoretical results are in a good agreement with the experimental data on quasi-periodic damage of Si surfaces by laser pulses of various durations.**

***Keywords*:** *self-organisation of surface fields, laser-induced surface damage, thermodeformation structures.*

## 1. Introduction

Laser-induced formation of micro- and nanometre periodic defect-deformation (DD) structures on surfaces of strongly absorbing solids is a topical problem of laser physics. To date, the formation of DD structures involving vacancies, interstitials, and electron–hole pairs has been studied (see review [1]).

The universal mechanism of the laser-induced formation of surface DD structures is the diffusion-deformation instability, which appears when the concentration of laser-induced defects exceeds the critical one, so that the deformation-induced flow of defects becomes greater than counter diffusion flow, resulting in self-localisation of defects in self-consistent deformation wells. A positive DD feedback automatically appears for laser-induced point defects, whose energy of interaction with the self-consistent (defect-induced) deformation field is given by [2]

$$H = -\theta_{\mathrm{d}}\xi\,, \tag{1}$$

where $\theta_{\mathrm{d}}$ is the deformation potential of the defect; $\xi = \mathrm{div}\boldsymbol{u}$ is the deformation, and $\boldsymbol{u}$ is the displacement vector of the medium.

Apart from creating point defects, laser radiation heats the surface. The radiation-induced thermal field can be treated as consisting of 'defects' of an ideal lattice – thermal phonons. Owing to the cubic anharmonicity of an elastic continuum, thermal phonons create a self-consistent static deformation $\xi$ in a solid. The energy density of the cubic anharmonic interaction between thermal phonons and the deformation $\xi$ is given by the expression

$$U_{\mathrm{unh}} = -K\alpha\xi_{\mathrm{ph}}^2\xi, \tag{2}$$

where $K$ is the modulus of elasticity; $\alpha$ is a dimensionless positive constant of anharmonicity; and $\xi_{\mathrm{ph}}$ is the time-oscillating deformation caused by the displacement of medium in the phonon wave. It was shown in Ref. [3] that the coupling energy between a thermal phonon and the deformation field $\xi$ obtained from Eqn (2) coincides with energy (1). Therefore, thermal phonons belong to the class of lattice defects that can produce DD instability. This instability can be conveniently described in terms of the temperature field $T \sim \xi_{\mathrm{ph}}^2$. The temperature-deformation coupling, described by expression (2), then gives rise to a positive feedback between the fields of temperature and deformation. This can lead to the development of thermodeformation (TD) instability, which is similar to surface DD instability [1].

In this work, we construct the theory of laser-induced TD instability leading to the formation of periodic TD fields on solid surfaces. We consider the 'anisotropic film on a substrate' model, in which the near-surface layer heated by a laser pulse is treated as the 'film' connected with the lower part of the sample (the 'substrate'). We take into account the elastic anisotropy of the crystal surface, which leads to the formation of periodic TD structures (TD lattices) whose geometry corresponds to the crystallographic symmetry of the crystal.

## 2. Model of a heated elastically anisotropic film on a substrate

Consider a laser pulse of duration $\tau_{\mathrm{p}}$ incident on the (100) surface of a strongly absorbing cubic crystal (with an optical absorption coefficient $\alpha_0$). The $z = 0$ plane coincides with this surface, and the $z$ axis is directed inside the medium. When the pulse ends, the temperature distribution along the $z$ axis is given by [4]

$$T(z) = T_0\,\mathrm{i}\,\mathrm{erfc}\frac{z}{2(\chi\tau_{\mathrm{p}})^{1/2}} \approx T_0\exp(-\gamma z), \tag{3}$$

where $\chi$ is the temperature diffusivity; $\gamma = 2(\chi\tau_{\mathrm{p}})^{-1/2}$, and $T_0$ is a spatially uniform surface temperature which is assumed a fixed (externally controlled) parameter. Expression (3) is valid if $(\chi\tau_{\mathrm{p}})^{1/2} \gg \alpha_0^{-1}$.

Let us define the effective thickness $h$ of the heated near surface layer by the condition

$$T(z = h) = T_{\mathrm{c}}, \tag{4}$$

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where $T_c$ is the critical temperature of the appearance of the TD instability (see expression (31) below). Then, equations (3) and (4) yield

$$h = (\chi\tau_p)^{1/2}\ln\frac{T_0}{T_c}. \tag{5}$$

We will treat this heated layer of thickness $h$ as a film, having density $\rho$ and elasticity modulus $K$, that is rigidly connected with the 'substrate', i.e., the remaining part of the crystal, which is characterised by parameters $\rho_s$ and $K_s$. For convenience, we shift the coordinate origin $z = 0$ to the plane of the film–substrate interface (Fig. 1) and direct the $x$ and $y$ axes along the [100] and [010] crystallographic directions, respectively. If $\lambda\tau_p \gg 1$, where $\lambda$ is the increment (growth rate) of the TD instability, we can neglect the heat redistribution along the $z$ axis at lengths of the order of the film thickness $h$. Then, assuming that at times of the instability development ($t \sim \lambda^{-1}$) the heat redistributes along the film only, and using Eqn (3), we obtain

$$T(\boldsymbol{r}, z, t) = T(\boldsymbol{r}, t)\exp[-\gamma(z-h)].$$

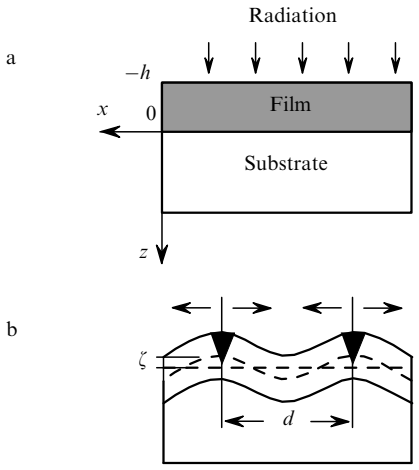


**Figure 1.** 'Heated film on a substrate' model: (a) initial state in which the film is undistorted and the thermal phonons are distributed uniformly. (b) Owing to the TD instability, the film bends periodically, with the regions of maximum heating (dark) coinciding with the regions of maximum extension. The arrows show the periodic displacements of surface points that give rise to the periodic damage (cracking) of the surface (see Eqn (36) in the text).

The flow of heat along the surface consists of a diffusion part and a deformation-induced part. Based on the results of Ref. [3] and using of Eqn (2), we can express the heat flow as

$$Q = -\chi\mathrm{grad}_{\|}T - \chi\alpha\mathrm{grad}_{\|}(T\mathrm{div}\,\boldsymbol{u}_f). \tag{6}$$

Hereafter, the $\|$ subscript denotes differentiation with respect to $x$ and $y$; $\boldsymbol{u}_f$ is the displacement vector of the medium in the film. Starting from the equation of continuity for the heat flow, using expression (6), and taking the deformation-induced flow into account, we arrive at

$$\frac{\partial T}{\partial t} = \chi\Delta_{\|}(T) - \alpha\chi\mathrm{div}_{\|}(T\mathrm{grad}_{\|}(\mathrm{div}\,\boldsymbol{u}_f)). \tag{7}$$

The deformation of the film $\mathrm{div}\,\boldsymbol{u}_f = \xi_f$ can be expressed in terms of the bending coordinate $\zeta$ of the film, which specifies the displacement of the film's middle plane from its equilibrium position along the $z$ axis (Fig. 1):

$$\xi_f = -v\left(z + \frac{h}{2}\right)\Delta_{\|}\zeta, \tag{8}$$

where $v = (1-2\sigma)/(1-\sigma)$; and $\sigma$ is Poisson's ratio of the film.

For a film parallel to the (100) plane, the coordinate $\zeta$ satisfies the equation which can be derived by generalising the conventional bending equation of an elastically anisotropic free film [5, 6]:

$$\frac{\partial^2\zeta}{\partial t^2} + l_0^2c^2\left(\frac{\partial^4}{\partial x^4} + 2A\frac{\partial^4}{\partial x^2\partial y^2} + \frac{\partial^4}{\partial y^4}\right)\zeta + \frac{v\theta_T}{\rho h}\int_{-h}^{0}\left(z + \frac{h}{2}\right)\Delta_{\|}T\mathrm{d}z = \frac{\sigma_{\perp}}{\rho h}, \tag{9}$$

where $l_0^2 = h^2/12$; $c^2 = S_{11}S_{44}/\rho\left(S_{11}^2 - S_{12}^2\right)S_{44}$; $A = c_3^2/c^2$ is the coefficient of elastic anisotropy; $c_3^2 = (-S_{12}S_{44} + 2S_{11}^2)/\rho\left(S_{11}^2 - S_{12}^2\right)S_{44}$; $\theta_T = \alpha k_B n$; $n$ is the concentration of atoms; $k_B$ is the Boltzmann constant; and $\sigma_{\perp}$ is the stress normal to the film surface (substrate reaction). The elastic compliance constants $S_{ij}$ can be expressed in terms of the constants of elastic hardness $c_{11}, c_{12}$, and $c_{44}$, which are usually given in the literature:

$$c_{44} = S_{44}^{-1},\ c_{11} - c_{12} = (S_{11} - S_{12})^{-1},$$

$$c_{11} + 2c_{12} = (S_{11} + 2S_{12})^{-1}.$$

The bending of the film induces displacements in the substrate. Neglecting the anharmonicity, the displacement vector $\boldsymbol{u}$ satisfies the equation

$$\frac{\partial^2\boldsymbol{u}}{\partial t^2} = c_{\perp}^2\Delta\boldsymbol{u} + \left(c_{\|}^2 - c_{\perp}^2\right)\mathrm{grad}(\mathrm{div}\,\boldsymbol{u}). \tag{10}$$

We have the following boundary conditions at the film-substrate interface. The displacement along the $z$ axis is continuous:

$$u_z(z=0) = \zeta. \tag{11}$$

The tangential stress due to the spatially uniform temperature distribution in the film is compensated by the shearing stress in the substrate at $z = 0$:

$$\left(\frac{\partial u_{x_i}}{\partial z} + \frac{\partial u_z}{\partial x_i}\right)_{z=0} = \frac{\theta_T}{\mu_s}\frac{\partial}{\partial x_i}\int_{-h}^{0}T\mathrm{d}z, \tag{12}$$

where $x_1 = x$, $x_2 = y$; and $\mu_s$ is the sheer modulus of the crystal near the substrate-film interface.

The normal stress in the substrate determines the force acting on the film along its normal

$$\left[\frac{\partial u_z}{\partial z} + (1-2\beta)\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right)\right]_{z=0} = \frac{\sigma_{\perp}(x,y)}{\rho_s c_{\|}^2}, \tag{13}$$

where $\beta = c_{\perp}^2/c_{\|}^2$; $c_{\|}$ and $c_{\perp}$ are the transverse and longitudinal sound speeds in the substrate, respectively.

Starting from Eqn (7) and using Eqn (8), we derive the equation for the temperature $T(\boldsymbol{r},t)$ on the free surface $z=-h$:

$$\frac{\partial T(\boldsymbol{r},t)}{\partial t}=\chi\Delta_{\|}(T(\boldsymbol{r},t))-\alpha\chi\mathrm{div}_{\|}\left(T(\boldsymbol{r},t)\mathrm{grad}_{\|}v\frac{h}{2}\Delta_{\|}\zeta(\boldsymbol{r},t)\right). \quad (14)$$

The system of equations (9)–(14), together with relations (8) and $T(\boldsymbol{r},z,t)=T(\boldsymbol{r},t)\exp[-\gamma(z-h)]$, represents a closed system of equations, which describes the TD instability in the model of an elastically anisotropic film on a substrate.

## 3. System of kinetic thermodeformation equations in mode representation

Let us expand the bending deformation of the film in a Fourier series:

$$\zeta(\boldsymbol{r},t)=\sum_{\boldsymbol{q}}\zeta_{\boldsymbol{q}}\exp(\mathrm{i}\boldsymbol{q}\boldsymbol{r}) \quad (\zeta_{\boldsymbol{q}}=\zeta_{-\boldsymbol{q}},\ \zeta_{\boldsymbol{q}}=\zeta_{\boldsymbol{q}}(t)). \quad (15)$$

Here, $\boldsymbol{q}=\{q_x,q_y\}$ is the wave vector, whose modulus lies in the interval $0\leqslant q\leqslant q_{\mathrm{c}}$, where $q_{\mathrm{c}}=\pi/h$ is the maximum wave number. Similarly, we represent $T(r,t)$ in the form

$$T(\boldsymbol{r},t)=T_0+\sum_{\boldsymbol{q}\neq 0}T(\boldsymbol{q})\exp(\mathrm{i}\boldsymbol{q}\boldsymbol{r}) \quad (16)$$

where $(T(\boldsymbol{q})=T(-\boldsymbol{q})$, and $T(\boldsymbol{q})=T(\boldsymbol{q},t))$. Inserting Eqns (15) and (16) in Eqn (14), we obtain the system of nonlinear kinetic equations for the Fourier amplitudes:

$$\frac{\partial T(\boldsymbol{q})}{\partial t}+\gamma(\boldsymbol{q})T(\boldsymbol{q})= -A\sum_{\boldsymbol{q}_1}\left[\boldsymbol{q}(\boldsymbol{q}-\boldsymbol{q}_1)\right]|\boldsymbol{q}-\boldsymbol{q}_1|^2T(\boldsymbol{q}_1)\zeta_{\boldsymbol{q}-\boldsymbol{q}_1}, \quad (17)$$

where

$$\gamma(q)=\chi q^2;\quad A=v\chi\alpha\frac{h}{2}\frac{\theta_T}{k_{\mathrm{B}}T}.$$

To close the system of equations (17), we will derive a relation between $T(\boldsymbol{q})$ и $\zeta_{\boldsymbol{q}}$. To this end, we need to solve the boundary value problem (10)–(13) in the mode representation. In this representation, we have

$$\sigma_{\perp}(\boldsymbol{r},t)=\sum_{\boldsymbol{q}}\sigma_{\boldsymbol{q}}(t)\exp(\mathrm{i}\boldsymbol{q}\boldsymbol{r}+\varepsilon t), \quad (18)$$

in Eqn (13), where $\varepsilon$ is an auxiliary parameter [at the end of the calculation we take the $\varepsilon\to 0$ limit (see below)].

The displacement vector in the substrate that satisfies equation (11) can be represented as the sum of transverse and longitudinal parts,

$$\boldsymbol{u}=\boldsymbol{u}_{\|}+\boldsymbol{u}_{\perp},$$

which satisfy the conditions

$$\mathrm{rot}\,\boldsymbol{u}_{\|}=0,\ \mathrm{div}\,\boldsymbol{u}_{\perp}=0. \quad (19)$$

We will write the components of the corresponding solutions. For the longitudinal component we have

$$u_{\|x_{\alpha}}=-\mathrm{i}q_{x_{\alpha}}R(t)\exp(\mathrm{i}\boldsymbol{q}\boldsymbol{r}-k_{\|}z+\varepsilon t),\qquad u_{\|z}=k_{\|}R(t)\exp(\mathrm{i}\boldsymbol{q}\boldsymbol{r}-k_{\|}z+\varepsilon t), \quad (20)$$

for the transverse component,

$$u_{\perp z_{\alpha}}=-\mathrm{i}\frac{q_{x_{\alpha}}}{q}k_{\perp}Q(t)\exp(\mathrm{i}\boldsymbol{q}\boldsymbol{r}-k_{\perp}z+\varepsilon t),\qquad u_{\perp z}=qQ(t)\exp(\mathrm{i}\boldsymbol{q}\boldsymbol{r}-k_{\perp}z+\varepsilon t), \quad (21)$$

where $k^2_{\|,\perp}=q^2+\varepsilon^2/c^2_{\|,\perp}$, and $R(t)$, and $Q(t)$ are some functions of time. The solution in the form (20), (21) automatically satisfies conditions (19). Note that for $\varepsilon=-\mathrm{i}\omega$, with $\omega$ real, solution (20), (21) describes surface Rayleigh waves [5]. Here, we will consider a solution of another type, when $\varepsilon\to 0$ ($\omega\to 0$), i.e., the solution of the boundary value problem (10)–(13) in the adiabatic approximation.

To this end, we will first express $\sigma_{\boldsymbol{q}}$ in terms of $T(\boldsymbol{q})$ and $\zeta_{\boldsymbol{q}}$ using the system of the following four equations. Starting from Eqn (10) and using of Eqns (20) and (21), we derive the first equation

$$k_{\|}^2-q^2=\left(k_{\perp}^2-q^2\right)\beta,$$

where $\beta$ is defined in (13). Inserting expressions (20) and (21) in the boundary condition (11), we obtain the second equation

$$Rk_{\|}+qQ=\zeta_{\boldsymbol{q}}.$$

The boundary condition (13) together with Eqns (19)–(21) yields the third equation

$$R\left(k_{\perp}^2+q^2\right)+2qk_{\perp}Q=-\frac{\sigma_{\boldsymbol{q}}}{\rho_{\mathrm{s}}c_{\perp}^2}.$$

Finally, inserting Eqns (19)–(21) to Eqn (12), we obtain the fourth equation

$$2k_{\|}q_{x_{\alpha}}R+\left(q^2+k_{\perp}^2\right)Q=\frac{\theta_T}{\mu_{\mathrm{s}}}hq_{x_{\alpha}}T(\boldsymbol{q}).$$

Solving this system of four equations, we obtain the relation

$$\frac{1}{k_{\|}}\frac{2k_{\|}k_{\perp}-k_{\perp}^2-q^2}{q^2-k_{\perp}^2}\left[\frac{\theta_T}{\mu_{\mathrm{s}}}hq^2T(\boldsymbol{q})-\zeta_{\boldsymbol{q}}\left(q^2+k_{\perp}^2\right)\right] -2\zeta_{\boldsymbol{q}}k_{\perp}=\frac{\sigma_{\boldsymbol{q}}}{\rho_{\mathrm{s}}c_{\perp}^2}.$$

Expanding the expression in the brackets in powers of $\varepsilon^2/c^2_{\|,\perp}q^2$ (in the limit $\varepsilon\to 0$), we obtain the required relation:

$$\frac{\sigma_{\boldsymbol{q}}}{\rho_{\mathrm{s}}c_{\perp}^2}=\zeta_{\boldsymbol{q}}2q(\beta-1)-\beta\frac{\theta_T}{\mu_{\mathrm{s}}}hqT(\boldsymbol{q}). \quad (22)$$

Inserting Eqns (15), (16), and (18) in Eqn (9), we obtain

$$\frac{\partial^2\zeta_{\boldsymbol{q}}}{\partial t^2}+\Phi(\theta)c^2l_0^2q^4\zeta_{\boldsymbol{q}}+\frac{v\theta_T}{2\rho}hq^2T(\boldsymbol{q})=\frac{\sigma_{\boldsymbol{q}}}{\rho h}. \quad (23)$$

Here, the factor $\Phi(\theta)=\left(\cos^4\theta+2A\cos^2\theta\sin^2\theta+\sin^4\theta\right)$ describes the angular dependence of the film hardness, and

$\theta$ the angle between vector $\boldsymbol{q}$ and the $x$ axis (the [100]-direction in the plane of the film).

Now, starting from Eqn (23), using the adiabatic approximation ($\partial^2\zeta/\partial t^2 = 0$) and Eqn (22), we find the relation between $\zeta_{\boldsymbol{q}}$ and $T(\boldsymbol{q})$:

$$\zeta_{\boldsymbol{q}} = T(\boldsymbol{q})\eta_{\mathrm{d}}(\boldsymbol{q}), \tag{24}$$

where the coefficient of the defect-bending coupling is given by

$$\eta_{\mathrm{d}}(\boldsymbol{q}) = -\frac{\theta_T}{2\mu_{\mathrm{s}}} h(\nu q h + 2\beta)\left[2(1-\beta) + \frac{\rho c^2}{\mu_{\mathrm{s}}} h l_0^2 q^3 \Phi(\theta)\right]^{-1}.$$

We derived this expression, by using of the relation $\mu_{\mathrm{s}} = \rho_{\mathrm{s}} c_{\mathrm{t}}^2$.

Inserting (24) into Eqn (17), we obtain a closed system of kinetic equations for the Fourier amplitudes $T(\boldsymbol{q})$ of the surface temperature:

$$\frac{\partial T(\boldsymbol{q})}{\partial t} + \gamma(\boldsymbol{q})T(\boldsymbol{q}) = -\sum_{\boldsymbol{q}_1}\left[\boldsymbol{q}(\boldsymbol{q}-\boldsymbol{q}_1)\right]|\boldsymbol{q}-\boldsymbol{q}_1|^2 \times A\eta(\boldsymbol{q}-\boldsymbol{q}_1)T(\boldsymbol{q}_1)T(\boldsymbol{q}-\boldsymbol{q}_1). \tag{25}$$

Below, we perform a linear analysis of the system of equations (25) and compare the obtained theoretical results with the experimental data.

## 4. Characteristics of surface TD lattices

Assuming that the amplitude of the zeroth harmonic of the laser-induced surface temperature $T_0$ is much greater than the amplitudes of all other harmonics ($T_0 \gg T(\boldsymbol{q})$) and retaining the terms that are linear in $T(\boldsymbol{q}_1 = 0) = T_0$ in equation (25), we obtain

$$\frac{\partial T(\boldsymbol{q})}{\partial t} + \gamma(\boldsymbol{q})T(\boldsymbol{q}) = -AT_0\eta(\boldsymbol{q})T(\boldsymbol{q})q^4. \tag{26}$$

By representing the solution of Eqn (26) in the form

$$T(\boldsymbol{q}) = T(\boldsymbol{q}, t) = \mathrm{const}\exp(\lambda t),\ \zeta_{\boldsymbol{q}} = \mathrm{const}\exp(\lambda t) \tag{27}$$

and assuming that $q \geqslant h^{-1}$, we derive the expression for the increment as a function of the modulus $q$ and direction of the wave vector $\boldsymbol{q} = \{q_x, q_y\}$ ($q_x = q\cos\theta,\ q_y = q\sin\theta$):

$$\lambda(\boldsymbol{q}) = -\chi q^2 + \chi q^4 \frac{h^2}{2}\nu\alpha\beta_{\mathrm{s}}\frac{\theta_T T_0}{2(1-\beta)\mu_{\mathrm{s}} + \rho c^2\Phi(\theta)hl_0^2q^3}. \tag{28}$$

Fig. 2 shows the dependence $\lambda(\boldsymbol{q})$ (28) of the increment on the wave vector direction. The maximum increment is achieved for modes whose wave vectors are directed along the [110] and [010] crystallographic directions (along the $x$ and $y$ axes).

The dependence $\lambda(q)$ of the increment of the TD lattice given by formula (28) at $\theta = 0$, for the wave vector directed along one of the [100] axis is plotted in Fig. 3. At temperatures $T_0$ near the threshold of the TD lattice formation ($T_{\mathrm{c}} \approx 900$ K, see expression (31) below), the maximum value $\lambda = \lambda_{\mathrm{m}}$ is reached at $q = q_{\mathrm{m}}$ (see Fig. 3a), where, taking into account (5), we have

$$q_{\mathrm{m}} = \frac{3\nu\beta_{\mathrm{s}}}{(\chi\tau_{\mathrm{p}})^{1/2}\ln(T_0/T_{\mathrm{c}})}\frac{\alpha^2 k_{\mathrm{B}} n T_0}{\rho c^2}, \tag{29}$$

and the maximum increment is

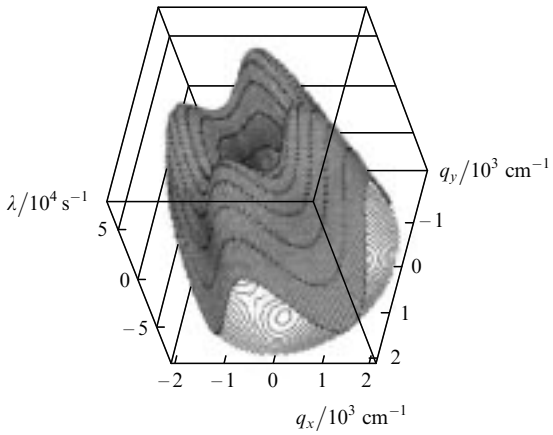


**Figure 2.** Dependence $\lambda(q_x, q_y)$ of the increment of the surface TD lattice on the components of its wave vector calculated according to formula (28) for the parameter values given in Section 4, $T_0 = 1500$ K, and $\tau_{\mathrm{p}} = 10^{-3}$ s.

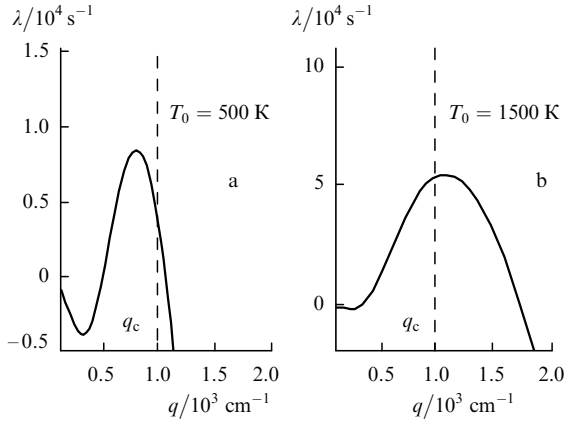


**Figure 3.** Dependence of the increment of the surface TD lattice on the modulus of its wave vector $\lambda(q)$ calculated according to formula (28) for $\theta = 0$, the parameter values given in Section 4, $\tau_{\mathrm{p}} = 10^{-3}$ s, $T_0 = 500$ (a) and 1500 K (b).

$$\lambda_{\mathrm{m}} = \chi q_{\mathrm{m}}^2. \tag{30}$$

To estimate the critical temperature of the appearance of the TD instability, we will use the relation $\lambda_{\mathrm{m}} = \chi q_{\mathrm{m}}^2 - \gamma_T$, instead of Eqn (30), where $\gamma_T = \chi/h^2$ accounts for the heat loss from the film to the substrate. Then, it follows from the critical condition $\lambda_{\mathrm{m}} \geqslant 0$ that the TD instability appears when the spatially uniform temperature $T_0$ exceeds the critical temperature $T_{\mathrm{c}}$:

$$T_0 > T_{\mathrm{c}} = \frac{\rho c^2}{3\nu\alpha\beta_{\mathrm{s}}\theta_T} = \frac{\rho c^2}{3\alpha^2\nu\beta_{\mathrm{s}}k_{\mathrm{B}}n}. \tag{31}$$

For the parameter values characteristic of Si $\nu \approx 0.3$, $\beta_{\mathrm{s}} \approx 0.4$, $\chi = 10^{-1}$ cm$^2$ s$^{-1}$, $\rho c^2 = 1.2 \times 10^{12}$ erg cm$^{-3}$, $n = 10^{22}$ cm$^{-3}$, $\alpha \approx 10^2$), equation (31) yields the critical temperature $T_{\mathrm{c}} \approx 900$ K.

Thus, the development of the TD instability in a linear regime ($T_0 \geqslant T_{\mathrm{c}}$) on a surface gives rise to a temperature lattice (16), (27) and the concomitant deformation lattice (15), (24), (27) with $q$ lying inside the amplification band, $\lambda(q) > 0$ (Fig. 3a). The lattice with the greatest increment has a period of $d = 2\pi/q_{\mathrm{m}}$.

Fig. 3b shows the dependence $\lambda(q)$ given by expression (28) at higher laser heating temperatures ($T_0 \gg T_c$). One can see that $q_m > q_c$, so that the maximum increment is achieved for the mode with $q = q_c = \pi/h$. In this case, the period of the TD lattices, taking into account (5), is

$$d_c = \frac{2\pi}{q_c} \approx 2h \approx 2(\chi\tau_p)^{1/2} \ln\frac{T_0}{T_c}. \quad (32)$$

Based on the above discussion, one should expect that the TD self-organisation occurs at two stages. At the first stage, those TD modes are selected whose wave vectors are directed along the two perpendicular [100] crystallographic directions; at the second stage, the periods of the TD modes are selected. Therefore, a two-dimensional TD lattice should appear, which is described by a superposition of two orthogonal lattices

$$T(x_i) = T(q)\cos(qx_i),\ \zeta(x_i) = \zeta_q\cos(qx_i), \quad (33)$$

where $q = q_c$; $x_i = \{x, y\}$. The relation between the Fourier amplitudes of the bending coordinate $\zeta_q$ and the temperature $T(q)$ is specified by expression (24) with the coupling coefficient $\eta(q)$ taken in the limit $q > h^{-1}$:

$$\zeta_q = -\frac{6\theta_T(vqh + 2\beta)}{\rho c^2 h^2 q^3} T(q). \quad (34)$$

Therefore, the maxima of the film relief ($\zeta_q < 0$) coincide with the temperature maxima (see Fig. 1b).

According to Eqns (8), (33), and (34), the crystal surface also features a two-dimensional deformation lattice representing the superposition of two lattices of the type

$$\xi(x_i) = v\frac{h}{2}\frac{\partial^2\zeta}{\partial x_i^2} = v\frac{h}{2}q^2|\zeta_q|\cos(qx_i). \quad (35)$$

The component $u_{x_i}(x_1 = x, x_2 = y)$ of the lateral displacement vector of the surface is related to the deformation $\xi$ by the expression $\xi = \partial u_{x_i}/\partial x_i$. Taking into account (35), we obtain

$$u_{x_i} = \int \xi dx_i = v\frac{h}{2}q|\zeta_q|\sin(qx_i). \quad (36)$$

Therefore, the lines of zero lateral displacement of the surface, determined by expression (36), coincide with the lines of maximum temperature determined by Eqn (33), so that the displacements on the two sides of these lines are oppositely directed (see Fig. 1b).

## 5. Comparison with experiment and conclusions

Thus, as the critical temperature $T_c$ (31) is exceeded, two lattices of temperature variation are formed on the crystal surface. Their grooves are perpendicular to the [100] crystallographic axes and the distance between the maximum temperature lines is specified by formula (32). Simultaneously, two deformation lattices of the type (35) are formed on the surface, with the lines of zero displacement of the medium directed along the [100] axes and being coincident with the lines of maximum temperature [cf. (36) and (33)]. For this reason, upon the development of the TD instability, when the wave amplitude of static lateral displacements of the surface becomes sufficiently large, the periodic damage (cracking) of the surface occurs along the lines of maximum temperature.

The theory of TD instability predicts two-dimensional cracking (into square fragments) of the irradiated surface at temperatures close to the melting temperature. The sides of these square fragments should be parallel to the [001] crystallographic axes, and their linear size should increase with increasing laser pulse duration in accordance with Eqn (32).

In connection with this prediction, note that the authors of Ref. [7] have observed quasi-periodic laser-induced fissuring of crystalline Si, with the characteristic period (the fragment size) increasing with the laser pulse duration $\tau_p$. In the experiments, the radiation intensity was close to the melting intensity. Fig. 4 shows the dependence of the period of the TD structure calculated from (32), as well as the experimental data of Ref. [7]. One can see that the predictions of the theory of TD instability agree with the experimental data.

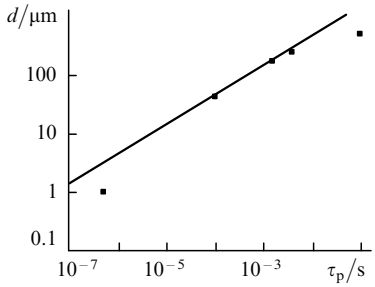


**Figure 4.** Experimental dependence of the period of the surface-fissuring lattice on the laser pulse duration (dots) for Si irradiated by 1.06-μm laser pulses of intensities near the melting threshold [7], and the theoretical line corresponding to formula (32) for the same parameters as in Fig. 2.

The geometry of TD structures (lattices) considered in this paper is determined by the crystallographic symmetry of the surface. In the case of polycrystals or amorphous media, the geometry of the structures can be determined by the symmetry of the external influence, namely, the symmetry of the original surface stresses or the symmetry of the transverse intensity distribution of the laser beam. Depending on the specific symmetry of the external influence, one can expect the formation of periodic TD fields having the form of one- or two-dimensional lattices, concentric circles, or radial structures.

## References

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