Quantum Electronics **31** (2) 164 – 166 (2001)

NONLINEAR OPTICAL PHENOMENA

PACS numbers: 42.50.Dv; 42.65.Yj
DOI: 10.1070/QE2001v031n02ABEH001910

The possibility of enhancing light squeezing by cascade parametric oscillation

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Abstract. **Quantum fluctuations of the output radiation amplitude in a cascade of unidirectional ring optical parametric oscillators (OPOs) operating upon pumping below the threshold are studied theoretically. It is shown that, by using a cascade of OPOs, one can reduce the spectral density of the fluctuations due to more efficient light squeezing.**

Keywords: *optical parametric oscillator, light squeezing, spectrum of amplitude fluctuations.*

Optical parametric oscillators (OPOs) are one of the main sources of squeezed light [1 – 5]. At the generation threshold, quantum fluctuations of the output radiation amplitude can, in principle, be totally suppressed by eliminating the energy dissipation inside the cavity. In the low-frequency region, the spectral density of the amplitude fluctuations, equal to the ratio between the spectral densities of quantum noise and shot noise at the generation threshold, is given by [5]

$$V = \frac{L}{L+T}, \tag{1}$$

where T is the transmission coefficient of the output mirror and L is all other losses per a transit in the cavity. To reduce the quantum noise, one should increase T and decrease L, the efficiency of this procedure being limited by the dissipative losses caused by absorption inside a nonlinear crystal.

However, in real OPOs, the limit related to the energy dissipation inside the cavity has not been achieved. To date, the best degree of squeezing, obtained for $T = 7.2$ % and $L = 0.41$ %, was 7.2 dB [5], which is considerably below the theoretical limit of 12 dB determined by expression (1) for these parameters. As the generation threshold was approached in the region below the threshold in [5], the noise intensified and the stationary regime became unstable.

In its methodological and conceptual aspects, the problem of quantum noise transformation in multicascade OPOs belongs to a wide range of problems recently discussed in the literature (see [3] and references therein).

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Received 5 July 2000; revision received 8 September 2000
Kvantovaya Elektronika **31** (2) 164 – 166 (2001)
Translated by I V Bargatin

A quantum system formed by two subsystems is considered. The first subsystem generates nonclassical light (squeezed light, light with photon antibunching, etc.). The output radiation of the first subsystem is directed to the input of the second subsystem and controls it. In [3], a general approach was developed for describing such systems, which we use in this work.

Consider a cascade of two unidirectional circular OPOs pumped below the generation threshold. A vacuum field is incident on the input mirror of the first OPO, and the output radiation of the first OPO is directed to the input of the second OPO. The creation (a_j^+) and annihilation (a_j) operators of intracavity photons of each OPO ($j = 1, 2$) are described by the following system of Langevin equations [1 – 3]:

$$\tau_c \dot{a}_j = -\frac{k_j}{2} a_j + \frac{\varepsilon_j}{2} a_j^+ + F_j, \tag{2}$$

$$\tau_c \dot{a}_j^+ = -\frac{k_j}{2} a_j^+ + \frac{\varepsilon_j^*}{2} a_j + F_j^+. \tag{3}$$

Here, k_j are the total losses per a round-trip transit of the jth cavity; τ_c is the round-trip transit time for light in the cavity; ε_j is the effective complex amplitude of the pump; F_j and F_j^+ are the Langevin forces. We assume in (2) and (3) that the two resonators have the same natural frequency ω_{1c}, and that the pump frequency ω_p satisfies the condition $\omega_p = 2\omega_{1c}$.

Creation ($a_{\text{out}\,1}^+$) and annihilation ($a_{\text{out}\,1}$) operators of photons emerging from the first OPO ($j = 1$) through a semitransparent mirror with the transmission coefficient T are determined by the boundary conditions on this mirror [1, 2]:

$$a_{\text{out}\,1} = \sqrt{T} a_1 - a_{\text{in}}, \quad a_{\text{out}\,1}^+ = \sqrt{T} a_1^+ - a_{\text{in}}^+, \tag{4}$$

where a_{in} (a_{in}^+) is the annihilation (creation) operator of the vacuum field photons incident on the input mirror of the first OPO. Since the input field for the second cascade ($j = 2$) is the output field of the first OPO, the boundary conditions on the output mirror of the second cascade (which has the same transmission coefficient T) assume the following form, different from Eqns (4):

$$a_{\text{out}\,2} = \sqrt{T} a_2 - a_{\text{out}\,1}, \quad a_{\text{out}\,2}^+ = \sqrt{T} a_2^+ - a_{\text{out}\,1}^+. \tag{5}$$

We will assume that the two cascades are identical: $k_1 = k_2 = T + L$ and $\varepsilon_1 = \varepsilon_2 = \varepsilon$. The effective complex amplitude ε of the pump can be represented in the form

$$\varepsilon = \frac{1}{2}(T+L)\eta e^{-i\vartheta}, \tag{6}$$

where ϑ it is the phase of the pump and η is the ratio of the pump amplitude to the threshold amplitude. The generation threshold corresponds to $\eta = 1$; upon pumping below the threshold considered here, we have $\eta < 1$.

It is known that the OPO squeezes the quadrature component $a + a^+$ when the phase of the pump $\vartheta = \pi$. In the following, we will be interested in squeezing only; therefore, we set $\vartheta = \pi$. In this case, the expression for ε takes the form $\varepsilon = -(T+L)\eta/2$.

By solving the system of equations (2) and (3) with the help of a Fourier transform and using Eqns (4), we obtain the following formulas for the quadrature components of the intracavity and output fields of the first cascade:

$$a_1(\omega) + a_1^+(\omega) = \frac{F_1(\omega) + F_1^+(\omega)}{p(\omega)}, \tag{7}$$

$$a_{\text{out}\,1}(\omega) + a_{\text{out}\,1}^+(\omega) = \sqrt{T}\,[a_1(\omega) + a_1^+(\omega)]$$

$$- a_{\text{in}}(\omega) - a_{\text{in}}^+(\omega), \tag{8}$$

where $p(\omega) = (T+L)(1+\eta)/2 - i\omega\tau_c$.

We will represent the Langevin force $F_1(t)$ of the first cascade in the form [1, 2]

$$F_1(t) = \sqrt{T} a_{\text{in}}(t) + \sqrt{L}\, b_1(t). \tag{9}$$

The first term in the right-hand side of Eqn (9) corresponds to the external (vacuum) field that enters the OPO through the semitransparent mirror, whereas the second term corresponds to the quantum noise due to the dissipative loss in the first cascade. The sources of the noise are delta-correlated:

$$\langle a_{\text{in}}(t) a_{\text{in}}^+(u)\rangle = \delta(t-u), \quad \langle a_{\text{in}}^+(t) a_{\text{in}}(u)\rangle = 0,$$
$$\langle b_1(t) b_1^+(u)\rangle = \delta(t-u), \quad \langle b_1^+(t) b_1(u)\rangle = 0. \tag{10}$$

The noise operators a_{in} and b_1 are uncorrelated.

Similarly to Eqns (7) and (8), we obtain the following formulas for the Fourier components of the field in the second cascade [3]:

$$a_2(\omega) + a_2^+(\omega) = \frac{F_2(\omega) + F_2^+(\omega)}{p(\omega)}, \tag{11}$$

$$a_{\text{out}2}(\omega) + a_{\text{out}2} + (\omega) = \sqrt{T}[a_2(\omega) + a_2^+(\omega)]$$

$$-\sqrt{T}[a_1(\omega) + a_1^+(\omega)] + a_{\text{in}}(\omega) + a_{\text{in}}^+(\omega). \tag{12}$$

We will represent the Langevin force $F_2(t)$ of the second cascade in the form [3]

$$F_2(t) = \sqrt{T}[\sqrt{T} a_1(t) - a_{\text{in}}(t)] + \sqrt{L}\, b_2(t). \tag{13}$$

The term enclosed in square brackets in the right-hand side of this expression corresponds to the output field of the first cascade, whereas the last term describes the quantum noise due to the dissipative loss in the second cascade. The operator b_2 has the same correlation properties as b_1; it does not correlate with a_{in} and b_1.

Using relations (5)–(13), we can express the Fourier component of the output field $a_{\text{out}\,2}(\omega) + a_{\text{out}\,2}^+(\omega)$ in terms of the spectral components of operators a_{in}, b_1, and b_2. Taking the correlation properties of these operators into account, we obtain the following formula for the spectral density of the fluctuations of the amplitude $V(\omega) = \langle |a_{\text{out}\,2}(\omega) + a_{\text{out}\,2}^+(\omega)|^2\rangle$ in a cascade of two identical OPOs:

$$V(\omega) = \frac{A^2}{B^2} + \frac{TL}{B} + \frac{TLA}{B^2}, \tag{14}$$

where $A = [(T+L)(1+\eta)/2 - T]^2 + \omega^2\tau_c^2$; $B = [(T+L)(1+\eta)/2]^2$.

Expression (14) is valid in the frequency region $|\omega| \ll (T+L)/\tau_c$. It follows from this expression that, upon pumping below the threshold, we can obtain better light squeezing in a cascade of two identical OPOs than in a single OPO. Fig. 1a shows a typical dependence of $V(\omega = 0)$ on the relative amplitude η of the pump for a cascade of two OPOs (solid lines) and a single OPO (dashed lines). Curves *1* correspond to the experimental parameters of Ref. [5]: $L = 0.41$ %, $T = 7.2$ %, $\tau_c = 4.8 \times 10^{-10}$ s [$(L+T)/(2\pi\tau_c) = 25$ MHz]. For curves *2*, the transmission coefficient $T = 20$ % whereas L and τ_c are the same as for curves *1*. Fig. 1b shows the dependences of $V(\omega)$ on η for the spectral component

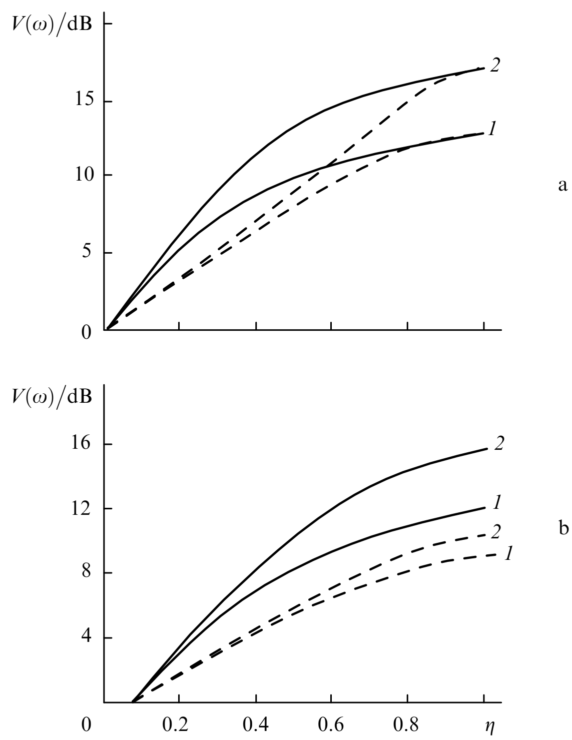

Figure 1. Dependences of the spectral density $V(\omega)$ of the amplitude fluctuations on the relative amplitude η of the pump for the spectral components with $\omega/2\pi = 0$ (a) and 6.5 MHz (b) for a cascade of two OPOs (solid lines) and a single OPO (dashed lines) for $L = 0.41$ %, $c = 4.8 \times 10^{-10}$ s, and $T = 7.2$ % [5] (*1*) and 20 % (*2*).

with $\omega/2\pi = 6.5$ MHz. The authors of Ref. [5] performed light squeezing measurements at this frequency.

It follows from Eqn (14) (as well as from Fig. 1a) that, at the parametric generation threshold ($\eta = 1$), the spectral density $V(0)$ for the spectral components with $\omega = 0$ is the same in a cascade and in a single OPO. This result also applies to a system of many cascades: the ultimate spectral density $V(0)$ at the threshold ($\eta = 1$) is the same as in the case of a single OPO. It is determined by the noise caused by the intracavity dissipative loss in the last cascade.

At $\omega = 0$, the use of a cascade is advantageous only if the pump amplitude is below the threshold. It follows from Eqn (14) that in the region of spectral components with $\omega \neq 0$, the use of a cascade can enhance light squeezing in the entire range of pump intensities, including the threshold one (see Fig. 1b). By employing a cascade of OPOs, one can significantly broaden the frequency band in which the quantum noise is suppressed. In a single OPO, the spectral density $V(\omega)$ is proportional to ω^2 in the low-frequencies region $|\omega| \ll (T + L)/\tau_c$. In a system of N cascades, $V(\omega)$ is proportional to ω^{2N}.

Thus, our analysis of the fluctuation spectrum of the output radiation amplitude in a cascade of unidirectional ring OPOs showed that one can enhance light squeezing by employing a cascade of identical OPOs operating upon pump amplitude below the threshold.

Acknowledgements. This work was supported by the Russian Foundation for Basic Research (Grant No. 00-02-16041) and jointly by the Belarussian and Russian Foundations for Basic Research (Grant No. 00-02-810400 Belarus, 2000-a).

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