

PACS numbers: 42.65.Hw; 42.70.Nq
DOI: 10.1070/QE2001v031n02ABEH001911

Self-excitation of mutually phase-conjugated light waves in a cubic gyrotropic photorefractive crystal subjected to a square-wave electric field

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Abstract. **Stationary four-wave mixing in a shifted photorefractive transmission grating formed in cubic gyrotropic crystals of the 23 symmetry is considered in the case of low contrasts of the original optical interference pattern. Expressions for the transmission and reflection coefficients for the phase conjugation of weak light beams in arbitrarily cut samples are obtained by solving exactly the equations for the coupled waves that include the effects of the natural circular birefringence and the linear birefringence induced by the external field. The conditions for the generation of phase-conjugated waves are determined for the mixing at 633 nm in $Bi_{12}SiO_{20}$ and $Bi_{12}TiO_{20}$ samples in the case when the grating vector is parallel to the [110] axis and the incident pump waves propagate in the (001) crystal plane and have arbitrary polarisations.**

Keywords: *photorefractive grating, four-wave mixing, gyrotropic crystal.*

Degenerate four-wave mixing (DFWM) in photorefractive crystals is efficiently used for phase conjugation of light beams [1 – 10]. The reflectivity for the phase-conjugated wave depends on the electrooptical properties of the crystal and the formation mechanism of the photorefractive grating. In crystals with diffusion nonlinearity, the intensity of the phase-conjugated wave does not exceed a few percent of the intensity of the incident probe wave [1, 6, 7, 9, 10]. The drift mechanism of charge separation, realised by applying a constant electric field to the crystal, made it possible to achieve the phase-conjugated reflectivity $R = 500\,\%$ in a nominally pure lithium tantalate [1].

However, it is difficult to use a constant field for increasing the photorefractive response because of the need to uniformly illuminate the interelectrode space in the crystal. The nonstationary mechanism of the formation of photorefractive gratings, when the crystal is subjected to an alternating external field, is free of this drawback [2 – 4, 9]. If the period of the applied voltage T is much shorter than the time of the Maxwell relaxation τ_{di} in the illuminated region, the space charge has no time to shield the external field. The amplitude of the formed photorefractive grating increases with increasing external field, making it optimal to use a voltage of the square-wave shape [9, 11].

Note that the photorefractive response reduces drastically when the external field period approaches the lifetime τ_R of the photoexcited charge carriers [12, 13]. In cubic crystals ($Bi_{12}GeO_{20}$, $Bi_{12}SiO_{20}$, and $Bi_{12}TiO_{20}$) at typical intensities of the light beams from 1 to 100 mW cm^{-2}, the inequality $\tau_{di} \gg \tau_R$ is valid, and the frequency $f = 1/T$ of the applied voltage can be varied from hundreds of hertz to several kilohertz. Similarly to the case of the diffusion mechanism, the transmission grating of the variation $\Delta\varepsilon$ in the dielectric constant formed under such conditions is shifted with respect to the interference pattern by a quarter of the grating spacing.

In the traditional transmission geometry [1 – 10], the phase conjugation efficiency for weak light beams is determined by the mutual spatial shift of the two photorefractive gratings that are formed in the crystal. The first of this gratings is due to the interaction between an intense pump beam I_1, incident on the input face of the sample, and a weak phase-conjugated beam I_3 (see inset in Fig. 1). The second grating is formed upon diffraction of the pump beam I_2 from the first grating, which is incident on the opposite crystal face and propagates in the opposite direction to the beam I_1. The diffracted wave I_4, phase conjugated with respect to the beam I_3, is amplified if the secondary transmission grating formed by beams I_4 and I_2 coincides with the original one. This regime of 'positive feedback' is realised in selenite crystals cut along the (110) plane and subjected to an alternating field, for example, when the photorefractive grating vector $\boldsymbol{K}$ is parallel to the $[\bar{1}10]$ crystallographic axis and orthogonal to the polarisations of pump waves I_1 and I_2 [2 – 4, 9]. The authors of Ref. [2] have demonstrated in experiments with a $Bi_{12}TiO_{20}$ crystal that the self-excitation of mutually phase-conjugated waves can take place in this geometry.

To analyse theoretically the DFWM in cubic photorefractive crystals in the case when the secondary grating is in phase with the original one, the authors of Ref. [3, 4] used the scalar model of the mixing and the approximation of undepleted pump. They derived simple analytic expressions for the reflectivity of the phase-conjugated beam and determined the conditions for the generation of mutually phase-conjugated waves in the geometry considered. The analytic solution of the self-consistent variant of this problem was obtained in Refs [14, 15].

However, the scalar model of the interaction, used in Refs [2 – 4, 8, 14, 15], does not take the natural circular birefringence of selenite crystals and the linear birefringence

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Received 17 May 2000; revision received 9 November 2000
Kvantovaya Elektronika **31** (2) 167 – 172 (2001)
Translated by I V Bargatin

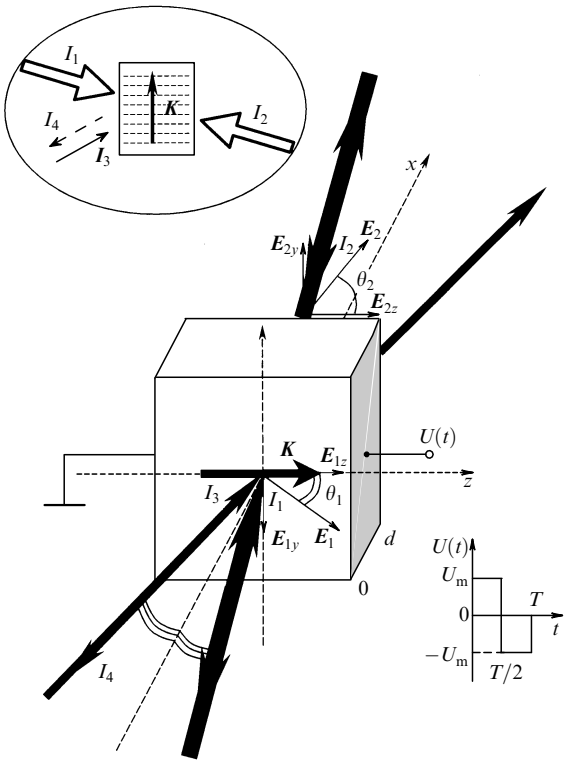

Figure 1. Geometry of the four-wave mixing on the photorefractive transmission grating in a cubic photorefractive crystal that is subjected to an external electric field, and the time dependence of the applied voltage.

induced by the external field into account. Note that, for the case of a constant applied field and two special orientations of the grating vector, the authors of Ref. [5] have derived the equations that describe the DFWM in optically active cubic crystals and include the effects of the induced linear birefringence and the vector nature of the interaction. The numerical analysis performed in [5] for a $Bi_{12}GeO_{20}$ crystal showed that, in the case of a constant applied field, the reflectivity R of the phase-conjugated wave does not exceed 12 %.

The purpose of this work is to analyse the vector model of the DFWM in the traditional phase-conjugation scheme in optically active cubic selenite crystals subjected to an external alternating square-wave electric field.

Fig. 1 shows the scheme of the DFWM in a transmission grating that is formed in a crystal subjected to an external electric field. The pump (I_1) and signal (I_3) waves, symmetrically incident on the sample boundary $x = 0$, have in general arbitrary polarisations and form a photorefractive transmission grating whose vector $\boldsymbol{K}$ is parallel to the z axis, along which the external field is applied. The second pump wave I_2, incident on the boundary $x = d$, is also arbitrarily polarised and propagates in the opposite direction to wave I_1, giving rise to the phase-conjugated wave I_4 due to the diffraction from the grating with the vector $\boldsymbol{K}$.

In a gyrotropic crystal with an externally induced linear birefringence, each of the interacting waves is elliptically polarised. Within the paraxial approximation and neglecting the absorption, we can represent their optical fields as superpositions of the eigenwaves:

$$\boldsymbol{E}_1 = \left\{ C_{11}(x)\boldsymbol{e}_1 \exp\left[-\mathrm{i}\left(kn_1 x - \frac{K}{2}z\right)\right] + C_{12}(x)\boldsymbol{e}_2 \exp\left[-\mathrm{i}\left(kn_2 x - \frac{K}{2}z\right)\right]\right\} \exp(\mathrm{i}\omega t),$$

$$\boldsymbol{E}_2 = \left\{ C_{21}(x)\boldsymbol{e}_1^* \exp\left[\mathrm{i}\left(kn_1 x - \frac{K}{2}z\right)\right] + C_{22}(x)\boldsymbol{e}_2^* \exp\left[\mathrm{i}\left(kn_2 x - \frac{K}{2}z\right)\right]\right\} \exp(\mathrm{i}\omega t),$$

$$\boldsymbol{E}_3 = \left\{ C_{31}(x)\boldsymbol{e}_1 \exp\left[-\mathrm{i}\left(kn_1 x + \frac{K}{2}z\right)\right] + C_{32}(x)\boldsymbol{e}_2 \exp\left[-\mathrm{i}\left(kn_2 x + \frac{K}{2}z\right)\right]\right\} \exp(\mathrm{i}\omega t),$$

$$\boldsymbol{E}_4 = \left\{ C_{41}(x)\boldsymbol{e}_1^* \exp\left[\mathrm{i}\left(kn_1 x + \frac{K}{2}z\right)\right] + C_{42}(x)\boldsymbol{e}_2^* \exp\left[\mathrm{i}\left(kn_2 x + \frac{K}{2}z\right)\right]\right\} \exp(\mathrm{i}\omega t), \quad (1)$$

Here, $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ ($\boldsymbol{e}_1^*$ and $\boldsymbol{e}_2^*$) are the polarisation vectors of the eigenwaves propagating in the positive (negative) direction along the x axis; n_1 and n_2 are their refractive indices [16–19]. The amplitudes $C_{sj}(x)$ of the eigenwaves are assumed to depend on x because of the wave mixing; $k = 2\pi/\lambda$ is the wave number of light in vacuum; $K = 2\pi/\Lambda$; Λ is the photorefractive grating spacing. In Appendix, we present analytic expressions for the polarisation vectors $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ and refractive indices n_1 and n_2 of the eigenwaves for an arbitrary orientation of the crystal and the grating vector $\boldsymbol{K}$.

Pairwise interference between co-propagating waves I_1, I_3 and I_2, I_4 gives rise to an intensity grating with the contrast

$$m(x) = m_{13}(x) + m_{24}(x),$$

$$m_{13}(x) = 2\frac{C_{11}(x)C_{31}^*(x) + C_{12}(x)C_{32}^*(x)}{I_0}, \quad (2)$$

$$m_{24}(x) = 2\frac{C_{21}^*(x)C_{41}(x) + C_{22}^*(x)C_{42}(x)}{I_0},$$

where $I_0 = I_1 + I_2 + I_3 + I_4$ is the total intensity of the opti-cal field in the crystal. In the approximation of a small cont-rast $m \ll 1$, the space charge of the photorefractive crystal, subjected to a square-wave electric field $E_0(t)$, creates a field whose amplitude can be written as [9, 11]

$$\mathscr{E}_1 = \mathrm{i}mE_{\mathrm{sc}},\ E_{\mathrm{sc}} = E_{\mathrm{q}}\frac{(E_\mu + E_{\mathrm{d}})E_{\mathrm{d}} + E_{\mathrm{m}}^2}{(E_{\mathrm{q}}+E_{\mathrm{d}})(E_\mu + E_{\mathrm{d}}) + E_{\mathrm{m}}^2}, \quad (3)$$

where $E_{\mathrm{d}} = 2\pi k_{\mathrm{B}}T/\Lambda e$ is the diffusion field; $E_\mu = \Lambda/2\pi\mu\tau_{\mathrm{R}}$ is the drift field; $E_{\mathrm{q}} = eN_{\mathrm{a}}\Lambda/2\pi\varepsilon$ is the trap saturation field; E_{m} is the amplitude of the applied field; N_{a} is the concentration of acceptors in the crystal; T is the absolute temperature; μ and τ_{R} are the mobility and the lifetime of electrons in the conduction band; e is the elementary electric charge; ε is the static dielectric constant of the crystal.

In the following, we will restrict our analysis of the four-wave mixing to the approximation of undepleted pump waves I_1 and I_2. In this case, we can neglect the variation in amplitudes C_{11}, C_{12}, C_{21}, and C_{22} along the coordinate x. Employing the standard approximation of slowly varying amplitudes and the relations describing the modulation of the crystal optical properties due to the linear electrooptical effect, we can derive the following coupled-wave equations from the wave equation for gyrotropic media:

$$\frac{\mathrm{d}C_{31}(x)}{\mathrm{d}x} = \frac{\gamma}{4} m_{\mathrm{u}}^{*}(x)[g_{11}C_{11} + g_{12}\exp(\mathrm{i}\Delta k x)C_{12}], \tag{4}$$

$$\frac{\mathrm{d}C_{32}(x)}{\mathrm{d}x} = \frac{\gamma}{4} m_{\mathrm{u}}^{*}(x)[g_{12}^{*}\exp(-\mathrm{i}\Delta k x)C_{11} + g_{22}C_{12}], \tag{5}$$

$$\frac{\mathrm{d}C_{41}(x)}{\mathrm{d}x} = \frac{\gamma}{4} m_{\mathrm{u}}(x)[g_{11}C_{21} + g_{12}^{*}\exp(-\mathrm{i}\Delta k x)C_{22}], \tag{6}$$

$$\frac{\mathrm{d}C_{42}(x)}{\mathrm{d}x} = \frac{\gamma}{4} m_{\mathrm{u}}(x)[g_{12}\exp(\mathrm{i}\Delta k x)C_{21} + g_{22}C_{22}], \tag{7}$$

where

$$m_{\mathrm{u}}(x) = \frac{2}{I_0}\left(C_{11}C_{31}^{*}(x) + C_{12}C_{32}^{*}(x) + C_{21}^{*}C_{41}(x) + C_{22}^{*}C_{42}(x)\right) \tag{8}$$

is the contrast of the interference pattern; $\gamma = 2\pi n^3 r_{41}^{\mathrm{u}} E_{\mathrm{sc}}/\lambda$ is the coupling constant; n is the refractive index of an unperturbed medium; r_{41}^{u} is the electrooptical coefficient of the clamped crystal; $\Delta k = (n_1 - n_2)k$. The tensor convolutions describing the contribution of intramodal ($g_{11} = \boldsymbol{e}_1^{*}\cdot\boldsymbol{g}\cdot\boldsymbol{e}_1$, $g_{22} = \boldsymbol{e}_2^{*}\cdot\boldsymbol{g}\cdot\boldsymbol{e}_2$) and intermodal ($g_{12} = \boldsymbol{e}_1^{*}\cdot\boldsymbol{g}\cdot\boldsymbol{e}_2$) mixing processes [16–19] can be calculated with the help of relations (A2) and (A3) presented in Appendix.

Within the approximations considered, system (4)–(8) is also applicable to the general case of the stationary DFWM in a cubic photorefractive crystal of symmetry 23, when two plane optical waves are incident on the boundary $x = d$ (Fig. 1). Like equations derived in Ref. [5] by expanding the optical field in TE- and TM-components for two specific interaction geometries, this system of equations takes the anisotropy of the linear electrooptical effect and the natural circular birefringence into account. However, the system of coupled equations (4)–(8) allows us to analyse the DFWM within the approximation of undepleted pump in arbitrarily cut samples and for arbitrary orientations of the photorefractive grating vector $\boldsymbol{K}$ with respect to the crystallographic system of coordinates.

Considering the phase conjugation efficiency of wave I_3, we will use the condition of the absence of wave I_4 at the crystal boundary: $m_{24}(d) = 0$. In this case, we can obtain the following solution of equations (4)–(7) for the contrast of the interference pattern:

$$m_{\mathrm{u}}(x) = \frac{2m_{13}(0)\exp(G(x)x)}{1 + \exp(G(d)d) - \Phi(d)}, \tag{9}$$

where

$$G(x) = \frac{\gamma}{2I_0}\left\{g_{11}\left(|C_{11}|^2 + |C_{21}|^2\right) + g_{22}\left(|C_{12}|^2 + |C_{22}|^2\right) - 2\mathrm{Im}\left[g_{12}\left(C_{11}^{*}C_{12} + C_{21}C_{22}^{*}\right)\frac{1 - \exp(\mathrm{i}\Delta k x)}{\Delta k x}\right]\right\}; \tag{10}$$

$$\Phi(x) = \frac{\gamma}{2I_0}\int_0^x \exp(G(\xi)\xi)\left\{g_{11}\left(|C_{11}|^2 - |C_{21}|^2\right) + g_{22}\left(|C_{12}|^2 - |C_{22}|^2\right) + 2\mathrm{Re}\left[g_{12}\left(C_{11}^{*}C_{12} - C_{21}C_{22}^{*}\right)\exp(\mathrm{i}\Delta k\xi)\right]\right\}\mathrm{d}\xi. \tag{11}$$

If the dependence $m_{\mathrm{u}}(x)$ is known, system (4)–(7) can be easily integrated. In the case of coincident polarisations of beams I_1 and I_3, the transmission coefficients for the probe wave, $K_3 = |\boldsymbol{E}_3(d)|^2/|\boldsymbol{E}_3(0)|^2$, and the phase-conjugated wave, $K_4 = |\boldsymbol{E}_4(0)|^2/|\boldsymbol{E}_3(0)|^2$, can be written as

$$K_3 = \frac{4\exp(2G(d)d) + |\Phi_3(d)|^2}{[1 + \exp(G(d)d) - \Phi(d)]^2}, \qquad K_4 = \frac{I_1}{I_2}\frac{[1 - \exp(G(d)d) + \Phi(d)]^2 + |\Phi_4(d)|^2}{[1 + \exp(G(d)d) - \Phi(d)]^2}, \tag{12}$$

where

$$\Phi_3(x) = \frac{\gamma}{I_0}\left(\frac{I_1}{I_{30}}\right)^{1/2}\int_0^x \exp(G(\xi)\xi)[g_{11}C_{11}C_{32}(0) - g_{22}C_{12}C_{31}(0) + g_{12}C_{12}C_{32}(0)\exp(\mathrm{i}\Delta k\xi) - g_{12}^{*}C_{11}C_{31}(0)\exp(-\mathrm{i}\Delta k\xi)]\mathrm{d}\xi;$$

$$\Phi_4(x) = \frac{\gamma}{I_0}\int_0^x \exp(G(\xi)\xi)[(g_{11} - g_{22})C_{21}C_{22} + g_{12}^{*}C_{22}^2\exp(-\mathrm{i}\Delta k\xi) - g_{12}C_{21}^2\exp(\mathrm{i}\Delta k\xi)]\mathrm{d}\xi.$$

Consider DFWM in the widely used orientation of photorefractive crystals when the optical beams are incident on the (110) face and the grating vector $\boldsymbol{K}$ is parallel to the $[\bar{1}10]$ axis. If the incident pump waves have the same intensity and are mutually orthogonally polarised, forming angles of $+45^{\circ}$ and -45° with the $[\bar{1}10]$ axis, the phase-conjugated reflectivity can be expressed analytically as

$$K_4^{[1\bar{1}0]} = \frac{\gamma^2 d^2\left\{\chi^2(1 + \chi^2) + \left[\sin^2|\rho|d\sqrt{1 + \chi^2}/\rho^2 d^2\right]\right\}}{\left\{4(1 + \chi^2) - \gamma d\left(\chi^2 + \left[\sin\left(2|\rho|d\sqrt{1 + \chi^2}\right)\Big/2|\rho|d\sqrt{1 + \chi^2}\right]\right)\right\}^2}, \tag{13}$$

where $\chi = \pi n^3 r_{41}^{\mathrm{T}} E_0/\rho\lambda$ and r_{41}^{T} is the electrooptical coefficient of an unclamped crystal.

The generation of mutually phase-conjugated waves sets in when the denominator

$$1 + \exp(G(d)d) - \Phi(d) = 0, \tag{14}$$

in the general expressions (9) and (12) for the contrast m_{u}, the phase-conjugated reflectivity K_4, and the transmission coefficient K_3, or the denominator

$$4(1 + \chi^2) - \gamma d\left\{\chi^2 + \frac{\sin\left[2|\rho|d(1 + \chi^2)^{1/2}\right]}{2|\rho|d(1 + \chi^2)^{1/2}}\right\} = 0, \tag{15}$$

in expression (13), for K_4 in the case when the grating vector $\boldsymbol{K}$ is parallel to the $[\bar{1}10]$ axis in the (110) plane, vanishes [20].

The self-excitation condition (14) is similar to that derived in Refs [2–4, 9]; however, it includes the effects of the induced linear birefringence and the optical activity in selenite crystals subjected to an external field. The self-excitation is caused by the amplification of weak optical waves by the both counterpropagating two-wave processes, which leads to a positive feedback in the DFWM process in photorefractive crystals [2–10].

It follows from the general relation (14) that the achievement of the generation regime in the crystals under study depends not only on the product γd, but also on the polarisation states of the incident pump waves and the birefringence Δk. One can use relation (14) to determine the optimal conditions for the phase conjugation in cubic photorefractive crystals subjected to an external square-wave electric field in the case when the threshold value of γ is not reached. In particular, the maximum phase-conjugated reflectivities for the phase-conjugated wave should be observed for orientations of polarisation vectors of pump waves that are optimal for the generation of mutually phase-conjugated beams.

When the grating vector $\boldsymbol{K}$ is parallel to the $[\bar{1}10]$ axis in the (110) plane, the angles θ_1 and θ_2 between the $[\bar{1}10]$ axis and the polarisation vectors of the incident pump waves I_1 and I_2, respectively, at which the condition (14) of self-excitation of mutually phase-conjugated waves is satisfied at 633 nm in $Bi_{12}SiO_{20}$ and $Bi_{12}TiO_{20}$ crystals is determined by closed curves in Fig. 2. In the calculations, we assumed that the thickness $d = 1$ cm and parameters $r_{41}^{u} = 5$ pm V^{-1}, $r_{41}^{T} = 5.7$ pm V^{-1}, and $\mu\tau_R = 10^{-11}$ m^2 V^{-1} were the same for both crystals. For the bismuth silicate, we used the interference pattern period $\Lambda = 35$ μm and the acceptor concentration $N_a = 10^{21}$ m^{-3}; for the bismuth titanate, $\Lambda = 10$ μm and $N_a = 20^{22}$ m^{-3}.

One can see from Fig. 2 that condition (14) can be satisfied when the amplitude of the external field exceeds the threshold value $E_{min} = 9.62$ kV cm^{-1} in $Bi_{12}SiO_{20}$ and 3.61 kV cm^{-1} in $Bi_{12}TiO_{20}$. This agrees qualitatively with the threshold dependence of the intensity of the phase-conjugated wave on the external field amplitude experimentally observed in $Bi_{12}SiO_{20}$ [8] and $Bi_{12}TiO_{20}$ [2]. The lesser value of the specific rotary power ρ and the greater concentration N_a of acceptors in $Bi_{12}TiO_{20}$ crystals result in a lower threshold than in $Bi_{12}SiO_{20}$ crystals.

As the amplitude of the external field exceeds E_{min}, the critical point in the plane of angles θ_1 and θ_2 transforms to an enclosing curve. In this case, the generation regime becomes possible for a whole set of polarisation vectors of the pump waves. The coordinates of the critical points, (49°, −47°) for $Bi_{12}SiO_{20}$ and (55.5°, −39.7°) for $Bi_{12}TiO_{20}$, are close to the angles at which the maximum gain of the partial interference patterns with contrasts m_{13} and m_{24} is realised.

It follows from conditions (14) and (15) that, due to the influence of the natural circular birefringence and externally induced linear birefringence, the threshold value of the product $(\gamma d)_{ex}$ of the coupling constant by the interaction length, which is sufficient for self-excitation of mutually phase-conjugated waves, can be different for crystals that have coincident photorefractive parameters but different thicknesses d. Fig. 3 shows the dependence of the threshold value $(\gamma d)_{ex}$ on the thickness d of $Bi_{12}SiO_{20}$ and $Bi_{12}TiO_{20}$

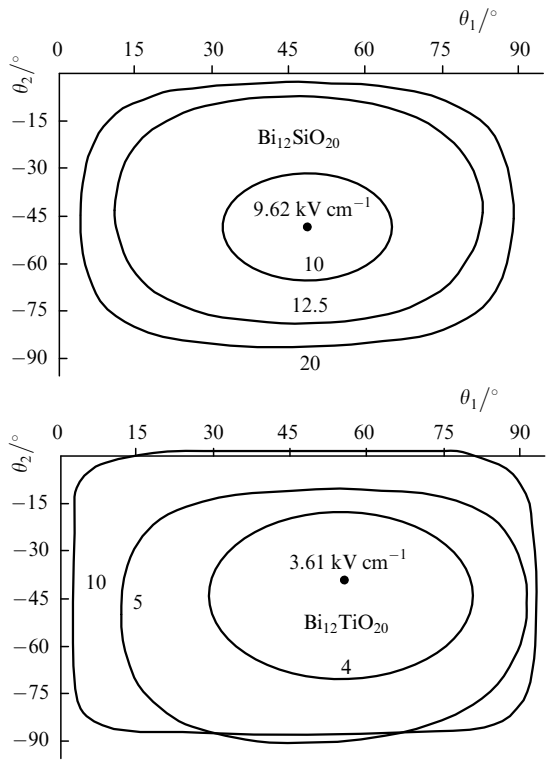

Figure 2. Curves corresponding to the angles θ_1 and θ_2 between the polarisation vectors of pump waves I_1 and I_2 and the $[\bar{1}10]$ axis of $Bi_{12}SiO_{20}$ and $Bi_{12}TiO_{20}$ crystals, that satisfy the condition for self-excitation of mutually phase-conjugated waves at various amplitudes of the applied field.

crystals in the case when the grating vector $\boldsymbol{K}$ is parallel to the $[\bar{1}10]$ axis in the (110) plane. The curves were calculated for the external field amplitude $E_m = 10$ kV cm^{-1} and the incident pump waves polarised at angles of 45° and −45° with respect to the $[\bar{1}10]$ axis.

One can see from Fig. 3 that $(\gamma d)_{ex} = 4$ at $d \to 0$. In the cited papers [2–4, 9], this threshold value was found by neglecting the birefringence and was independent of the interaction length. In the case of $Bi_{12}TiO_{20}$, which has a small specific rotary power $\rho = 6.3°$ mm^{-1}, an increase in d slightly increases the absolute value of $(\gamma d)_{ex}$. However, in the case of $Bi_{12}SiO_{20}$ ($\rho = 22°$ mm^{-1}), $(\gamma d)_{ex}$ strongly depends on the crystal thickness.

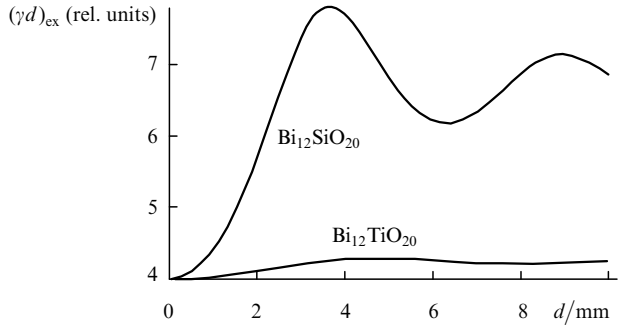

Figure 3. Threshold value $(\gamma d)_{ex}$ sufficient for self-excitation of mutually phase-conjugated waves as a function of the interaction length d in $Bi_{12}SiO_{20}$ and $Bi_{12}TiO_{20}$ crystals for the applied field amplitude $E_m = 10$ kV cm^{-1} and the pump waves polarised at 45° and −45° with respect to the $[\bar{1}10]$ axis.

For a given polarisation of the pump waves, the phase conjugation efficiency of weak optical waves is primarily determined by the spatial period Λ of the photorefractive grating, which enters the coupling constant γ, and the amplitude E_m of the external field, which enters both γ and the induced linear birefringence δn. Fig. 4 shows the dependence of the phase-conjugation reflectivity K_4 on the amplitude E_m for the DFWM geometry considered above (see expression (13)) in 1-cm-thick $Bi_{12}SiO_{20}$ and $Bi_{12}TiO_{20}$ crystals. Fig. 4 does not show the decaying tails of the curves observed when the amplitude E_m exceeds the threshold value ($E_m > E_{min}$). In this case, the solution for the phase-conjugated wave is unstable [21]; therefore, the DFWM remains in the generation regime when the external field exceeds the threshold $E_{min} = 7.0, 3.68, 12.13$, and 9.64 kV cm^{-1} for curves *1*, *2*, *3*, and *4*, respectively.

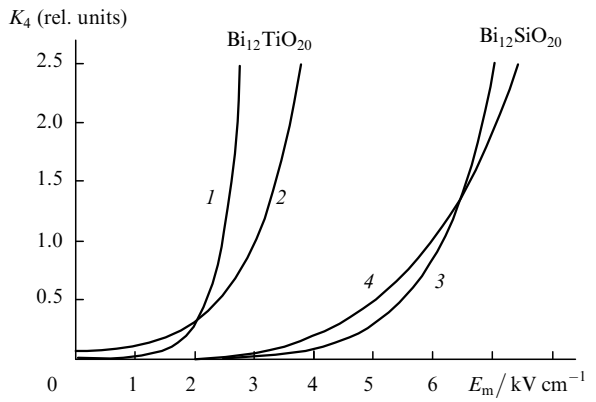

Figure 4. Dependences of the reflection coefficients on the external field amplitude for the DFWM in $Bi_{12}SiO_{20}$ and $Bi_{12}TiO_{20}$ crystals for the crystal thicknesses $d = 1$ cm and the photorefractive grating period $\Lambda = 1$ (*1*), 10 (*2*), 15 (*3*) and 35 μm (*4*).

One can see that the phase-conjugation reflectivity K_4 strongly reduces with decreasing amplitude of the external field. In the case of small values of E_m, the optical waves interact on the diffusion photorefractive grating, whose amplitude increases with decreasing spatial period Λ. Accordingly, in the case of $E_m < 2$ kV cm^{-1}, curve *2* in Fig. 4, which corresponds to $\Lambda = 1$ μm, lies above curves *2*, *3*, and *4*, which correspond to the grating periods $\Lambda = 10$, 15, and 35 μm.

In the absence of the external field ($E_m = 0$), the expression for the phase-conjugated reflectivity assumes the simple form $K_4 = 4\gamma^2 \sin^2(|\rho|d)/[8|\rho| - \gamma \sin(2|\rho|d)]^2$. It follows from this expression that, for a given coupling constant and crystal thickness, the maximum of the phase-conjugated reflectivity, $K_4 = \gamma^2 d^2/[4 - \gamma d]^2$, is reached at $\rho = 0$. Therefore, the threshold amplitude of the external field is lower in the $Bi_{12}SiO_{20}$ crystal, which has a lower specific rotary power, than in the $Bi_{12}TiO_{20}$ crystal. Note that the last expression coincides with the one derived in Refs [2–4, 9] by solving the scalar equations for coupled waves.

The external field shifts the optimal spatial periods Λ towards longer wavelengths in both crystals, resulting in a reduction in the lasing threshold Λ with increasing E_{min}.

Thus, by analysing the DFWM on photorefractive transmission gratings in cubic gyrotropic crystals subjected to a square-wave electric field, we have derived expressions that determine the optimal phase conjugation conditions and include the effects of the natural circular birefringence and the linear birefringence induced by the external field. For the special interaction geometry when the grating vector is parallel to the $[\bar{1}10]$ axis and the optical waves propagate in the (001) plane, we have calculated the threshold conditions for the generation of phase-conjugated light in 1-cm-thick $Bi_{12}SiO_{20}$ and $Bi_{12}TiO_{20}$ crystals pumped by non-orthogonally polarised waves. In the case of orthogonally polarised pump beams, we have analysed the dependence of the threshold value of $(\gamma d)_{ex}$ on the interaction length d and the dependence of the reflection coefficient on the external field amplitude.

Appendix

Following Refs [15–18], we represent the refractive indices $n_{1,2}$ and polarisation vectors $\boldsymbol{e}_{1,2}$ of the eigenwaves of a cubic non-centrosymmetrical gyrotropic crystal subjected to an external electric field $\boldsymbol{E}_0$ (Fig. 1) as

$$n_{1,2} = n + \delta n \frac{\boldsymbol{x}^o \cdot \boldsymbol{g} \cdot \boldsymbol{x}^o}{2} \pm \left\{ \left(\frac{\rho}{k} \right)^2 + \left[\left(\frac{\boldsymbol{x}^o \cdot \boldsymbol{g} \cdot \boldsymbol{x}^o}{2} \right)^2 - \boldsymbol{x}^o \cdot \left(\boldsymbol{g}^2 - \boldsymbol{\delta} \right) \cdot \boldsymbol{x}^o \right] \delta n^2 \right\}^{1/2}, \tag{A1}$$

$$\boldsymbol{e}_1 = \frac{(1 - i\zeta)\boldsymbol{y}^o + ir\boldsymbol{z}^o}{(1 + \zeta^2 + r^2)^{1/2}}, \quad \boldsymbol{e}_2 = \frac{r\boldsymbol{y}^o + (\zeta - i)\boldsymbol{z}^o}{(1 + \zeta^2 + r^2)^{1/2}}, \tag{A2}$$

where

$$\boldsymbol{g} = \begin{bmatrix} 0 & z_3^o & z_2^o \\ z_3^o & 0 & z_1^o \\ z_2^o & z_1^o & 0 \end{bmatrix};$$

$$r = \frac{1}{\rho} \left\{ \rho^2 + \left[\left(\frac{\boldsymbol{x}^o \cdot \boldsymbol{g} \cdot \boldsymbol{x}^o}{2} \right)^2 - \boldsymbol{x}^o \cdot \left(\boldsymbol{g}^2 - \boldsymbol{\delta} \right) \cdot \boldsymbol{x}^o \right] (\delta n k)^2 \right\}^{1/2} - \left(\frac{\boldsymbol{x}^o \cdot \boldsymbol{g} \cdot \boldsymbol{x}^o}{2} + \boldsymbol{z}^o \cdot \boldsymbol{g} \cdot \boldsymbol{z}^o \right) \frac{\delta n k}{\rho}; \tag{A3}$$

$\delta n = n^3 r_{41}^T E_0/2$; $\zeta = \delta n k (\boldsymbol{y}^o \cdot \boldsymbol{g} \cdot \boldsymbol{z}^o)/\rho$; ρ is the specific rotary power; z_k^o are the components of the unit vector $\boldsymbol{z}^o$ in the crystallographic system of coordinates; $\boldsymbol{y}^o = \boldsymbol{z}^o \times \boldsymbol{x}^o$; $\boldsymbol{\delta}$ is the unit tensor.

Acknowledgements. The authors thank B N Poizner for useful discussions of this paper.

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