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Synthesis of the spectrum in ring lasers with coupling of counterpropagating waves in a spectral filter and the filtering by real spectral lines

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***Abstract*. The formation of radiation with a complex spectrum in a ring titanium laser, with counterpropagating waves coupled in an extracavity spectral filter, is studied. It is shown experimentally that when absorption lines in a flame are used for filtering, weak lenses formed in the flame cause distortions of the spectrum, which can be described only within the framework of a modified method for calculating coupling coefficients. The mechanism of this effect is shown to be associated with a narrow peak of matching fields of counterpropagating waves in unstable cavities in the vicinity of the zero power of a lens in the coupling channel. An optimum regime of the synthesis of the spectrum without distortions but with a high efficiency is found.**

Keywords: *ring titanium laser, coupling, spectral filter, distortions.*

1. Introduction

Lidar probing and laser spectroscopy, data recording, storage, and transfer, and other problems, which become especially topical because of the burning problems related to ecology, computer networks, multichannel communication systems, and new physicochemical systems of medical diagnostics persistently call for the development of advanced methods and facilities. Some of them may be based on multifrequency lasers. This statement is supported by numerous examples from present-day scientific literature showing that the use of multifrequency emission enables one to increase the efficiency of these methods and facilities in various applications [1 – 4].

An important stage on this way is the development of lasers with the feasibility of electronic synthesis of emission with a prescribed structure of the spectrum (multifrequency or continuous). The methods developed on the basis of intracavity acousto-optical elements [5, 6] are efficient only in dye lasers with a short-pulse coherent pump and, because of high losses, are unsuitable for lasers operating on new media (titanium-doped sapphire, forsterite, etc.), including others pumping methods (using flashlamps, laser diodes, etc.). However, it is reasonable to expect a substantial progress precisely for the latter systems because they have an uniquely wide tuning range and offer promise for operation in long-wavelength ranges, which are important for applications. The papers available in this area of research are, as a rule, devoted to separate specific questions (for instance, two-frequency femtosecond lasers [7] and titanium lasers amplifying a signal of diode emitters [8], but do not solve the whole problem. The solution of the problem calls for new approaches and principles.

Because of this, we have proposed in Ref. [9] a new method and reported there the first results of its study. The method is based on the unidirectional selective coupling of counterpropagating waves of a ring laser in a complex extracavity spectral filter. This approach is universal because it enables one to form multifrequency emission and emission with a complex analogue spectrum and develop new methods of its tuning, in particular, develop complex filters using real absorption (amplification) lines of a complex form. The possibility of retaining in the spectrum details and features that are inevitably lost in the case of an artificial synthesis determines an independent importance of our method. It is also evident that the method is important for practice. In addition to applications in metrology (spectroscopy etc.), one can use it to form emission for an optimum action on objects via a medium with a complex absorption spectrum etc.

The aim of this work is to realise and study specific features of the synthesis of the spectrum by introducing real substances in filters and to seek for optimum ways of realisation of the method (to study factors that affect the wave coupling). In particular, the comparison of the theory with the experiment called for the refinement and modification of determination and calculation of mode coupling coefficients.

2. Experiment

First, recall the main advantages of the method proposed to form the spectrum, which are described in detail in Ref. [9]:

(1) the removal of complex and, frequently, cumbersome devices for the spectrum formation (synthesise) from a cavity enables one to decrease losses and pass to the solution of such problems in lasers with a lower gain of an active medium (for instance, with electron – phonon transitions) and even with flashlamp pumping;

(2) a change to the formation of the spectrum on a large number of round trips in the cavity allows one to use spectral filters with higher losses;

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(3) the additive injection of a formed signal from a coupling channel into a cavity eliminates the multiplicative competitive interaction, which distorts the laser spectrum, and provides its maximum similarity to the filter spectrum.

Let us descibe our results. A Ti^{3+}:Al_2O_3 laser (Fig. 1) with a crystal 30 mm long and 8 mm in diameter, which had Brewster windows, was used as a base laser. It was longitudinally pumped by 20-ns pulses of the second harmonic of an LTI-401 laser at 553 nm. Lenses *2*, with a variable spacing between them, matched the excitation region in the active medium to other elements of the cavity. The wave reflected from the face OO′ of a glass prism *3* was used to form a channel providing coupling between counterpropagating waves. It included a totally reflecting mirror *5* and a spectral filter *4*. The prism *3* provided control of the coupling coefficient and its constancy in a wide frequency range. The prism asymmetry, despite the fact that the field structure became more complicated, allowed us to eliminate losses at the second face. The laser spectrum was recorded by a linear CCD array placed at the output of an UF-90 camera. A computer working in line with a detector analysed spectra of separate pulses, and a special program was able to average them.

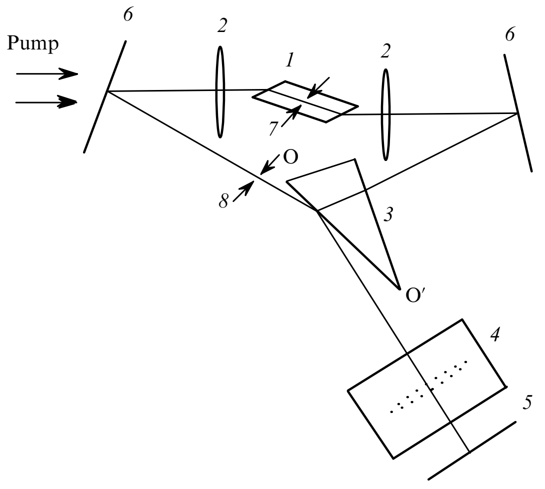

Figure 1. Schematic optical diagram of the laser: (*1*) active medium; (*2*) lenses; (*3*) prism; (*4*) spectral filter; (*5*) totally reflecting mirror; (*6*) mirrors; (*7*, *8*) equivalent apertures.

In contrast to the systems with dispersion cavities, the spectrum in the laser proposed here is formed during a large number of round trips (gradually) and therefore one needs laser pulses with large duration τ_g to achieve the desired efficiency of spectrum formation. In the titanium laser, this condition is fulfilled because its emission pulses are sufficiently long despite a small duration of pump pulses ($\tau_p \simeq$ 20 ns, whereas $\tau_g = 50 - 500$ ns) [10].

We studied different variants of the filter *4*: a cell with a gas mixture, a Fabry–Perot interferometer, a flame of an alcohol burner, etc. Unexpected results were obtain in the latter case, and they will be discussed in greater detail below. It is known that a flame contains different elements [11] whose absorption lines may be used as filters. Fig. 2 presents results of forming a spectrum with two potassium absorption lines with $\lambda = 769.90$ nm (the $4^2S_{1/2} - 4^2P_{1/2}$ transition with the oscillator strength $F = 0.347$) and 766.49 nm ($4^2S_{1/2} - 4^2P_{3/2}$, $F = 0.684$), which lie in the range of titanium laser emission.

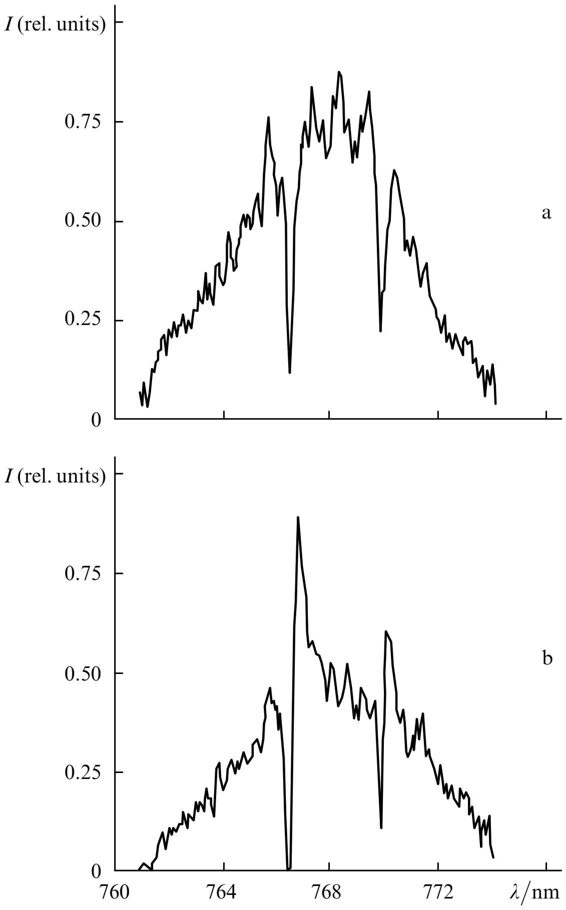

Figure 2. Experimental spectra with the flame filter without distortions (a) and with distortions (b).

In most cases, we clearly observed the aforementioned advantages of the method, and the similarity (Fig. 2a) between the spectra of laser emission and the flame filter was retained (an additional structure is associated with the contribution of absorption lines of oxygen, ozone, etc.).

However, in some cases, the spectrum was substantially distorted (a typical result is shown in Fig. 2b, note that spectra with stronger distortions were also observed). This argued against the aforementioned advantage of the methods associated with the similarity of the transformation. It was desired to find out a cause of this discrepancy and find a regime without distortions. It is most likely that distortions are caused by a lens in the flame in the coupling channel, whose power P_s, as well as the refractive index, depends on λ in the vicinity of the absorption (amplification) line [12]. As showed our estimates, P_s of these lenses was of the order of 0.1 D and they were expected to have no substantial effect and especially to cause no such distortions in the method proposed here. (The condensation of the spectrum that is observed in laser spectroscopy in the case of a flame found inside a cavity is caused by the multiplicative accumulation of difference in loss produced by a lens with a frequency-dependent power [13], whose effect rapidly increases with increasing number of round trips of radiation in the cavity. In our case, this accumulation is in principle absent, i. e., the spectrum is formed through interference additive summation of the signals injected from the coupling channel into the cavity.) To understand the cause of this anomaly, we studied the behaviour of the coupling coefficient in the laser filtering channel.

3. Coupling coefficient for the modes of counterpropagating waves

The resulting coupling coefficient is determined by the following factors: (1) the reflectivity of a wave from the face of prism 3 (the angle of incidence); (2) losses in the filtering channel; (3) matching of the spatial structure of modes of counterpropagating waves upon the transformation of one of the waves in the coupling channel. Let us discuss the third factor.

The distortions observed in the experiment cannot be explained by using the commonly accepted classical expressions obtained in [14] because they are valid only for freely travelling Gaussian beams and cannot be used for radiation in a cavity, where aperture factors are of primary importance, which is typical of many real systems.

Let us define more exactly the calculation method, taking into account the fact that the linear integral equations of laser cavities are non-Hermitian and their modes are nonorthogonal in the sense of complex conjugation (this orthogonality was used in Ref. [14]). For simplicity, without loss of generality, we consider a two-dimensional cavity whose characteristics are independent of the transverse variable y, whose axis is perpendicular to the cavity plane (Fig. 1). The general approach consists in constructing two systems of eigenfunctions $\{u_j\}$ and $\{v_j\}$ which are solutions of the basic equation and the conjugate one.

These functions have the same eigenvalues and are orthogonal in the following sense:

$$\int u_n(x)v_m(x)\mathrm{d}x = W_n\delta_{nm}, \tag{1}$$

where W_n are normalisation factors and x is the transverse spatial coordinate. Such functions should be constructed for each cavity of the system. In our case, the fields of the same ring cavity are coupled, and the modes of counterpropagating waves E_n^- and E_m^+ fulfil the role of the basic and conjugate functions.

Comparing in a common reference plane the mode fields $E_n^-(x)$ of the wave travelling in the cavity in the clockwise direction (Fig. 1) with the fields $E_m^+(x)$ travelling in the same direction and formed by the modes of the counterpropagating wave upon its return to the cavity from the coupling channel, we obtain the general relation for the coupling coefficients of different modes of these waves

$$T_{nk} = \int \mathrm{d}x \int K_{\mathrm{c}}(x, x_1)E_n^+(x_1)E_k^+(x)\mathrm{d}x_1, \tag{2}$$

where $K_{\mathrm{c}}(x, x_1)$ is the Green function of wave transformation in the coupling channel in the Fresnel approximation.

Without loss of generality, we present results for the modes with a low transverse index (longitudinal modes) for a cavity with Gaussian apertures *7* and *8*, which simplify calculations. Note that aperture *7* models the excitation region in the active medium, whereas aperture *8* takes into account all other elements bounding the field (see Fig. 1). Describing the action of cavity elements by the transformation matrix with elements A, B, C, and D, we find expressions for the Gaussian beams of modes of counterpropagating waves. Substituting them in (2), we obtain

$$T_{00} = 2\mathrm{i}\,\mathrm{Im}\frac{1}{q_0}\left\{B_{\mathrm{s}}\left[\left(\frac{1}{q_0}+\frac{A_{\mathrm{s}}}{B_{\mathrm{s}}}\right)^2-\left(\frac{1}{B_{\mathrm{s}}}\right)^2\right]\right\}^{-1}, \tag{3}$$

where

$$\frac{1}{q_0} = \pm\frac{\sqrt{(A+D)^2-4}}{2B};$$

and A_{s}, B_{s} are elements of the transformation matrix of the coupling channel. The sign of the root is chosen from the condition that the field at infinity be equal to zero. (In the derivation of (3), the normalisation factor W_0, on which the coupling coefficient depends, corresponded to the unit mode energy.)

Recall that fields may differ in phase and amplitude distributions. Coefficients (2) and (3) characterise the degree of matching of these distributions, which increases with increasing $|T_{nn}|$ in accordance with the physical sense of the processes under study. From this point of view, three regimes of laser operation are possible: (1) total matching in the case of $|T_{nn}|^2 = 1$, which is characterised by the maximum coupling of modes with the same transverse index; (2) complete mismatch in the case of $|T_{nn}|^2 = 0$ and $|T_{mn}|^2 \neq 0$ for $n \neq m$, when coupling is observed only for modes with different indices (this regime is contrary to point 1); (3) intermediate regime.

By using expression (3), we analysed in detail the dependences of $|T_{00}|^2$ on different parameters entering in expressions for the matrix elements (A, B, C, D, A_{s}, B_{s}, C_{s}, D_{s}), which are determined in the standard way [12] in accordance with a laser scheme (Fig. 1).

Among these parameters are the coefficient M characterising the field expansion by the prism *3*, the spacing L between lenses *2* (by varying L, M, and the configuration parameter g, we were able to realise all types of stable and unstable cavities), the width a of aperture *8*, the power P of the lens in the feedback loop, its position, etc.

Our analysis showed that the largest information content was given by the dependences of $|T_{00}|^2$ on P for different fixed g (the remaining parameters were chosen the same as in the experimental setup in Fig. 1). Fig. 3 illustrates the evolution of the form of these dependences with changing g,

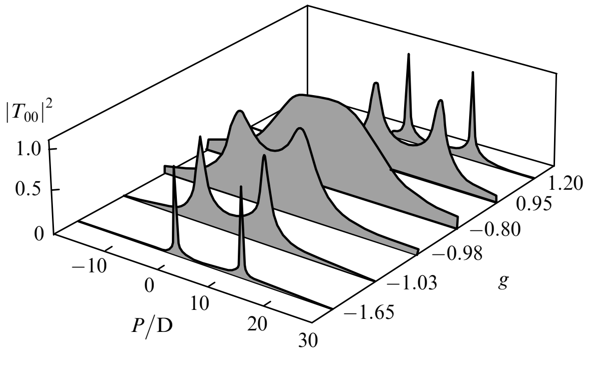

Figure 3. Dependences of $|T_{00}|^2$ on the lens power P of the lens in the coupling channel for different g.

and Fig. 4 compares the results of the modified calculation (solid lines) with the results obtained by using relations from Ref. [14] (dashed lines). One can see that the refinement of the calculation method caused not only quantitative, but also substantial qualitative changes. For stable cavities, both methods give wide matching regions, which differ insignificantly ($g = -0.42$, Fig. 4c). Distinctions (in the form of regions and their overlap) increase on approaching the boundary of the stability region ($g = -1.03$, Fig. 4b). In the instability domain ($g = -1.65$, Fig. 4a), the regions are transformed into narrow peaks, whose numbers and positions are different according to (3) (two peaks at $P \simeq 0$ and 12) and relations from Ref. [14] (one peak at $P \simeq 6$).

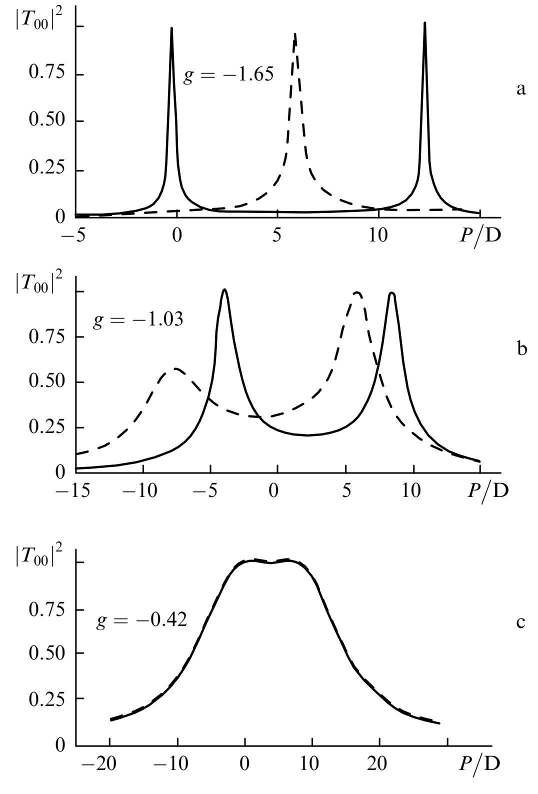

Figure 4. Comparison of the methods used for calculating $|T_{00}|^2$ as a function of the lens power P in the coupling channel for different g. The dashed lines illustrate the modified calculation method, and the solid lines present the results of calculations by the formulas from [14].

4. Distortions of the spectrum and the optimum mode of laser operation

The modification of the calculation led to the appearance of a narrow peak at $P \simeq 0$ (0.25 D wide for $a = 4$ mm and 0.15 D for $a = 6$ mm) in the dependence of $|T_{00}|^2$ on P. Owing to the presence of this peak, changes in the optical strength P_s of weak lenses in a flame with wavelength cause strong variations in the coupling coefficient, which distort the spectrum. The mechanism is as follows. When $\Delta = L/2 - f = 0$ (f is the focal length of lens 2), the dependence of $|T_{00}|^2$ on P is symmetric about the point $P = 0$. Small deviations Δ (300 μm for $a = 4$ mm and 150 μm for $a = 6$ mm), which are inevitably present in real systems, shift the point $P = 0$ to the wing of the peak. This disturbs the line symmetry (like in Fig. 2b) because a change in the sign of P in different line wings leads to different $|T_{00}|^2$.

Fig. 5a presents the spectra calculated in the vicinity of the absorption lines of the flame in which a lens with the power $P_s \approx \pm 0.15$ D is formed (the radius of curvature of the surface is $R \sim 7$ mm, the equivalent lens thickness is $b \sim 0.5$ mm, and the logarithmic absorption coefficient at the maximum $\sigma \sim 5$). The results are presented for $a = 4$ mm and different Δ. The dependence for $\Delta = 320$ μm quantitatively and qualitatively agrees in the character of distortions with the experimental curve (Fig. 2b).

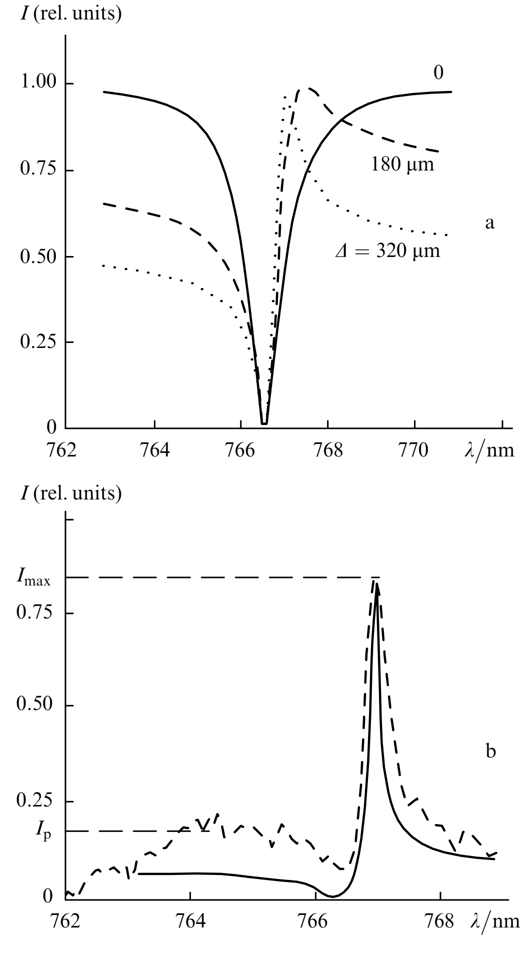

Figure 5. Calculated spectra in the vicinity of absorption lines (a) and the comparison of the calculated spectrum (solid curve) with the experimental spectrum (dashed curve) for the position of the point $P = 0$ on the pedestal of the coupling peak and $\Delta = 800$ μm (b).

Additional arguments in favour of the given mechanism are the results of control experiments. In the first experiment, we observed only undistorted spectra when an additional lens with power $P_d = 20\ \mathrm{cm}^{-1}$ was placed in the feedback loop (the working point was strongly displaced from the point $P = 0$). In the second experiment, we introduced a small additional shift Δ and displaced the point $P = 0$ further on the pedestal of the coupling peak. In this case, we observed stronger distortions in some spectra (see Fig. 5b). For comparison, we also show the calculated spectrum (solid line) for $a = 4$ mm and a lens in the flame with $\sigma \sim 25$, $R = 5$ mm, and $b = 0.4$ mm.

The experimental spectrum (Fig. 5b) simultaneously gives an answer to the question of principle importance: What profit is obtained by matching the cavity? The maximum intensity $I_{\max}$ corresponds to the total matching, and the intensity $I_{\rm p}$ in the other wing corresponds to the total mismatch. The intensity ratio $I_{\max}/I_{\rm p}$ is about 4–5. This result is extremely important for practice because it is very difficult or even impossible to estimate theoretically in our case the laser efficiency corresponding to $I_{\rm p}$ in regimes *2* and *3* (with mismatched modes).

It is clear from the above results that lasing in the regime of total matching is optimum for the synthesis of the emission spectrum in the laser (Fig. 1), i.e., one can see from Figs 3 and 4c that the operation in the stability region (it is realised by the choice of $\varDelta$ and, if necessary, $P_{\rm d}$) provides not only the absence of distortions, but also an advantage in energy by a factor of 8–10 (this regime eliminates the loss through mismatch and decreases the cavity loss by 40–50 %). Note that one can obtain a high quality of the spatial structure of a laser beam, i.e., a low divergence, a high degree of focusing, etc., only in this regime.

5. Conclusions

The laser proposed here provides the efficient generation of emission with rather complex synthesised spectra. Simple spectral components are formed there by active methods (using dispersion elements with spatial-angular modulators), and complex elements (fronts, dips, internal structure), which cannot be formed by other methods, are formed by real absorption lines.

The mechanism responsible for distortions of the spectrum is associated with the nature of mismatch in the unstable region, so that there exists the operation mode of a laser in which the maximum laser efficiency is achieved for the maximum similarity of spectra of the laser and the filter used in the coupling channel.

We found an advantage of the matching regime, which is of practical importance for numerous systems with ring cavities in which the coupling between counterpropagating waves is used for controlling their parameters or is present as an inevitable factor. This result is important not only for lasers with ring cavities, but also for other systems with open cavities because a quantitative theoretical analysis of a strongly mismatched regime is practically impossible.

Our results theoretically and experimentally show that accounting for the non-Hermitian nature of operators of such systems leads not only to corrections, but also to other results. Thus, the modes with orthogonality (1) are a physical reality and not a mathematical abstraction because they are responsible for the appearance of new effects.

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