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Regenerative regime in optomechanical transducer with modulated pump

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***Abstract*. The problem of detection of weak classical forces by an optomechanical transducer with modulated pump is considered. For increasing the sensitivity of the scheme, it is proposed to realise the parametric regeneration of the optical cavity by introducing an additional quadratic nonlinear element driven at the doubled frequency. The ultimate sensitivity of such a transducer is estimated, and the prospects of its practical realisation are discussed.**

Keywords: *optomechanical transducers, detection of classical forces, modulated pump, parametric regeneration.*

1. Introduction

The problem of detection of weak perturbations of mechanical systems that have vanishingly small dissipation is extremely important for experiments with test bodies. Usually, an electromechanical [1 – 3] or optomechanical [4 – 7] transducer is used to detect classical forces. The optomechanical transducer has a higher potential for increasing the sensitivity because in the optical region the noise in the transducer is quantum and therefore of fundamental nature, whereas in the UHF and even more so in the radio frequency range, the noise is mostly thermal, with the energy $\hbar\omega$ of a quantum being orders of magnitude lower than the energy kT of thermal motion.

In the traditional phase-insensitive regime, electrodynamic transducers can detect external perturbations whose amplitude is not below the standard quantum limit [1, 2],

$$F_{\mathrm{sql}} = (\zeta/\hat{\tau})(M\hbar\omega_{\mu})^{1/2}, \tag{1}$$

where $\hat{\tau}$ is the duration of the external perturbation (force); M and ω_{μ} are the mass and frequency of the mechanic oscillator, respectively; and ζ is a parameter of the order of a few units. According to the postulates of quantum mechanics, a classical variable (force) can be measured with arbitrary precision. Therefore, the phase-insensitive regime of the optomechanical transducer is not optimal for detecting a classical external force.

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To improve the sensitivity of electrodynamic transducers, authors [1, 2] proposed to use a modulated (two-component) pump of the form

$$E(t) = E_{\mathrm{las}}\cos(\omega_{\mu}t + \varphi)\cos\omega_0 t = \frac{E_{\mathrm{las}}}{2}\{\cos[(\omega_0 + \omega_{\mu})t + \varphi] + \cos[(\omega_0 - \omega_{\mu})t - \varphi]\}, \tag{2}$$

where ω_0 is the frequency of the optical (electrical) circuit. This pump consists of two modes tuned to the both sides of the resonance curve of the high-frequency circuit. A constant phase shift φ, appearing in expression (2), was introduced to compensate for the retardation of the response of the circuit with respect to the driving force for the two modes. A pump of this form leads to phase-sensitive detection of the external perturbation because only one quadrature of the external force, determined by the form of the pump, is measured, which enhances the sensitivity of the system. In this case, the minimum detectable amplitude of the external force is given by [1, 2, 4]

$$F_0 \geqslant F_{\mathrm{sql}}\left(\frac{\delta_{\mathrm{e}}}{2\omega_{\mu}}\right)^{1/2}, \tag{3}$$

where δ_{e} is the width of the resonance curve of the optical (electrical) circuit. The sensitivity is increased at the expense of the capability to reconstruct the shape of the external force since the information about one of the quadrature components is lost in the measurement process. (At least two identical systems detecting different quadrature components of the external force are needed for the exact reconstruction of its shape.)

According to expression (3), the potential for increasing the sensitivity is determined by the $\delta_{\mathrm{e}}/2\omega_{\mu}$ factor. In the case of electromechanical transducers, one can make the decay rate of the high-frequency circuit, δ_{e}, several orders of magnitude smaller than the resonance frequency of the mechanical oscillator, ω_{μ}. In the optical region, in contrast, it is extremely difficult to make δ_{e} much less than ω_{μ} for technical reasons [6]. Even for the best available mirrors with reflection coefficient $r \approx 1 - 10^{-6}$, the decay rate δ_{e} is comparable to ω_{μ}. In this connection, alternative methods for increasing the sensitivity of optomechanical transducers with modulated pump are of interest.

The sensitivity of schemes with two-component pump is limited by the noise at frequencies $\omega_0 \pm 2\omega_{\mu}$ that the transducer introduces into the mechanical system. According to Eqn. (3), the improvement factor is exactly the degree to which this noise is suppressed by the narrow-band resonance curve of the high-frequency circuit. Therefore, by decreasing somehow the decay rate of the high-frequency circuit (e.g., with the help of parametric regeneration), we can improve the sensitivity of the setup to external forces.

The purpose of this work is to analyse the sensitivity of an optomechanical (electromechanical) transducer with modulated pump that operates in the regenerative regime (below the generation threshold) for detecting an external classical force acting on a high Q mechanical oscillator.

For the definiteness, we will perform the analysis for the case of optomechanical transducers, although all conclusions and results apply equally to electromechanical transducers with correspondingly adjusted parameters.

2. Model and basic equations

Fig. 1 shows the scheme of the measuring system. The optomechanical transducer includes a high Q oscillator with the resonance frequency ω_μ, decay rate δ_μ, and mass M. The movable mirror *1* of the Fabri – Perot interferometer is attached to the oscillator, whereas mirror *2* of the optical cavity is rigidly fixed [6]. The mechanical oscillator experiences the force to be measured, F_s, as well as a Nyquist force, F_μ, characterising the thermal oscillation of the oscillator within the Langevin approach.

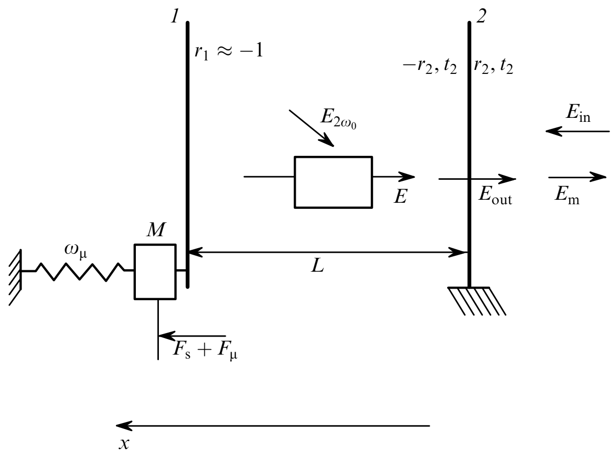

Figure 1. Scheme of the measuring system: (*1*, *2*) mirrors of the Fabri – Perot interferometer, (*3*) nonlinear optical element.

For simplicity, we assume that mirror *1* is almost perfectly reflecting and the interferometer is pumped through the unmoveable mirror *2*. To realise the parametric regeneration, an optical element with quadratic nonlinearity, driven by electromagnetic field at frequency $2\omega_0$, is placed inside the cavity. For determinacy, we assume that, for a complex field amplitude, the coefficient of reflection from the outer surface of mirror *2* is positive, $r_2 > 0$, whereas the coefficient of reflection from the inner surface is negative and equal to $-r_2$. The transmission coefficient, t_2, is then a positive quantity [8]. Obviously, this case corresponds to a high-finesse optical cavity in which the inner surfaces of mirrors are covered with multilayer dielectric coatings and the outer surface of mirror *2* is covered with an antireflection coating.

Suppose that the field $E_{\rm in}(t)$ incident on the system has the form

$$E_{\rm in}(t) = E'_c(t)\cos\omega_0 t - E'_s(t)\sin\omega_0 t, \tag{4}$$

where $E'_c(t)$ and $E'_s(t)$ are the quadrature components. Within the single-mode approximation, the field $E_i(t)$ inside the interferometer (a travelling wave) satisfies the following equation [9]:

$$\ddot{E}_i(t) + 2\delta_e\dot{E}_i(t) + \omega_1^2(1 + m\sin 2\omega_0 t)E_i(t)$$
$$= 2\omega_0 t_2 \tau_i^{-1}[-E'_s(t)\cos\omega_0 t + E'_c(t)\sin\omega_0 t]. \tag{5}$$

Here, $m \ll 1$ is the degree of modulation, determined by the characteristics of the quadratic nonlinear optical element and the driving field of the doubled frequency; $\tau_i = 2L/c$ is the round-trip time of the optical cavity of length L; c is the speed of light; $\delta_e = (1 - r_2)\tau_i^{-1} = c(1 - r_2)/(2L)$ is the decay rate of the cavity (assuming that $r_2 \approx 1$); $\omega_0 = n\pi c/L$ is the resonance frequency of the Fabri – Perot interferometer with mirror *1* fixed in the equilibrium position (the excursion $x = 0$); and $\omega_1 = n\pi c/(L + x) \simeq \omega_0(1 - x/L)$ is the slowly varying frequency of the interferometer for moving mirror *1* (the ratio $x/L \ll 1$). In experiment, one can realise the degeneration of the optical cavity using, for example, the scheme of Ref. [10].

The field entering the measuring device (a homodyne detector) is given by

$$E_m(t) = t_2 E_i(t) + E_{\rm in}(t), \tag{6}$$

where the first term in the right-hand side is the radiation decoupled from the cavity, whereas the second term arises from the direct reflection of the incident field from mirror *2* (again, we assume $r_2 \approx 1$).

Suppose that the emission of the pumping laser is of the form (2). Then, the quadrature components, $E'_c(t)$ and $E'_s(t)$, read

$$E'_c(t) = E_{\rm las}\cos(\omega_\mu t + \varphi) + E_c(t), \quad E'_s(t) = E_s(t), \tag{7}$$

where operators $E_c(t)$ and $E_s(t)$ describe the quantum fluctuations of the quadrature components of the field that is incident on the interferometer. If this field is in a coherent state, $E_c(\omega)$ and $E_s(\omega)$ are uncorrelated, and their spectral densities are given by [11, 12]

$$\langle|E_c^2(\omega)|\rangle = \langle|E_s^2(\omega)|\rangle = N_0, \tag{8}$$

where $N_0 = \pi\hbar\omega_0/(cS)$; S is the cross section of the pumping beam. For simplicity, we neglect the excess noise of the emission of the pumping laser and the proper noise of the nonlinear element in the following.

The field equation (5) should be complemented by the equation of motion of the mechanical oscillator,

$$\ddot{x} + 2\delta_\mu\dot{x} + \omega_\mu^2 x = \frac{F_{\rm pr}}{M} + \frac{F_s}{M} + \frac{F_\mu}{M} = f_{\rm pr} + f_s + f_\mu, \tag{9}$$

where

$$F_{\rm pr} = \frac{SE_i^2(1 + r_2)}{4\pi} \simeq \frac{SE_i^2}{2\pi} \tag{10}$$

is the light pressure that the intracavity field exerts on mirror *1*.

In a coherent state, the fluctuations of the quadrature components of the field, $E_c(t)$ and $E_s(t)$, are usually much smaller than the oscillation amplitude of the pumping laser, E_{las}, allowing us to linearise equations of motion (5), (9), and (10) in the fluctuations.

The driving field inside the interferometer satisfies equation

$$\ddot{E}_p(t) + 2\delta_e\dot{E}_p(t) + \omega_0^2(1 + m\sin 2\omega_0 t)E_p(t) = 2\omega_0 t_2\tau_i^{-1}E_{las}\cos(\omega_\mu t + \varphi)\sin\omega_0 t. \quad (11)$$

The field of the induced oscillation is then given by

$$E_p(t) = \frac{-t_2E_{las}}{\tau_i[(\delta_e - m\omega_0/4)^2 + \omega_\mu^2]^{1/2}} \times\cos(\omega_\mu t + \varphi + \vartheta)\cos\omega_0 t, \quad (12)$$

where $\vartheta = \arctan[\omega_\mu/(\delta_e - m\omega_0/4)]$ is the retardation of the response (the intracavity field) with respect to the driving force (the pumping field). Choosing for determinacy $\varphi + \vartheta = 0$, we obtain the field of induced oscillations inside the interferometer

$$E_p(t) = \frac{-t_2E_{las}}{\tau_i[(\delta_e - m\omega_0/4)^2 + \omega_\mu^2]^{1/2}}\cos\omega_\mu t\cos\omega_0 t = E_0\cos\omega_\mu t\cos\omega_0 t. \quad (13)$$

Linearising expressions (5), (9), and (10), we finally arrive at the following system of equations ($E = E_i - E_p$):

$$\begin{aligned}
&\ddot{E}(t) + 2\delta_e\dot{E}(t) + \omega_0^2(1 + m\sin 2\omega_0 t)E(t) \\
&\quad = -2\omega_0^2 xL^{-1}E_0\cos\omega_\mu t\cos\omega_0 t \\
&\quad + 2\omega_0 t_2\tau_i^{-1}(E_s(t)\cos\omega_0 t + E_c(t)\sin\omega_0 t), \\
&\ddot{x} + 2\delta_\mu\dot{x} + \omega_\mu^2 x = -SE_0(\pi M)^{-1}\cos\omega_\mu t\,\overline{E(t)\cos\omega_0 t} \\
&\quad + f_s(t) + f_\mu(t), \\
&E_m(t) = t_2E_i(t) + E_{in}(t),
\end{aligned} \quad (14)$$

where the line over expression $E(t)\cos\omega_0 t$ denotes time averaging.

3. Sensitivity of optomechanical transducer

To solve system (14), we will use the method of slowly varying amplitudes. Suppose that the field $E(t)$ inside the interferometer can be written as

$$E(t) = E_1(t)\cos\omega_0 t - E_2(t)\sin\omega_0 t, \quad (15)$$

where E_1 and E_2 are the quadrature components of the field. Then, according to Eqn. (14) the optomechanical transducer is described by the following reduced equations of motion:

$$\begin{aligned}
&\dot{E}_1(t) + \delta_1 E_1(t) = -t_2\tau_i^{-1}E_c(t), \\
&\dot{E}_2(t) + \delta_2 E_2(t) = -t_2\tau_i^{-1}E_s(t) + \omega_0 L^{-1}E_0 x\cos\omega_\mu t, \\
&\ddot{x} + 2\delta_\mu\dot{x} + \omega_\mu^2 x = -SE_0(2\pi M)^{-1}E_1\cos\omega_\mu t + f_s + f_\mu,
\end{aligned} \quad (16)$$

with $\delta_1 = \delta_e - m\omega_0/4$ and $\delta_2 = \delta_e + m\omega_0/4$. As seen from equations (16), the equivalent decay rate is different for the two quadrature components of the field inside the interferometer, which is a consequence of the parametric regeneration of the optical cavity. The information about the x-coordinate of the mechanical oscillator is acquired by component E_2 of the field inside the interferometer (and, accordingly, the same quadrature component of the output field), while component E_1 introduces fluctuations into the mechanical system.

Suppose that the signal, $f(t)$, has the form:

$$f(t) = f_0\sin\omega_\mu t, \quad 0 \leqslant t \leqslant \hat{\tau}, \quad \omega_\mu\hat{\tau} \geqslant 1 \quad (17)$$

and $f(t) = 0$ at all other instants of time. Changing over to the frequency representation, we derive the following equations for the coordinate of the mechanical oscillator and the quadrature components of the field, E_1 and E_2, from system (16):

$$\begin{aligned}
&E_1(\omega) = -t_2\tau_i^{-1}E_c(\omega)(\delta_1 - i\omega)^{-1}, \\
&E_2(\omega) = -t_2\tau_i^{-1}E_s(\omega)(\delta_2 - i\omega)^{-1} \\
&\quad + \omega_0 L^{-1}E_0(\delta_1 - i\omega)^{-1}[x(\omega + \omega_\mu) + x(\omega - \omega_\mu)], \\
&x(\omega) = G(\omega)\Bigg\{SE_0 t_2(4\pi M\tau_i)^{-1} \\
&\quad \times\left[\frac{E_c(\omega - \omega_\mu)}{\delta_1 - i(\omega - \omega_\mu)} + \frac{E_c(\omega + \omega_\mu)}{\delta_1 - i(\omega + \omega_\mu)}\right] + f_s(\omega) + f_\mu(\omega)\Bigg\},
\end{aligned} \quad (18)$$

where $G(\omega) = (\omega_\mu^2 - \omega^2 - 2i\delta_\mu\omega)^{-1}$ is the transfer function of the mechanical oscillator. Then, inserting the expression for $x(\omega)$ from the third equation of system (18) to the second equation, and making use of relation (6), we obtain the following expression for the sine quadrature component of the measured field:

$$\begin{aligned}
E_{m2}(\omega) &= -E_s(\omega)\frac{\delta_1 + i\omega}{\delta_2 - i\omega} + \frac{S\omega_0E_0^2\delta_e}{2\pi ML(\delta_2 - i\omega)} \\
&\quad \times\Bigg\{\frac{E_c(\omega)}{\delta_1 - i\omega}[G(\omega + \omega_\mu) + G(\omega - \omega_\mu)] + \\
&\quad + \frac{G(\omega-\omega_\mu)}{\delta_1 - i(\omega-2\omega_\mu)}E_c(\omega-2\omega_\mu) + \frac{G(\omega+\omega_\mu)}{\delta_1 - i(\omega+2\omega_\mu)}E_c(\omega+2\omega_\mu)\Bigg\} \\
&\quad + \frac{\omega_0E_0t_2}{L(\delta_2 - i\omega)}\big\{[f_s(\omega + \omega_\mu) + f_\mu(\omega + \omega_\mu)]G(\omega + \omega_\mu) \\
&\quad + [f_s(\omega - \omega_\mu) + f_\mu(\omega - \omega_\mu)]G(\omega - \omega_\mu)\big\}.
\end{aligned} \quad (19)$$

Suppose that the measurement employs the low-frequency part, $\omega \ll \omega_\mu$, of the spectrum of quadrature component E_{m2}. For simplicity, we will also assume that the decay rate, δ_μ, of the mechanical system is vanishingly small, so that we can neglect the terms containing δ_μ as well as the fluctuation forces, f_μ, in the equations. Then, simple transformations of expression (19) yield the modulus squared of the spectrum of quadrature component E_{m2},

$$|E^{s}_{m2}(\omega)|^2 = \left(\frac{\omega_0 E_0 t_2 f_0 \hat{\tau}}{4\pi L \omega_\mu \omega}\right)^2 \left(\delta_2^2 + \omega^2\right)^{-1}, \tag{20}$$

as well as the spectral density of noise in E_{m2},

$$\langle |E^{n}_{m2}(\omega)|^2\rangle = N_0 \left\{ \frac{\delta_1^2 + \omega^2}{\delta_2^2 + \omega^2} + \frac{S^2 \omega_0^2 E_0^4 \delta_e^2}{16\pi^2 M^2 L^2 \omega_\mu^2 (\delta_2^2 + \omega^2)} \right.$$
$$\left. \times \left[\omega_\mu^{-2}\left(\delta_1^2 + \omega^2\right)^{-1} + 2\omega^{-2}\left(\delta_1^2 + 4\omega_\mu^2\right)^{-1}\right]\right\}. \tag{21}$$

The appearance of frequency ω in the denominators of expressions (20) and (21) does not mean that the expressions diverge as $\omega \to 0$: For simplicity, we have omitted the terms containing δ_μ, which limit the increase in the signal and noise intensities at low frequencies.

Let us define the signal-to-noise ratio μ in the usual way [13]:

$$\mu = \pi^{-1} \int_0^\infty |E^{s}_{m2}(\omega)|^2 d\omega / \langle |E^{n}_{m2}(\omega)|^2 \rangle. \tag{22}$$

Then, inserting expressions (20) and (21) in Eqn (22), we obtain

$$\mu = \left(\frac{\omega_0 E_0 t_2 f_0 \hat{\tau}}{4\pi L \omega_\mu}\right)^2 (N_0 \pi)^{-1}$$
$$\times \int_0^\infty d\omega \left\{ \omega^4 + \left[\delta_1^2 + \frac{\alpha}{(\delta_1^2 + \omega^2)\omega_\mu^2}\right]\omega^2 + \frac{2\alpha}{\delta_1^2 + 4\omega_\mu^2} \right\}^{-1}, \tag{23}$$

where we introduce the simplified notation $\alpha = [S\omega_0 E_0^2 \times \delta_e/(4\pi M L \omega_\mu)]^2$. One can easily see that the current frequency cancels out in the denominators.

In the general case, the signal-to-noise ratio (23) cannot be calculated analytically. Note, however, that μ increases with decreasing amplitude E_0 of the pump (decreasing parameter α). In addition, a decrease in E_0 narrows the filtering band. Therefore, it is natural to assume that the filtering band is narrower than δ_1 (we will verify this condition in the following) or, in other words, we can choose the principal value of the integral in the expression for μ from near-zero frequencies, $\omega \leqslant \delta_1$. In this case, we can neglect ω^2 with respect to δ_1^2 in the denominator of the expression in square brackets in Eqn (23). (The resulting value of μ is smaller than the actual one and therefore provides a lower bound on the signal-to-noise ratio).

The expression for μ then becomes a standard integral, yielding

$$\mu = \left[\frac{\omega_0 E_0 t_2 f_0 \hat{\tau}}{4\pi L \omega_\mu}\right]^2 (4N_0)^{-1} \left\{ \frac{\alpha}{\delta_1^2 + 4\omega_\mu^2} \left[\delta_1^2 + \frac{\alpha}{\delta_1^2 \omega_\mu^2} \right.\right.$$
$$\left.\left. + \left(\frac{2\alpha}{\delta_1^2 + 4\omega_\mu^2}\right)^{1/2}\right]\right\}^{-1/2}. \tag{24}$$

The structure of expression (24) shows explicitly that with increasing amplitude of the pump, E_0, the signal-to-noise ratio first increases and then remains virtually constant. This takes place at

$$\alpha \leqslant \delta_1^4 \omega_\mu^2. \tag{25}$$

Therefore, optimising the pump in accordance with condition (25), we find the maximum signal-to-noise ratio to be

$$\mu = \mu_{sql}\left(1 + \frac{4\omega_\mu^2}{\delta_1^2}\right)^{1/2} = \mu_{sql}\left[1 + \frac{4\omega_\mu^2}{(\delta_e - m\omega_0/4)^2}\right]^{1/2}, \tag{26}$$

where μ_{sql} is the signal-to-noise ratio for the measurements at the level of the standard quantum limit (1). Thus, parametric regeneration in an optomechanical detector with modulated pump can indeed enhance the signal-to-noise ratio, and the stronger is the regeneration, the smaller is the minimum detectable amplitude of the force: $F_0 = F_{sql}\delta_1/(2\omega_\mu) = F_{sql}(1-R)\delta_e/(2\omega_\mu)$ with $R = m\omega_0/(4\delta_e)$ the coefficient of regeneration.

Interestingly, for the amplitude of the pump given by expression (25) one can simultaneously realise a near-maximal signal-to-noise ratio μ and the broadest filtering band. Indeed, the filtering band, $\Delta\omega_f$, is defined as the frequency span between the points where the integrand reaches a certain fraction of its maximum (one half, for example). Then, expressions (23) and (25) yield

$$\Delta\omega_f^2 = 2\alpha\omega_\mu^2\delta_1^2\left[\left(\delta_1^2 + 4\omega_\mu^2\right)\left(\alpha + \delta_1^4\omega_\mu^2\right)\right]^{-1}. \tag{27}$$

As parameter α (pumping amplitude E_0) increases to approximately $\delta_1^4\omega_\mu^2$, the filtering band $\Delta\omega_f$ first broadens and then saturates at $\delta_1/2$ with $\delta_1 \ll \omega_\mu$, confirming the assumption made during the estimation of the signal-to-noise ratio (23). All other things being equal, the maximisation of the filtering band is necessary to reduce the influence of the thermal noise F_μ, which was neglected in the present analysis for simplicity. Thus, the optimal amplitude of the pump is determined by the equality in expression (25).

4. Discussion

The practical realisation of the proposed method for increasing the sensitivity of the optomechanical transducer involves the problem of suppressing the excess noise rather than that of attaining the required degree of regeneration, R, of the optical cavity. The experimental realisation of optical parametric oscillators [14] leaves no doubts that $R = 1$ can be reached, and formally as $R \to 1$, the gain in the sensitivity can become arbitrarily large, as seen from expression (26).

Note, however, that our scheme requires not only to increase coefficient R to unity, but also to preserve the minimum possible level of various types of excess noise, which can arise, for example, from a mismatch between the wave fronts of the optical beams, nonoptimal phase matching, various losses in the system etc. Similar difficulties arise in the problem of generating squeezed states of light (a decrease in the noise dispersion of one quadrature field component at the expense of the other) [10, 15]. Although there are no fundamental limits on the degree of squeezing, only relatively weak squeezing has been realised in experiment because of the limitations of the presently available technology.

To estimate the sensitivity gain that is feasible with the presently available technology, it is reasonable to use the results of experiments on the generation of squeezed states of light. For the parameters of the optical scheme that was used in Ref. [10] (the interferometer pumped at 532 nm, the $LiNbO_3$ nonlinear element pumped at 1064 nm, and the effective number of reflections in the cavity of the order of 100), we can expect a gain in the sensitivity of the order of a few decibel [10, 15]. Therefore, the margin is relatively small. However, one cannot ignore the fundamental importance of the experimental proof of the possibility to detect a classical force with a sensitivity that surpasses the standard quantum limit (1), as this problem has not been solved so far.

Note that the parametric regeneration in an optomechanical transducer with modulated pump is in some sense similar the situation when squeezed light, which can come, for example, from a degenerate parametric amplifier [10], is used to pump an optical cavity [16]. Indeed, in this work we analyse the case when nonclassical (squeezed) states of the field are automatically created inside the optical cavity of the transducer owing to the parametric regeneration. This may constitute an advantage of our scheme over the one considered in Ref. [16], since in our case there is no need to pass the fragile squeezed state through numerous optical elements of the setup. At the same time, the scheme with external squeezing is more flexible and, in principle, allows squeezing of different quadrature components at mixed frequencies ω_0 and $\omega_0 \pm 2\omega_\mu$, which further enhances the sensitivity [16]. The latter approach cannot be realised in the scheme with parametric regeneration because the same quadrature component is squeezed at frequencies ω_0 and $\omega_0 \pm 2\omega_\mu$ ($-E_2$ in our case, see Eqns (18) and (19)).

In conclusion, we note that the method proposed for increasing the sensitivity of an electrodynamic transducer can be especially interesting in the optical region, where the possibilities to decrease the decay rate of the cavity below the frequency of the mechanical oscillator are much limited.

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