

PACS numbers: 42.55.Wd; 42.60.Da; 42.60.Jf
DOI: 10.1070/QE2001v031n04ABEH001938

Multicore fibre laser phase locking by an external mirror

D V Vysotskii, A P Napartovich

***Abstract*. A theory of phase locking of radiation emitted by individual fibres of a multicore fibre laser with the help of an external mirror is developed. The Talbot effect in radiation from an annular set of emitters is described analytically. The theoretical results are compared with the results of experiments and numerical calculations. The method of collective mode selection in the multicore set with the help of a spherical mirror producing a nearly concentric cavity is analysed.**

Keywords: *fibre laser, phase locking, optical cavity, Talbot effect.*

1. Introduction

Modern technology makes it possible to produce optical multicore fibres (MCFs) containing a set of microcores circularly arranged [1] within the main fibre which serves as a waveguide for the pump radiation. Each microcore is a single-mode waveguide doped with Nd^{3+} ions. The absorption of diode pump radiation in such a structure is much more efficient than in a conventional fibre laser. The possibility of attaining a high output power easily suggests that MCF can be used as the basis for a compact fibre laser in which the set of microcores can operate in the regime of phase locking. We elaborated a mathematical program describing the propagation of radiation in the MCF to understand the mechanism of the development of supermodes in the set of microcores and to estimate the possibility of single-mode lasing of the entire set. The results of calculations of the field in the MCF excited by the radiation injection into one fibre are presented in [2].

The authors of [3] described an experiment on phase locking of radiation emitted by individual fibres in the MCF using the Talbot effect. Phase locking of radiation emitted by the fibre set was carried out using an external mirror placed at a certain distance from the output face of the MCF. The coincidence of the experimental data with the results of numerical calculations for the supermodes of the fibre set for certain distances between the mirror and the MCF confirmed at least partial phase locking of radiation emitted by the set of fibres.

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Received 16 November 2000
Kvantovaya Elektronika **31** (4) 298 – 302 (2001)
Translated by R Wadhwa

2. Analysis of the system consisting of the MCF and a plane mirror

Consider a cavity consisting of the MCF, containing N microfibres arranged along a circle of radius R_c, and of two plane mirrors. One of the mirrors, which is transparent for pump radiation and reflects laser radiation, is in contact with the end of the MCF, while the mirror opaque to pump radiation and semitransparent for laser radiation is at a distance L from the output end of the MCF (Fig. 1).

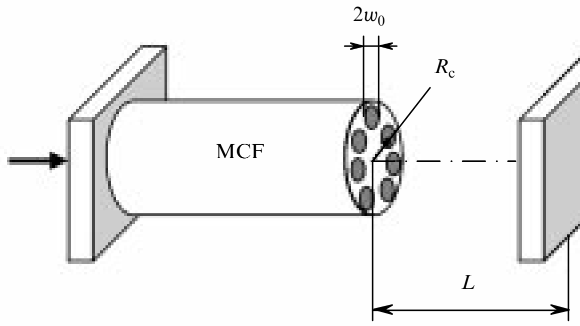

Figure 1. Schematic of the cavity.

To analyse phase locking of radiation from individual fibres of the MCF, we must consider the manifestation of the Talbot effect in the circular geometry. Some authors [4 – 8] applied the Talbot effect for phase locking of combinations of various types of lasers. In the given configuration, the classical Talbot effect is obviously absent. The annular set of emitters is similar, at short propagation lengths, to a certain extent to an infinite linear set, but diffraction in the radial direction leads to irreversible expansion of the region occupied by the field in this direction [9, 10]. In this connection, it is natural to expect that the self-reproducibility of the field distribution rapidly disappears when radiation propagates in empty space. However, after the diffracting field reaches the beam axis, further evolution of the field distribution cannot be analysed in analogy with a linear set of emitters. Our aim is to determine the conditions for a partial self-reproduction at various propagation lengths.

The simplest approach is to use the approximation in which the radiation emitted from the MCF is approximated by a combination of Gaussian beams having axes parallel to the axes of the waveguides and determined by the values of their phases. The transformation of Gaussian beams in empty space is well known: the normalised field amplitude distribution of a Gaussian beam after its propagation over a distance z from the waist can be written in the form

$$v_0(r,z)=\left(\frac{2}{w_0^2\pi}\right)^{1/2}\frac{1}{1+2\mathrm{i}z/(kw_0^2)}\exp\left[\frac{\mathrm{i}kr^2/(2z)}{1-\mathrm{i}kw_0^2/(2z)}\right], \quad (1)$$

where w_0 is the waist radius and k is the modulus of the wave vector of radiation in vacuum. The radiation field amplitude distribution at the MCF output and after reflection from the external mirror leading to the return to the plane of the output end can be written in the form

$$U_{\mathrm{out}}=\sum_{m=0}^{N-1}C_m v_0(|\boldsymbol{r}-\boldsymbol{R}_m|,0),$$
$$U_{\mathrm{in}}=\sum_{m=0}^{N-1}C_m v_0(|\boldsymbol{r}-\boldsymbol{R}_m|,2L). \quad (2)$$

Here, C_m is the amplitude of the Gaussian beam emitted by the mth microfibre; $\boldsymbol{r}$ are two-dimensional coordinates in the plane of the MCF end; $\boldsymbol{R}_m$ are the coordinates of the mth microfibre. The coupling coefficient of the radiation field arriving from the jth microfibre with the field in the l-th microfibre is equal to the projection of the incident beam intensity distribution on the initial Gaussian distribution:

$$M_{lj}=\int_{\boldsymbol{r}_i}U_{\mathrm{in}}(\boldsymbol{r},\boldsymbol{R}_j)U^*_{\mathrm{out}}(\boldsymbol{r},\boldsymbol{R}_l)\mathrm{d}\boldsymbol{r}$$
$$=\frac{-\mathrm{i}kw_0^2/(2L)}{1-\mathrm{i}kw_0^2/(2L)}\exp\left\{\frac{\mathrm{i}kR_{\mathrm{c}}^2}{L}\frac{\sin^2\left[\frac{\pi}{N}(l-j)\right]}{1-\mathrm{i}kw_0^2/(2L)}\right\}. \quad (3)$$

The expansion of the Gaussian beam due to diffraction is characterized by the quantity $kw_0^2/(2L)\equiv L_{\mathrm{R}}/L$, where L_{R} is the Rayleigh length of the beam. The coupling of the beams is characterised by the parameter kR_{c}^2/L. The amplitudes of Gaussian beams (C coefficients) are eigenvectors of the system of equations describing the condition for the reproduction of the microfibre field distribution during the round trip (MCF $\to$ plane mirror $\to$ MCF):

$$\gamma C_j=\sum_{l=1}^{N-1}M_{lj}C_l. \quad (4)$$

Here, γ is the eigenvalue of the mode of the cavity formed by the MCF and the external plane mirror. It follows from expression (3) that the matrix elements M_{lj} depend on the modulus of the difference between the microfibre numbers, i.e., $|l-j|$. Taking into account the periodicity condition for the coefficients C, we have the solution of system (4)

$$C_j^{(m)}=\exp(\pm 2\mathrm{i}jm\pi/N), \quad (5)$$

where m is the supermode number varying from zero to $(N-1)/2$ (N is assumed to be odd). It should be noted that all the supermodes of the MCF are degenerate (the eigenvalues of modes with numbers $+m$ and $-m$ are equal). The only exception is the synphase supermode of the set, for which all microfibres generate radiation with zero phase difference. In the case of an even N, the antiphase supermode for which the phase difference between adjacent waveguides is equal to π is also non-degenerate. However, this mode does not satisfy the periodicity condition $C_j^{(m)}(N)=C_j^{(m)}(0)$ for odd N.

The eigenvalues of supermodes can be obtained by substituting Eqn (5) into the system of equations (4):

$$\gamma_m=\sum_j M(j)\exp(2\mathrm{i}jm\pi/N). \quad (6)$$

In particular, for the synphase mode ($m=0$), we have

$$\gamma_0=\sum_j M(j)=N\frac{-\mathrm{i}L_{\mathrm{R}}/L}{1-\mathrm{i}L_{\mathrm{R}}/L}\exp\left(\frac{\mathrm{i}kR_{\mathrm{c}}^2}{2L}\frac{1}{1-\mathrm{i}L_{\mathrm{R}}/L}\right)$$
$$\times\sum_{l=-\infty}^{+\infty}(-\mathrm{i})^{Nl}J_{Nl}\left(\frac{kR_{\mathrm{c}}^2/2}{L-\mathrm{i}L_{\mathrm{R}}}\right), \quad (7)$$

where $J_{Nl}(z)$ is the Bessel function of the Nlth order. The estimate of the Rayleigh length for the beam emitted by a microfibre for $w_0=5$ μm, $R_{\mathrm{c}}=140$ μm and $N=61$ amounts approximately to 75 μm. On the other hand, the Rayleigh length for a beam with a waist radius of 140 μm is equal to 5.9 cm. With increasing L, the argument of Bessel functions in expression (7) decreases. For fairly large $L\geqslant L_N\approx kR_{\mathrm{c}}^2/(2N)\approx 1$ mm, the main contribution to the sum on the right-hand side of relation (7) comes from the term with the zeroth-order Bessel function. At such distances, we can write the following simple expression for the eigenvalue of the synphase mode:

$$\gamma_0=N\frac{-\mathrm{i}L_{\mathrm{R}}/L}{1-\mathrm{i}L_{\mathrm{R}}/L}\exp\left[\frac{\mathrm{i}kR_{\mathrm{c}}^2/(2L)}{1-\mathrm{i}L_{\mathrm{R}}/L}\right]J_0\left(\frac{kR_{\mathrm{c}}^2/2}{L-\mathrm{i}L_{\mathrm{R}}}\right). \quad (8)$$

In the limit of large distances, we have $J_0[kR_{\mathrm{c}}^2/(2L)\to 1$. After traversing the critical distance L_{cr}, when the argument of the Bessel function in Eqn (8) is equal to the first zero, the eigenvalue oscillations described by expression (8) disappear. In the range of distances from L_N to L_{cr}, the Bessel function can be replaced by an asymptotic expression with oscillations. At large distances $L>kR_{\mathrm{c}}^2/2$, the eigenvalue amplitude can be approximately written as $|\gamma_0|\approx NL_{\mathrm{R}}/L$.

For the experimental conditions described in [3], the propagation of a synphase mode in the MCF was calculated numerically using the method described in [2]. Numerical simulation makes it possible to determine the parameters of the emitted mode for the real profile of the refractive index. The obtained field distribution was taken as the initial distribution for calculating the propagation of radiation in vacuum. For each distance $2L$, the eigenvalue for the synphase mode was determined by projecting the distribution of the field being returned onto the distribution of the field generated by the MCF.

The application of the analytical model to the experiment in the actual MCF geometry gives rise to the problem of determining the parameters appearing in the approximation. Roughly speaking, we must determine the waist radius for the Gaussian beam imitating the actual field distribution in the microfibre. The eigenvalue was calculated by formula (7) as a function of the length of propagation in free space. The waist radius w_0 was used as a fitting parameter used for comparing the values of $|\gamma_0(L)|$ obtained analytically and numerically. It was found that the waist radius mainly determines the maxima in the dependence of the eigenvalue amplitude on $2L$. The result of matching the numerical calculations to the analytical model is presented in Fig. 2 ($w_0=4.5$ μm). One can see that the matching is almost perfect.

Formulas (3) and (6) lead to the following expression for the eigenvalue of the mth supermode:

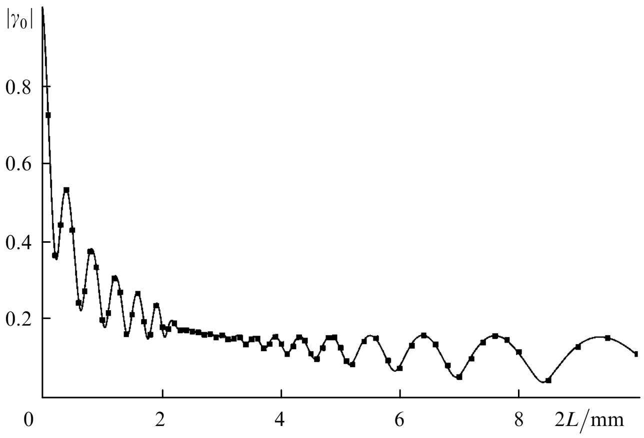

Figure 2. Dependence of the modulus of the synphase mode eigenvalue on the path length $2L$ calculated using formula (7); the results of numerical simulation are shown by squares.

$$\gamma_m = N \frac{-\mathrm{i}L_\mathrm{R}/L}{1-\mathrm{i}L_\mathrm{R}/L} \exp\left(\frac{\mathrm{i}kR_\mathrm{c}^2}{2L} \frac{1}{1-\mathrm{i}L_\mathrm{R}/L}\right)$$

$$\times \sum_{l=-\infty}^{+\infty} (-\mathrm{i})^{Nl+m} J_{Nl+m}\left(\frac{kR_\mathrm{c}^2}{2L - \mathrm{i}kw_0^2}\right), \tag{9}$$

This expression shows that the eigenvalues of modes with $m = 1, 2, \ldots$ exhibit similar behavior as functions of the propagation length. In the case of propagation over distances of the order of L_N and larger, only the term with $l = 0$ remains in the sum, and the corresponding Bessel function in the range $L_N < L < L_\mathrm{cr}$ can be replaced by its asymptotic expression. In particular, oscillations of Bessel's functions with even values of m occur in phase. Similarly, all functions with odd values of m oscillate synchronously with a $\pi/2$ shift relative to functions with even values of m.

In the case of independent lasing by individual microfibres, the feedback in the cavity is determined by the fraction of the diffracted beam power returning to the same microfibre. The corresponding eigenvalue is $\gamma_\mathrm{ind} = M(0) = |1 + \mathrm{i}L/L_\mathrm{R}|^{-1}$ for large distances to the mirrors. For large distances from the mirror, it is equal to $k\omega_0^2/L$; i.e., it is equal to the ratio of the squared waist radius to the squared radius of the diffracted beam after its propagation over a distance of $2L$.

The good agreement between the analytical and numerical approaches justifies the subsequent application of the analytical model. Fig. 3 shows the obtained dependences of the modulus of the supermode eigenvalues in the MCF on the propagation length $2L$ for the synphase mode, the mode with $m = 1$, and the 'nearly antiphase' mode (i.e., the closest to the antiphase mode forbidden in the given configuration) with $m = (N-1)/2 = 30$. The same figure shows for comparison the amplitude of the eigenvalue corresponding to the regime of independent lasing of microfibres as a function of the propagation length. Since the threshold amplification and the pumping power are proportional to $\ln(|\gamma|^{-1})$, the independent lasing regime for microfibres is virtually ruled out for a large distance from the mirror. A rapid decrease in $|\gamma_0(L)|$ with increasing distance can be attributed to a strong two-dimensional diffraction of radiation emitted by an individual Gaussian beam. The interference between various beams modifies the field distribution only in the azimuthal direction. In order to estimate the effect of interference, we may consider the situation when diffraction is allowed only in the radial direction ($\gamma_\mathrm{ind}(1\mathrm{D}) = [\gamma_\mathrm{ind}(2\mathrm{D})]^{1/2}$). Curve *5* in Fig. 3, which describes the dependence of $\gamma_\mathrm{ind}(1\mathrm{D})$ on the propagation length $2L$, is close to curve *1* for the 'nearly antiphase' mode ($m = (N-1)/2$) for $2L < 3$ mm and passes through the maxima of curve *2* for the synphase mode.

Fig. 3 illustrates the general features of the behavior of the eigenvalues of modes with small numbers ($m = 0, 1$): these values oscillate in the region between curves *4* and *5* with a period decreasing to zero for $L \sim L_N$ and increasing upon a further increase in L. An analysis of the field profiles at small path lengths indicates that the oscillations of γ are a manifestation of the Talbot effect. The eigenvalue of the 'nearly antiphase' mode also oscillates (although with a small amplitude) with a half as large period as in the theory of the Talbot effect for the 1D case.

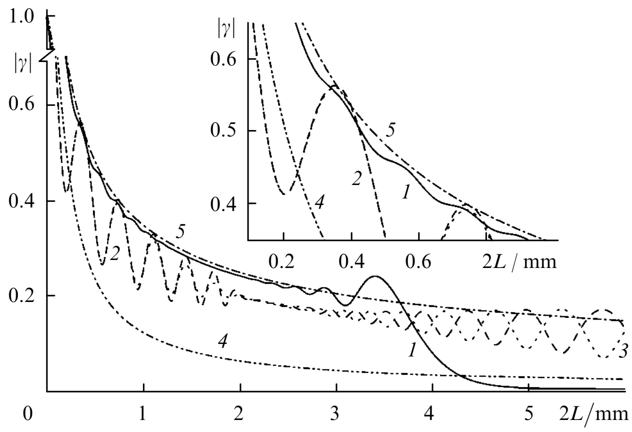

Figure 3. Eigenvalue moduli $|\gamma_m|$ for supermodes with $m = 30$ (*1*), 0 (*2*), and 1 (*3*) and the eigenvalue moduli $|\gamma_\mathrm{ind}|$ for unphased lasing of microfibres in the 2D (*4*) and 1D (*5*) cases as functions of the path length $2L$.

The modulation depth in the eigenvalue dependences is determined by the filling factor which in the given case is defined as the ratio of the microfibre diameter to the period of the set: $2\omega_0 N/(2\pi R_\mathrm{c}) > 0.6$. Such a large filling factor leads to a small modulation depth. The decrease in the period of self-reproduction of the field distribution in the case of the propagation in empty space differs from that in the 1D Talbot effect. At short distances, this period is virtually equal to the well-known Talbot length $L_\mathrm{T} = 2d^2/\lambda$, where d is the separation between microfibres. During the propagation in empty space, the radiation beams are efficiently displaced towards the axis, thus reducing the separation between the axes of individual beams, which leads to a decrease in the self-reproduction length.

For $L > L_N$, the behavior of the curves in Fig. 3 changes dramatically. According to our estimates, the radiation field for a certain path length reaches the axis of the system as a result of diffraction. The subsequent evolution of $|\gamma_m|$ for $m = 0, 1, 2, \ldots$ with increasing L is well described by expression (8) and its generalization to the case when $m \neq 0$. Oscillations with an increasing period are associated with the asymptotic behavior of the corresponding Bessel functions. The physical reason behind such a behavior of $|\gamma_m|$ is the propagation of the radiation from the MCF to the Fraunhofer diffraction region, leading to the formation of a system of concentric rings in the field distribution. These rings expand during the propagation of radiation so that the overlap integral of the system of rings with the ring of microcores oscillates. As regards the behavior of the mode with $m = 30$, the corresponding Bessel function $J_{N/2}[kR_c^2/(2L)]$ has a maximum near $L = 2L_N \approx 2$ mm and decreases rapidly to zero during the further propagation in proportion to $\sim [kR_c^2/(2L)]^{N/2}$.

The difference in the field distributions in the far-field zone for different supermodes is illustrated in Fig. 4, which shows the cross sections of these distributions for the synphase mode ($m = 0$), two adjacent modes ($m = 1, 2$), and the mode closest to the antiphase mode ($m = 30$). One can see that all the lower-order supermodes are characterized by the same order of the field distribution in the far-field zone. The only distinction of the synphase mode is the 'bright' peak at the axis. At the same time, the field distribution in the far-field zone for the antiphase mode ($m = 30$) is a narrow ring of radius $2\lambda/d \sim 38$ mrad, where $d = 2\pi R_c/N$.

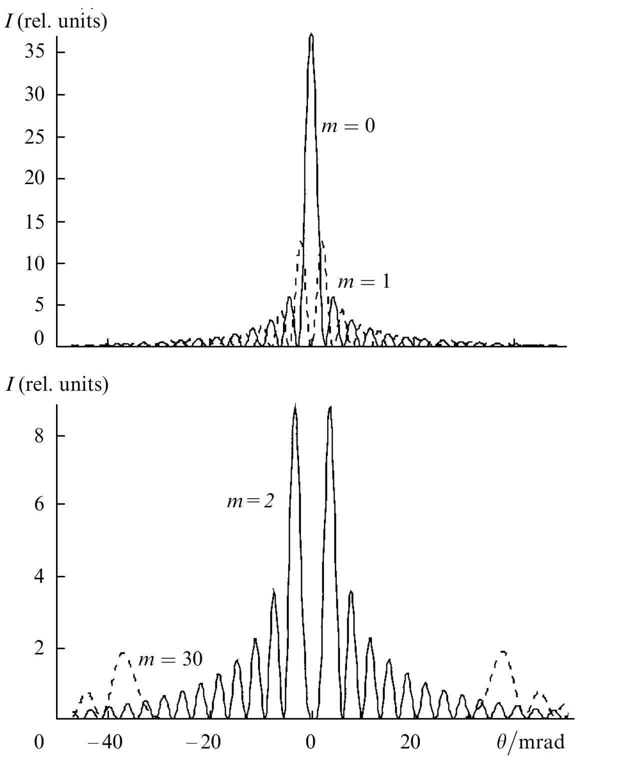

Figure 4. Cross sections of the intensity distribution I in the far-field zone for various supermodes of the MCF as functions of the observation angle θ.

Thus, we developed an analytical model of phasing of radiation emitted by individual fibres of the MCF by an external mirror, which describes the experimental results obtained in [3]. The perfect matching of the results of the model and exact numerical calculations of the radiation field propagation confirms the assumptions of the model. The experimentally observed generation of lower-order collective modes is in fact not a manifestation of the Talbot effect, but rather a consequence of better overlapping of the field returning to the MCF as compared to the case of unphased lasing. In this case, the selection of collective modes is achieved by incurring high diffraction losses.

3. Analysis of a system consisting of the MCF and a concave mirror

An analysis of a cavity with a plane external mirror shows that in order to suppress the independent lasing of individual waveguides, a considerable distance between the MCF and the mirror is required. The losses in the fundamental mode are large in this case. One of the ways to overcome this difficulty is to use a concave mirror with the radius of curvature R instead of the plane mirror. The expression for the coupling coefficients M_{lj} of the beams with the Gaussian approximation, which are emitted by an annular set of microfibres, can be written in the form

$$M(l-j) = \frac{-\exp[-\mathrm{i}kR_c^2/(AR)]}{1+\mathrm{i}L/L_R} \times \exp\left[\frac{\mathrm{i}kR_c^2}{AL}\frac{\sin^2\left(\frac{\pi}{N}(l-j)\right)}{1-\mathrm{i}L_R/L}\right], \tag{10}$$

where $A = 1 - (L - \mathrm{i}L_R)/R$. Substituting expression (10) into formula (6) for each supermode, we can again construct the dependence of the corresponding eigenvalue on the distance to the mirror. Fig. 5 shows the dependences of eigenvalues on L for the mirror radius $R = 25$ mm.

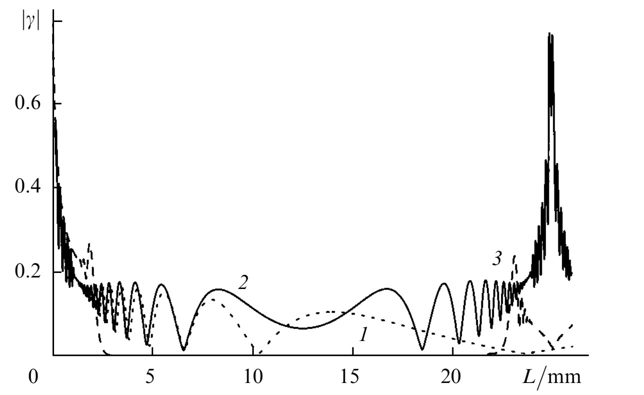

Figure 5. Dependence of the moduli of the eigenvalues for cavity supermodes on the position L of the output mirror for the synphase mode in a cavity with a plane mirror (*1*) as well as for the synphase mode (*2*) and the mode with $m = 30$ (*3*) in a cavity with a mirror with radius of curvature $R = 25$ mm.

A distinguishing feature of this system as compared to the system with a plane mirror is the reconstruction of the field distribution near $L = R$, which corresponds to a concentric cavity. For the exact equality, the distribution of the field returning to the MCF is a mirror image of the distribution of the emitted field. Since the field distributions in all microfibres for the synphase mode are identical, the synphase distribution for an odd set is reconstructed by a rotation through a half-period of the set. As a result, the overlapping of the field being returned and the emitted field is poor.

The situation can be improved if we recall that a system of images associated with the Talbot effect in converging beams is formed in the vicinity of the centre of the spherical mirror provided that the paraxial optics approximation is satisfied for periodic coherent emitters. The transition to the convergent beam geometry indicates, on qualitative level, that the Talbot length is now a variable quantity since the period of the arrangement of images on the circle decreases as we approach the center. In order to obtain the maximum degree of correlation, L must be detuned from $L = R$ by a quarter of the local Talbot length; then the field of the synphase mode will be inverted relative to the axis of the system and turned by half a period in the azimuthal direction after the roundtrip in the cavity [11]. Thus, almost the entire field emitted by the microfibre will fall into the microfibre located at the opposite side of the MCF. The maximum eigenvalue of the synphase mode, however, will be smaller than unity since the cavity is no longer concentric and the distribution of the field returned to the microfibre does not exactly coincide with the waveguide mode. At the same time, in the case of unphased lasing, the fields in symmetrically arranged microfibres are not coupled and this mode is suppressed.

For the mode with $m = 30$, close to the antiphase mode, the field distribution is reproduced almost completely without a rotation by half a period in the azimuthal direction upon a change in L by a quarter of the corresponding Talbot distance. Thus, the peak of the distribution of the field being returned to the MCF for the antiphase mode is located between microfibres so that the generation of this mode is strongly suppressed.

Upon a further detuning of L from the value corresponding to a concentric cavity, the behavior of the eigenvalues of lower-order supermodes is similar to that observed for a plane mirror for small values of L. As a result of the Talbot effect, the eigenvalues oscillate against the background of their general decrease (see Fig. 6). For the L detuning multiple to half the Talbot length, the value of $|\gamma_0|$ is larger than the remaining eigenvalues, and the generation of a single synphase mode can be expected. In this scheme, high accuracy of alignment of the elements is required, whose realization might be a serious problem.

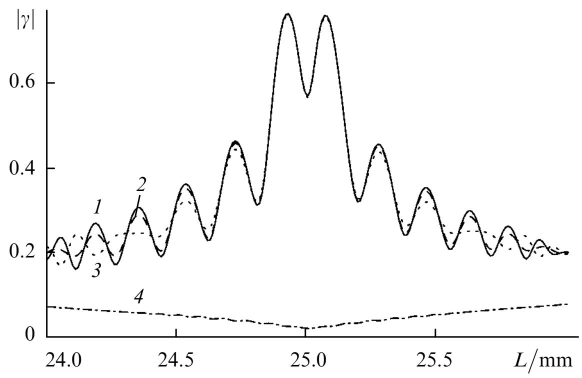

Figure 6. Dependence of the moduli of the eigenvalues for cavity supermodes on the position L of the output mirror with radius of curvature $R = 25$ mm for the synphase mode (*1*) and modes with $m = 1$ (*2*), 2 (*3*), and 30 (*4*).

4. Conclusions

An analysis of propagation of radiation emitted by an MCF in free space indicates that using an external mirror, one can attain a considerable degree of mode discrimination and obtain phase lasing from the MCF. The theory developed for a cavity formed by the MCF and a plane mirror is consistent with the results of experiments and numerical simulation. It is shown that when the plane mirror is replaced by a concave one, the same phase locking can be attained for much smaller parasitic losses (which makes it possible to obtain radiation with a small angular divergence). The main advantage of this system is the possibility of considerable reduction of the losses inherent in the scheme with a plane mirror.

Acknowledgements. The authors are grateful to N N Elkin for carrying out numerical calculations whose results were used to normalize the Gaussian approximation. This work was initiated by researchers from Max Born Institute (M Wrage, P Glas, et al.) and was partially supported by the Russian Foundation for Basic Research (Grant No. 99-02-17469).

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