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Polarisation capture of the wave front of depolarised speckle radiation by a holographic laser mode

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***Abstract*. The polarisation properties of a reference-free hologram inside which the diffraction of the recording depolarised radiation from the speckle inhomogeneities is insignificant, are studied. It is shown that such a hologram, used as the output mirror of a ring laser, has selective properties for the specklon waves comprising the lasing mode. The behaviour of the fundamental mode of a holographic laser with an inhomogeneous birefringent medium inside the cavity is analysed. Conditions for attaining amplitude-phase relations between mode specklons, which are required for vector phase conjugation of depolarised speckle signal, are determined.**

Keywords: *vector phase conjugation, depolarisation, speckle field, reference-free hologram, holographic laser*

1. Introduction

There are two main techniques for realising vector phase conjugation (VPC) of depolarised radiation. These techniques are based on the application of degenerate four-wave mixing with two circularly polarised reference pumping waves [1, 2] and induced backscattering [1, 3, 4]. In investigations relating to phase conjugation and the construction of lasers self-adapted to phase-polarisation distortions, considerable attention has been paid in recent years to the creation of the so-called holographic laser in which one of the mirrors is a dynamic hologram [5 – 17]. There are two possible ways of constructing such lasers. In the more traditional version, the holographic mirror as well as the ring cavity are formed by the controlling radiation injected into the loop circuit [5 – 9, 13 – 17], while a self-starting laser adapted to intracavity aberrations is realised in the other version [10 – 12].

In most works devoted to the investigation of such lasers, the possibility of correction for phase distortions along the path of the laser radiation was considered. In Ref. [12], the compensation for depolarisation in a self-starting loop circuit was reported. The polarisation properties of a hologram formed by an external depolarised speckle signal were analysed for the first time in Ref. [13]. The model proposed in Ref. [13] is developed in the present work which aims at determining the conditions for vector phase conjugation of depolarised speckle radiation in the mode of the holographic laser formed by this radiation.

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2. Mode structure in the polarisation model of a holographic laser with a short hologram and an inhomogeneous birefringent plate

A holographic ring cavity is formed as a result of recording of a hologram by the waves $\boldsymbol{E}_1$ and $\boldsymbol{E}_3$ (Fig. 1), where $\boldsymbol{E}_1$ is the signal wave arriving at the input of the loop circuit, and $\boldsymbol{E}_3$ is the signal wave obtained from wave $\boldsymbol{E}_1$ as a result of its traversal of the loop circuit elements. If the recording beams $\boldsymbol{E}_1$ and $\boldsymbol{E}_3$ (in particular, speckle beams) are essentially inhomogeneous, one has to deal with a reference-free hologram, when neither of the waves recording it can be a reference wave for the other, i.e., spatially homogeneous on the scale of its envelope. It is practically impossible to satisfy the traditional reference holographic appro-ximation in creating a holographic laser using the speckle beams $\boldsymbol{E}_1$ and $\boldsymbol{E}_3$. This would require the focusing of wave $\boldsymbol{E}_3$ at a speckle inhomogeneity of wave $\boldsymbol{E}_1$, leading to a considerable increase in the losses for the mode at the aperture of the holographic mirror formed in this way and thereby substantially increasing the excitation threshold of the holographic laser which even otherwise has a low Q-factor.

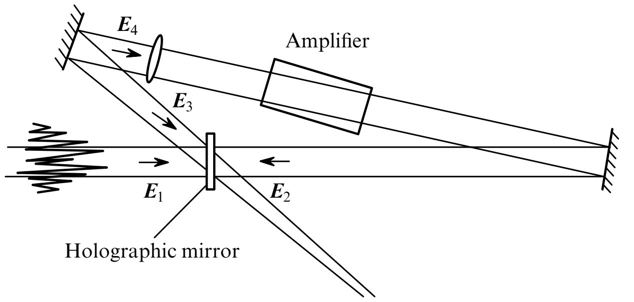

Figure 1. Ring laser with a holographic mirror formed by an external spatially inhomogeneous optical signal $\boldsymbol{E}_1 - \boldsymbol{E}_3$.

The ability of the reference-free hologram to single out from the noise of a ring laser a wave conjugated relative to the signal $\boldsymbol{E}_1 - \boldsymbol{E}_3$ in its mode $\boldsymbol{E}_2^{\text{mod}} - \boldsymbol{E}_4^{\text{mod}}$ counterpropagating to the signal was analysed in Refs [14 – 16]. In particular, we considered in Refs [15, 16] the case of a nondiffraction (or short) hologram, over whose thickness l_{h}

we can disregard the diffraction spreading of characteristic transverse inhomogeneities $\rho_{1,3}$ of the recording speckle field: $l_h < k\rho_{1,3}^2$, where $k = 2\pi/\lambda$, λ being the wavelength of the recording field. In view of the less stringent requirements on the thick-ness of the holographic medium, this approximation has obvious advantages over the reference-free diffraction hologram approximation considered earlier [14].

The selectivity of a short hologram is manifested in the case of a noncollinear convergence (at an angle φ) of the speckle beams $\boldsymbol{E}_1$ and $\boldsymbol{E}_3$ recording it, and is defined by the parameter $\mu \approx \rho_1/(l_h \sin\varphi)$ characterising the extent to which their speckles are intermixed in the hologram. For the optimal angle $\varphi_{\rm opt} \sim a_1/l_h$ (a_1 is the radius of the caustic cross section of the recording wave $\boldsymbol{E}_1$) for which such a mixing is most efficient, the parameter μ is approximately equal to the speckling factor ρ_1/a_1 of the signal field, and has a small value in the case of a developed speckle structure of this field (when $\rho_1/a_1 \ll 1$).

For a short hologram [15], the following selection mechanisms are operative. First, a short hologram discriminates the noise waves $\boldsymbol{E}_{2\rm n}$ which are uncorrelated in fine speckle structure with a signal field $\boldsymbol{E}_1$. This is due to their poorer reflection (by a factor of about $1/\mu$) from the selective hologram ($\mu \ll 1$) in comparison with the reflection of the conjugate component of the readout field $\boldsymbol{E}_{2\rm c} \sim \boldsymbol{E}_1^*$. Second, the small parameter μ also determines the relative fraction of the uncorrelated noise component $\boldsymbol{E}_{4\rm n}$ which always appears, together with the conjugate component $\boldsymbol{E}_{4\rm c} \sim \boldsymbol{E}_3^*$, in the scattered wave $\boldsymbol{E}_4$ upon reflection of even an exactly conjugated wave $\boldsymbol{E}_{2\rm c} \sim \boldsymbol{E}_1^*$ from the reference-free hologram.

Taking into account these properties of the short hologram used as the output mirror of a ring laser, the conjugated component $\boldsymbol{E}_{2\rm c,4c}^{\rm mod} \sim \boldsymbol{E}_{1,3}^*$ in its fundamental transverse mode $\boldsymbol{E}_{2,4}^{\rm mod}$ (with the highest Q-factor) is only slightly noise-polluted by the uncorrelated field $\boldsymbol{E}_{2\rm n,4n}^{\rm mod}$ [15, 16]:

$$\boldsymbol{E}_{2,4}^{\rm mod} = \boldsymbol{E}_{2\rm c,4c}^{\rm mod} + \boldsymbol{E}_{2\rm n,4n}^{\rm mod}, \frac{\int \left|\boldsymbol{E}_{2\rm n,4n}^{\rm mod}\right|^2 {\rm d}r}{\int \left|\boldsymbol{E}_{2\rm c,4c}^{\rm mod}\right|^2 {\rm d}r} \sim {\rm o}(\mu). \tag{1}$$

The latter relations are valid in the 'scalar' approximation, when the polarisation is uniform in the signal beam $\boldsymbol{E}_1 - \boldsymbol{E}_3$. This means that the wave fronts of the scalar fields E_{1x}, E_{1y} (where $\boldsymbol{E}_1 = E_{1x}\boldsymbol{x}_0 + E_{1y}\boldsymbol{y}_0$), obtained in the expansion of the vector field $\boldsymbol{E}_1$ in an arbitrary linear basis $(\boldsymbol{x}_0, \boldsymbol{y}_0)$, have the same structure: $K_{1x,1y} \equiv 1$, where

$$K_{1x,1y} = \frac{\langle E_{1x}(\boldsymbol{r}) E_{1y}^*(\boldsymbol{r})\rangle}{\left(\langle I_{1x}\rangle \langle I_{1y}\rangle\right)^{1/2}}$$

is the transverse correlation function of the fields E_{1x} and E_{1y}, the angle brackets indicate averaging over a statistical ensemble of realisations of the signal speckle radiation [1, 18]. In the polarisation-nondegenerate case, the fundamental mode under certain approximations [13] can be represented in the form of the superposition

$$\boldsymbol{E}_{2,4}^{\rm mod} = \sum_{i=1}^{4} \boldsymbol{E}_{2,4}^{(i)}. \tag{2}$$

In this equation, the weak uncorrelated waves $\boldsymbol{E}_{2\rm n,4n}^{\rm mod}$ have been omitted, and $\boldsymbol{E}_2^{(i)}$ are linearly polarised waves (or specklons) which are correlated exactly with one of the two scalar fields E_{1x} and E_{1y} and are defined as follows:

$$\boldsymbol{E}_2^{(1)} = a_\perp E_{1y}^* \boldsymbol{y}_0,\ \boldsymbol{E}_2^{(2)} = a_\parallel E_{1x}^* \boldsymbol{x}_0,$$

$$\boldsymbol{E}_2^{(3)} = b_\perp E_{1x}^* \boldsymbol{y}_0,\ \boldsymbol{E}_2^{(4)} = b_\parallel E_{1y}^* \boldsymbol{x}_0, \tag{3}$$

where $a_\parallel$, $a_\perp$, $b_\parallel$ and $b_\perp$ are complex constants which will be used in the following to refer to these waves as specklons of type $a_\parallel$, $a_\perp$, $b_\parallel$ and $b_\perp$, respectively (or specklons of type a and b).

Upon reflection from the reference-free hologram, specklons $\boldsymbol{E}_2^{(i)}$ will make contributions to analogous specklons $\boldsymbol{E}_4^{(j)}$ of the scattered wave $\boldsymbol{E}_4$ ($\boldsymbol{E}_4^{(1)} \sim E_{3y}^* \boldsymbol{y}_0$, $\boldsymbol{E}_4^{(2)} \sim E_{3x}^* \boldsymbol{x}_0$, $\boldsymbol{E}_4^{(3)} \sim E_{3x}^* \boldsymbol{y}_0$, $\boldsymbol{E}_4^{(4)} \sim E_{3y}^* \boldsymbol{x}_0$), as well as to the wave $\boldsymbol{E}_{4\rm n}$ that is not correlated with the signal wave $\boldsymbol{E}_3$ ($\langle \boldsymbol{E}_{4\rm n} \boldsymbol{E}_3\rangle = 0$) and participates in the formation of the weak nonconjugate component of mode $\boldsymbol{E}_{2\rm n,4n}^{\rm mod}$.

In the following analysis, we shall take into account the assumption used by us earlier [13] on the existence of a pair of unit vectors x_0, y_0 for which the scalar fields E_{1x} and E_{1y} are completely uncorrelated: $K_{1x,1y} = 0$. In particular, such a situation is realised when the signal wave $\boldsymbol{E}_1$ is formed in the presence of a randomly inhomogeneous birefringent plate at the input of the loop circuit (Fig. 2), which introduces for its ordinary and extraordinary waves deep ($\langle \psi_{\rm o,e}(\boldsymbol{r}) - \langle \psi_{\rm o,e}\rangle\rangle \geqslant 2\pi$), independent ($\langle \psi_{\rm o}(\boldsymbol{r}) - \psi_{\rm e}(\boldsymbol{r})\rangle = 0$) and rapidly varying over the cross section of the laser beam $\boldsymbol{E}_0$ fluctuations of the phase. In this case, one of the vectors $\boldsymbol{x}_0$ and $\boldsymbol{y}_0$ in the basis separated in this way will be collinear with the optical axis of the birefringent element.

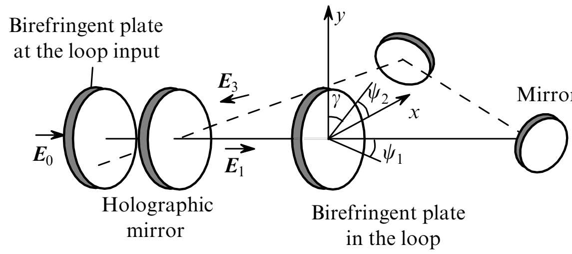

Figure 2. Holographic cavity with inhomogeneous birefringent plates at the input and at the feedback loop (the first plate is used for generating the depolarised input speckle-signal, while the second one is used for mode formation.

Thus, in the case under consideration, the spatially polarised mode structure depends strongly on the amplitude-phase relations between its constituent specklons. For realisation of the VPC, the following relations must be satisfied: $a_\parallel = a_\perp$, $b_\parallel = b_\perp = 0$. For this purpose, the feedback loop must contain a polarisation element providing an effective energy exchange between specklons during mode formation (Fig. 2). The birefringent medium plays the role of such an element. The amplitude-phase relations between specklons in a mode are determined by solving the system of equations

$$g_{ij}^{\rm pol} e_j = g e_i, \tag{4}$$

where $e_1 = a_\perp$, $e_2 = a_\parallel$, $e_3 = b_\perp$, $e_4 = b_\parallel$; g is the mode eigenvalue; $g_{ij}^{\rm pol}$ are elements of the 4×4 matrix $\hat{G}^{\rm pol}$ which determines the action of the polarisation elements on a- and b-specklons and is equal to the product of matrices:

$\hat{G}^{\rm pol} = \hat{T}\hat{R}$, where $\hat{T}$ is the operator of a birefringent plate, and $\hat{R}$ is the operator of reflection of specklons $\boldsymbol{E}_2^{(i)}$ from the holographic mirror.

In the above-described method of formation of depolarised radiation of the wave $\boldsymbol{E}_1$ (when $K_{1x,1y} = 0$), the matrix $\hat{R}$ is diagonal: $r_{11} = r_{44} = p$, $r_{22} = r_{33} = 1 - p$, where $p = \int\langle I_{1y}\rangle \mathrm{d}\boldsymbol{r} / \int\langle I_1\rangle \mathrm{d}\boldsymbol{r}$ is a parameter characterising the integral polarisation of the signal beam $\boldsymbol{E}_1$ in the basis $\boldsymbol{x}_0$, $\boldsymbol{y}_0$. In the present case, this parameter is the differential polarisation of the beam $\boldsymbol{E}_1$: $p = \langle I_{1y}(\boldsymbol{r})\rangle / \langle I_1(\boldsymbol{r})\rangle$.

The introduction of a homogeneous birefringent plate into the feedback loop does not solve the VPC problem in the holographic laser mode. It was shown in Ref. [13] that this obstruction is caused by a number of negative factors, notable among which is the presence of an equal number of 'parasitic' b-specklons along with the 'useful' ones in the fundamental mode. Attempts to neutralise these negative factors by introducing the dependence of the birefringent plate parameters $\gamma(r)$ and $\psi(r)$ (where γ is the angle at which the optical axis of this plate is oriented relative to a selected basis, and $\psi = \psi_2 - \psi_1$ is the phase shift between its normal waves) on the transverse coordinate r complicates the model.

Obviously, the efficiency of the proposed approach will depend on the characteristic scale of variation of these parameters in the cross section of the plate, as well as on the absolute magnitude of their fluctuations around a certain mean value. Consider the limiting case when the selected parameter (γ or ψ) is distributed uniformly (in the interval [0, 2π]) and homogeneously over the coordinate r, and the characteristic scale of its variation over the birefringent plate cross section is much smaller than the diameter of the signal beam in the same cross section. In this case, we can use the statistical approach. We will deal with the ensemble of realisations of a holographic laser in which a transition from one realisation to another is accompanied by a change in the fine structure of the holographic mirror determined by the speckle structure of the signal radiation forming it, as well as in the spatial distribution of the selected parameter of the birefringent plate, which randomly fluctuate in its cross section. Our aim is to determine the mean statistical mode parameters as functions of the determinate parameters of the holographic cavity.

In order to determine the fundamental mode parameters within the framework of the proposed model, we must find the modified matrix $\hat{T}^{\rm new}$ using the expression for operator $\bar{T}$ in the case of a homogeneous birefringent element [13]:

$$\hat{T} = \begin{bmatrix} A_1 & A_2 & B_1 & B_1^* \\ A_2 & A_1 & -B_1 & -B_1^* \\ B_1 & -B_1 & B_2 & A_2 \\ B_1^* & -B_1^* & A_2 & B_2^* \end{bmatrix}, \tag{5}$$

where

$$A_1 = 1 - A_2;\; A_2 = \frac{\sin^2 2\gamma}{2}(1 - \cos\psi);$$

$$B_1 = \frac{\sin 2\gamma}{2}\left[\cos 2\gamma - \cos^2\gamma\exp(\mathrm{i}\psi) + \sin^2\gamma\exp(-\mathrm{i}\psi)\right]; \tag{6}$$

$$B_2 = \frac{\sin^2 2\gamma}{2} + \cos^4\gamma\exp(\mathrm{i}\psi) + \sin^4\gamma\exp(-\mathrm{i}\psi).$$

For this purpose, we consider the result of a round trip in the holographic ring cavity for the wave $\boldsymbol{E}_2^{(0)}$ which is a superposition of four specklons:

$$\boldsymbol{E}_2^{(0)} = \sum_{j=1}^{4}\boldsymbol{E}_{2j}^{(0)} = \sum_{j=1}^{4} e_j^{(0)}\boldsymbol{E}_j,$$

where $e_1^{(0)} = a_\perp$, $e_2^{(0)} = a_\parallel$, $e_3^{(0)} = b_\perp$, $e_4^{(0)} = b_\parallel$; $\boldsymbol{E}_1 = E_{1y}^*\boldsymbol{y}_0$, $\boldsymbol{E}_2 = E_{1x}^*\boldsymbol{x}_0$, $\boldsymbol{E}_3 = E_{1x}^*\boldsymbol{y}_0$, $\boldsymbol{E}_4 = E_{1y}^*\boldsymbol{x}_0$. After a round trip in the cavity, the wave $\boldsymbol{E}_2^{(0)}$ is transformed into the wave

$$\boldsymbol{E}_2^{(1)} = \sum_{i=1}^{4}\boldsymbol{E}_{2i}^{(1)} = \sum_{i=1}^{4} e_i^{(1)}(r)\boldsymbol{E}_i,$$

where

$$e_i^{(1)}(r) = \sum_{j=1}^{4} t_{ij}(r) r_j e_j^{(0)},$$

t_{ij} being the elements of the operator $\hat{T}$. Thus, the spatial structure of the waves $\boldsymbol{E}_{2i}^{(1)} = e_i^{(1)}\boldsymbol{E}_i$ formed as a result of the action of the operator $\hat{G}^{\rm pol}(r)$ (inhomogeneous in the present case) on the wave $\boldsymbol{E}_2^{(0)}$, which is identical to the structure of the corresponding specklon $\boldsymbol{E}_i$, is modulated considerably by the inhomogeneous function $e_i^{(1)}(r)$. In order to obtain the elements $t_{ij}^{\rm new}$ of the modified matrix $\hat{T}^{\rm new}$, we must calculate the projections of waves $\boldsymbol{E}_{2i}^{(1)}$ on E_i:

$$\boldsymbol{E}_{2i}^{(1)} = c_{0i}\boldsymbol{E}_i + \boldsymbol{E}_{2{\rm n}i},\; \int \boldsymbol{E}_{2{\rm n}i}\boldsymbol{E}_i^*\mathrm{d}r \equiv 0, \tag{7}$$

where

$$c_{0i} = \frac{\int e_i^{(1)}(r) I_i \mathrm{d}r^2}{\int I_i \mathrm{d}r^2} \approx \frac{\int\langle e_i^{(1)}(r)\rangle\langle I_i\rangle \mathrm{d}r^2}{\int\langle I_i\rangle \mathrm{d}r^2} = \langle e_i^{(1)}\rangle. \tag{8}$$

The derivation of this equation is based on the principle of ergodicity which allows the replacement of the integrals of random functions with respect to transverse coordinates in each specific realisation from the ensemble by their mean integrals.

The ergodicity postulate is valid completely only for homogeneous statistics [18], which would be observed for an infinite envelope of the signal speckle field which makes the integration domain infinite. In the case of aperture-restricted speckle beam considered here, the applicability of the ergodicity postulate stems from the smallness of the speckle cross section in the signal field (and, hence, in the functions $I_i(r)$) and of the typical scale of variation of the function $e_i(r) \sim t_{ij}(r)$ over the integration domain.

Formula (8) was derived taking into consideration the statistical independence of the random functions $e_i(r)$ and $I_i(r)$ in the generalised ensemble, as well as the homogeneity of statistics of function $e_i(r)$ which leads to the independence of $\langle e_i(r)\rangle$ of the coordinate r. Taking this into account, we can reduce the procedure of determining the elements $t_{ij}^{\rm new}$ of the modified matrix $\hat{T}^{\rm new}$ to averaging of the corresponding matrix elements of the original operator $\hat{T}$ (5) over the spatially inhomogeneous parameter:

$$t_{ij}^{\rm new} = \frac{1}{2\pi}\int_0^{2\pi} t_{ij}(\omega)\mathrm{d}\omega, \tag{9}$$

where $\omega = \psi$ or γ. Thus, the above expression also takes into account the uniform distribution of the selected parameter ω in the interval [0, 2π].

Let us now elucidate the physical meaning of the procedure implemented in (7). The waves $\boldsymbol{E}_{2\mathrm{n}i}$ obtained as a result of such a procedure, which are orthogonal to the corresponding specklons $\boldsymbol{E}_{2\mathrm{c}i} \sim \boldsymbol{E}_i$, satisfy the requirement (which is quite significant) that they are not correlated with these specklons in the fine speckle structure: $\langle \boldsymbol{E}_{2\mathrm{n}i}\boldsymbol{E}_i^* \rangle = 0$. This means that upon reflection from the reference-free hologram, waves $\boldsymbol{E}_{2\mathrm{n}i}$ will contribute only to the uncorrelated components $\boldsymbol{E}_4$ of the scattered wave. Thus, waves $\boldsymbol{E}_{2\mathrm{n}i}$ will also participate in the formation of the uncorrelated mode component $\boldsymbol{E}_{2\mathrm{n}}^{\mathrm{mod}}$ as a result of superposition with the noise waves formed upon reflection from the reference-free hologram of specklons $\boldsymbol{E}_{2\mathrm{c}i}$.

However, waves of the type $\boldsymbol{E}_{2\mathrm{n}i}$ formed as a result of passage of specklons $\boldsymbol{E}_{4\mathrm{c}i}$ through the inhomogeneous medium in the cavity fundamentally differe from the noise waves generated by specklons $\boldsymbol{E}_{2\mathrm{c}i}$ reflected from the reference-free hologram. The difference is that the noise waves do not affect the absolute magnitude of the coefficient of transformation of the conjugate component of the mode $\boldsymbol{E}_{2\mathrm{c}}^{\mathrm{mod}}$ into itself upon a round trip of the resonator by the mode, which is manifested in the independence of the mode Q-factor on the selectivity of the hologram (on the parameter μ). The latter determines only the relative contribution of the noise component $\boldsymbol{E}_{2\mathrm{n}}^{\mathrm{mod}}$ in the mode [15–17].

On the contrary, the emergence of the waves $\boldsymbol{E}_{2\mathrm{n}i}$ efficiently decreases the above transformation coefficient, thus altering directly the mode Q-factor. For this reason, the mode striving to attain the maximum Q-factor tends to acquire the minimum possible amplitude of the noise wave $\boldsymbol{E}_{2\mathrm{n}i}$ [see (7)], which is ensured by certain amplitude-phase relations between a- and b-type specklons in the conjugate component $\boldsymbol{E}_{2\mathrm{c}}^{\mathrm{mod}}$.

3. Cavity with a birefringent plate inhomogeneous in phase shift between its normal waves

Consider now the situation when ψ is a random quantity and γ assumes a determinate value. The matrix $\hat{T}_{\psi}^{\mathrm{new}}$ differs from $\hat{T}$ in that the terms with factors $\exp(\pm \mathrm{i}\psi)$ and $\cos\psi$ must be 'nullified' in elements t_{ij}. As a result, the solution of the system of equations (4) yields to the following expressions for specklon coupling in modes and for their eigenvalues:

$$a_{\parallel} = a_{\perp}\frac{p-g}{g-(1-p)},\ b_{\parallel} = b_{\perp},\ b_{\perp} = a_{\perp}\frac{A_2(1-2g)}{2B_1(1-p-g)}, \quad (10)$$

$$g_{1,2} = 0,\ g_{3,4} = \frac{1 \pm |2\varepsilon|(A_1 - A_2)^{1/2}}{2}, \quad (11)$$

where $\varepsilon = p - 0.5$. For further analysis of the conditions for VPC, it is convenient to introduce dimensionless parameters characterising the amplitude-phase relations in the mode between 'useful' a-specklons and the relative energy contribution of 'parasitic' b-specklons to it: $\chi_1 = |a_{\perp}|^2/(|a_{\perp}|^2 + |a_{\parallel}|^2)$, $\chi_2 = |\arg(a_{\parallel}/a_{\perp})|$, $\chi_3 = (|b_{\perp}|^2 + |b_{\parallel}|^2)/(|a_{\perp}|^2 + |a_{\parallel}|^2 + |b_{\perp}|^2 + |b_{\parallel}|^2)$. Parameters χ_1, χ_2, χ_3 for the mode with the largest Q-factor (with the eigenvalue g_3) assume the following form in this case (see Fig. 3a):

$$\chi_1 = \frac{1}{2} + \operatorname{sgn}\varepsilon\frac{(A_1 - A_2)^{1/2}}{2A_1},\ \chi_2 = 0,\ \chi_3 = A_2. \quad (12)$$

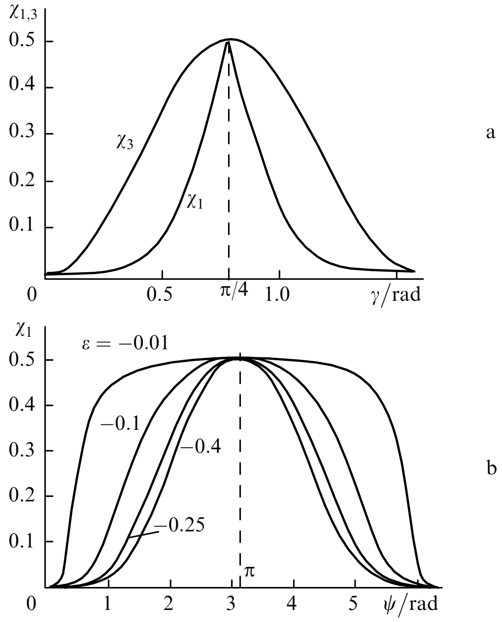

Figure 3. Dependences $\chi_1(\gamma)$ (for $\varepsilon < 0$) and $\chi_3(\gamma)$ for a birefringent plate with a random distribution of the phase shift ψ between an ordinary and an extraordinary waves (a), and dependences $\chi_1(\psi, \varepsilon)$ for a birefringent medium with a strongly random deviation γ of its optical axis in the plate cross section (b).

The obtained solution has the following characteristic features. First, there is no phase shift between specklons of the type $a_{\parallel}$ and $a_{\perp}$ over the entire range of variation of parameters γ and ε since $\chi_2 = 0$ for the mode with the highest Q-factor (with the eigenvalue g_3) as well as for the next nondegenerate mode (with eigenvalue g_4). However, in this case, the main drawback continues to be the presence of 'parasitic' specklons of the type $b_{\parallel}$ and $b_{\perp}$ in the mode, on a par with the useful modes, at the point $\gamma = 45°$ at which the parameter χ_1 has its optimal value 0.5 for VPC.

4. Cavity with a birefringent plate inhomogeneous in the angle of orientation of its optical axis

Consider now the case in which the angle γ is a random parameter for a fixed value of the parameter ψ. After averaging, we go over to the matrix $\hat{T}_{\gamma}^{\mathrm{new}}$ which is analogous to the matrix $\hat{T}$ if we put $A_1 = 3/4 + (1/4)\cos\psi$, $A_2 = 1 - A_1$, $B_1 = 0$, $B_2 = 1/4 + (3/4)\cos\psi$ in (6). Solving the system of equations (4), we obtain the following expressions connecting the specklons

$$a_{\parallel} = a_{\perp}\frac{p-g}{g-(1-p)} \quad (13)$$

with a pair of eigenvalues

$$g_{1,2} = \frac{A_1 \pm \left[A_1^2 - \left(1 - 4\varepsilon^2\right)(A_1 - A_2)\right]^{1/2}}{2} \quad (14)$$

and

$$b_{\parallel} = b_{\perp}\frac{g - (1-p)B_2}{pA_2} \quad (15)$$

with another pair of eigenvalues

$$g_{3,4} = \frac{B_2 \pm \left[B_2^2 - \left(1 - 4\varepsilon^2\right)\left(B_2^2 - A_2^2\right)\right]^{1/2}}{2}. \qquad (16)$$

In this case, the problem of discrimination of parasitic specklons of the type $b_{\parallel}$ and $b_{\perp}$ is solved a priori since, first, these specklons are independent of specklons of type $a_{\parallel}$ and $a_{\perp}$ (the modes containing a-type specklons do not contain specklons of type b, and vice versa). Second, the mode with the highest Q-factor (with the eigenvalue g_1) always contains only those a-specklons that are required for VPC and obviously satisfy the equality $\chi_3 = 0$. The phasing between specklons of the type $a_{\parallel}$ and $a_{\perp}$ required in this mode is also always ensured ($\chi_2 = 0$). Fig. 3b shows the functions $\chi_1(\psi, \varepsilon)$.

However, a practical realisation of the model situation described above is not easy. Thus, while inhomogeneity in parameter ψ is ensured by etching the homogeneous element in acid, a plate inhomogeneous in parameter γ (which is vital for VPC) can be formed only by breaking a thin homogeneous plate into fragments of size several hundred micrometers and then 'gluing' these pieces together. The optical axis, which always remains parallel to the face of the element formed in this way, will be oriented randomly in each fragment.

In actual practice, such a situation can be simulated, for example, with the help of two etched plates that are inhomogeneous in the parameter $\psi(r)$. Solving the corresponding system of equations (4) for this model situation in the case of a mode with the highest Q-factor, we arrive at the dependences $\chi_{1,3}(\gamma, \delta\gamma, \varepsilon)$ shown in Fig. 4 (where $\delta\gamma = \gamma_2 - \gamma_1$, $\gamma = \gamma_1$; γ_1 and γ_2 are the angles defining the orientation of the optical axes relative to the normal to the plane of the loop circuit in each of the two plates). In this mode the parameter $\chi_2(\gamma, \delta\gamma, \varepsilon)$ is always equal to zero.

We must pay attention to two peculiarities of the dependences $\chi_{1,3}(\gamma, \delta\gamma, \varepsilon)$ in the vicinity of point $\delta\gamma = \pi/4$, which are of fundamental importance for VPC. The peculiarity of point $\delta\gamma = \pi/4$ is that parameters $\chi_{1,3}$, which have optimal values for VPC at this point, are independent of ε as well as angle γ. The result depends only on mutual orientation of the optical axes of two etched plates, but not on their connection with the selected basis. It should be recalled that the selection of the latter depends on the extent to which the scalar fields E_{1x} and E_{1y} obtained in this basis for the signal field are mutually uncorrelated.

Thus, the attainment of the required amplitude-phase relations between 'useful' a-specklons in the fundamental mode and the suppression of 'parasitic' b-specklons in it at the point $\delta\gamma = \pi/4$ do not depend on the polarisation of the signal wave (parameter ε) or on the nature of depolarisation, i.e., on the extent of correlation between the scalar fields E_{1x} and E_{1y}, which may not be equal to zero, assuming arbitrary values between 0 and 1: $\min |\langle E_{1x}(r) E_{1y}^{*}(r)\rangle| / (\overline{I_{1x} I_{1y}})^{1/2} \in [0, 1]$. In other words, absolute 'polarisation capture' of signal radiation by the mode of spatially polarised structure occurs at the point $\delta\gamma = \pi/4$.

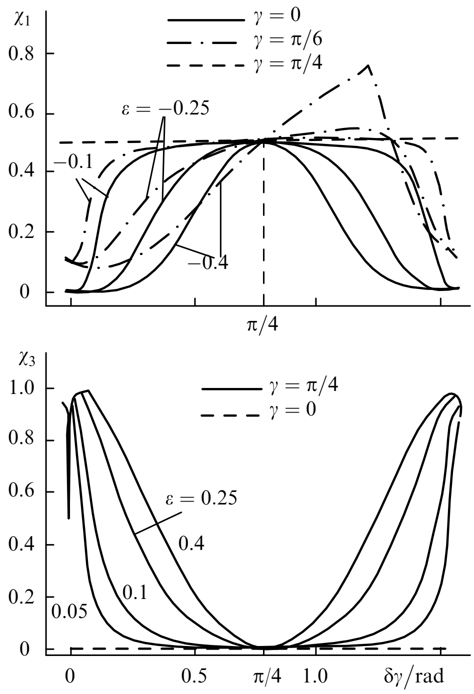

Figure 4. Dependences of χ_1 and χ_3 on γ, $\delta\gamma$, and ε for two birefringent plates random in the parameter ψ in the feedback loop of a holographic cavity.

5. Conclusions

Consider now the reasons for such a behaviour of the mode in a cavity with two etched plates in the vicinity of the point $\delta\gamma = \pi/4$. As the value of $\delta\gamma$ increases from zero to $\pi/4$, the normal waves of such a composite birefringent medium will become more and more elliptical. The eccentricity and orientation of the ellipses of polarisation of normal waves are highly random over the cross section of such a medium, but there exists an eccentricity of normal waves averaged over the ensemble, which has the same value at all points over the cross section.

While the point $\delta\gamma = 0$ corresponds to the limiting case when the cavity contains an inhomogeneous birefringent medium with linearly polarised normal (ordinary and extraordinary) waves and a random distribution of the parameter $\psi(r)$, the point $\delta\gamma = \pi/4$ corresponds to the other limit where the mean eccentricity of normal waves is 'nullified'. This limiting case is completely identical to the model situation considered above for a birefringent medium which is inhomogeneous in the parameter $\gamma(r)$ distributed uniformly in the interval [0, 2π]. Physically, this means that the interaction between a- and b-specklons in the polarisation element is weakened substantially with increasing $\delta\gamma$ and vanishes at the point $\delta\gamma = \pi/4$ (the intensity of energy exchange between $a_{\perp}$- and $a_{\parallel}$-specklons remains unchanged in this case, and does not depend on the extent and nature of inhomogeneity of the birefringent medium). This is the reason behind the independent existence of modes at the point $\delta\gamma = \pi/4$, which contain either a-specklons or b-specklons only.

The reason behind the attainment of optimal amplitude-phase relations between 'useful' $a_{\perp}$- and $a_{\parallel}$-specklons in the mode with the highest Q-factor at the point $\delta\gamma = \pi/4$ is the tendency of the mode to minimise the losses during passage through the feedback loop. Indeed, as the amplitude-phase 'misalignment' occurs between these specklons in the mode (relative to the signal field), the specklons passing through the birefringent medium in the cavity will impart a fraction of their energy to uncorrelated components $\boldsymbol{E}_{2ni}$ emerging on account of the inhomogeneity of the medium. This leads to a strong decrease in the coefficient of transformation of the conjugate mode component into itself upon a round trip in the ring cavity.

In the absence of such a 'misalignment' (and in the absence of any interaction between *a*- and *b*-specklons), the above-mentioned uncorrelated field components do not appear. This circumstance results in a decrease in the total losses for the fundamental mode in the cavity, the minimum losses at the point $\delta\gamma = \pi/4$ being achieved when the 'useful' *a*-specklons satisfy the amplitude-phase relations required for VPC.

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