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# A new Faraday rotator for high average power lasers

E A Khazanov

***Abstract*. The new design of a Faraday rotator is proposed which allows one to compensate partially the radiation depolarisation in magneto-optical elements induced by heating due to the laser radiation absorption. The new design is compared analytically and numerically with a conventional design for the cases of glass and crystal magneto-optical media. It is shown that a rotator, which provides the compensation for birefringence in active elements with the accuracy up to 1 % at the average laser radiation power of 1 kW in the rotator, can be created.**

***Keywords***: *Faraday rotator, compensation for birefringence, depolarisation.*

## 1. Introduction

In connection with an increase in the average laser power in recent years, studies of heating effects caused by the laser radiation absorption in Faraday isolators and rotators become more and more important [1 – 5]. In many applications, a combination of the high average power with a low depolarisation ratio and low introduced aberrations is required. The prominent examples are a laser interferometer for detection of gravitational waves [6] and a laser driver for laser fusion [7].

The radiation absorption in Faraday elements causes the inhomogeneous distribution of temperature over the cross section. This leads to a thermal lens, the inhomogeneous distribution of the angle of rotation of polarisation plane (because of the temperature dependence of the Verdet constant) and a linear birefringence caused by the photoelastic effect. Aberrations caused by the thermal lens do not lead to polarisation distortions and can be efficiently compensated by a phase-conjugate mirror [1, 2] or with the help of spherical optics [4]. Below, we assume that aberrations are absent, or are compensated.

Self-induced depolarisation of high-power radiation in a magneto-active medium was first studied in Refs [8 – 11], where it was shown that the photoelastic effect gives the greatest contribution into the depolarisation ratio, while the effect of the temperature dependence of the Verdet constant can be neglected. This conclusion was confirmed experimentally in Ref. [12].

In Ref. [10], two new designs of a Faraday isolator for high average power lasers were proposed and theoretically studied. They consist of two Faraday elements, each of which rotates the polarisation plane through 22.5°, and a reciprocal optical element between them. In this case, the polarisation distortions arising in a beam during its passage through the first element are partially compensated during the beam passage through the second element. Subsequent experiments [12] confirmed the high efficiency of these designs. In Ref. [13], these two new designs were compared with a conventional design from the viewpoint of obtaining the maximum depolarisation ratio and decreasing the beam aberration. The dependence of the isolation for different designs of a Faraday isolator on the orientation of a magneto-active crystal was studied in detail in Ref. [14]. The results of Refs [10 – 14] show that a reliable Faraday isolator for the average radiation power of 1 kW can be created.

At the same time, the problem of compensation for depolarisation in Faraday rotators has not been studied, and even has not been discussed to date. In contrast to an isolators, a Faraday rotator has no polarisers and is used not for optical isolation but, as a rule, for compensation for birefringence in active elements (AEs) of high-power laser systems (Fig. 1a). Indeed, after two transits through a Faraday rotator and a repeated transit through an AE, the linear polarisation of radiation is restored. Such a method of compensation for birefringence in AEs has been long used in amplifiers [1, 2, 7, 15], and recently was also used in oscil-

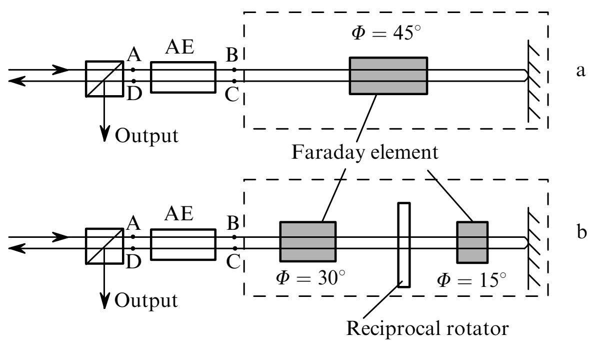


**Figure 1.** Compensation for birefringence in the AE in the conventional (a) and new (b) schemes of the Faraday isolator.

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lators [4, 16] and regenerative amplifiers [16]. It is obvious that, if a Faraday rotator itself introduces polarisation distortions (the depolarisation), the compensation for birefringence in an AE will be incomplete. In this paper, the dependences of the compensation efficiency on the laser radiation power, the heat release power in AEs and on other parameters are obtained.

In spite of a great similarity between a Faraday rotator and a Faraday isolator, they differ in two important aspects, which makes the use of new designs of the isolator, considered in [10] inefficient for the rotator. First, the isolation is affected only by the depolarisation during the second transit of radiation through the isolator, whereas in the rotator the polarisation distortions are accumulated during two transits. Aside from an obvious quantitative consequence, this fact has a more important qualitative consequence. In view of the nonreciprocity of the Faraday effect, new designs of the isolator, which provide the efficient compensation for depolarisation during the back transit, almost do not decrease depolarisation during the first transit [13].

Second, the radiation incident on the isolator is always polarised linearly (we assume that the polarisers are ideal) along a definite direction. Therefore, to achieve a good isolation, it is sufficient that only this linear polarisation would be slightly distorted during the back transit. However, the radiation incident on the rotator is already depolarised in an AE, and its polarisation at different points of the cross section can be arbitrary in the case of strong depolarisation. For this reason, to achieve an ideal compensation for birefringence in the AE, the rotator should rotate the polarisation ellipce through 90° during two transitss without distortion. An exact calculation shows that the use of new isolator designs [10] for the rotator allows one only to improve its parameters only slightly compared to the conventional design. In this paper, the new design of a Faraday rotator for high average power lasers is proposed and studied. The efficiencies of compensation for birefringence in an AE are compared for this design, a conventional design, and a recent design, containing a $\lambda/4$ plate [17].

## 2. New design of a Faraday rotator

The compensation for depolarisation in a Faraday rotator is performed by replacing the 45° Faraday element by two elements of different lengths and a reciprocal rotator of polarisation between them. In this case, two new free parameters appear: the angle $\beta$ of rotation by the reciprocal rotator and the ratio $t$ of the angle of rotation by the first Faraday element to the total angle $\Phi_0$ of rotation by both Faraday elements, which is equal to 45°. The angle $\beta$ can take any value from zero to $\pi$, and the parameter $t$ can take any value from zero to one.

Fig. 1 illustrates the compensation for birefringence in an AE using a conventional design of a Faraday rotator and the new design with compensation for polarisation distortion produced by the rotator itself. The combination of a Faraday rotator with a mirror (dashed rectangle in Fig. 1a, b) is called Faraday mirror. The radiation at the point A is polarised horizontally (in the plane of the figure). Because of the birefringence in an AE, the radiation at the point B becomes depolarised (the state of polarisation is constant in time, but changes over the cross section). In both of these designs in the absence of thermal effects in Faraday elements, the polarisation ellipse at the point C rotates through 90° after a beam reflection from a Faraday mirror, the ellipticity and the direction of polarisation rotation being unchanged (in the laboratory reference system). In this case, the radiation at the point D becomes vertically polarised (normally to the plane of figure) after a repeated transit through the AE and is reflected completely by the polarizer. This leads to a complete compensation for birefringence in the AE. However, the polarisation distortions induced by a temperature gradient in a magneto-active medium result in an incomplete compensation for birefringence in the AE, i.e., radiation with horizontal polarisation appears at the point D, which passes through the polariser.

It is obvious that the greater the Jones matrix $M$ of the Faraday mirror differs from the matrix of rotation through 90°, which is the matrix of an ideal Faraday mirror, the greater the effect of depolarisation of radiation in a magneto-active medium. According to Fig. 1, the Jones matrices of the Faraday mirror for the conventional (the subscript 'old') and the new (the subscript 'new') Faraday rotators are

$$M_{\rm old} = F(\Phi_0, \delta_1)F(\Phi_0, \delta_1), \qquad (1)$$

$$M_{\rm new} = F(t\Phi_0, t\delta_1)R(-\beta)F((1-t)\Phi_0, (1-t)\delta_1)$$
$$\times F((1-t)\Phi_0, (1-t)\delta_1)R(\beta)F(t\Phi_0, t\delta_1), \qquad (2)$$

where

$$R(\beta) = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix},$$
$$\qquad (3)$$
$$F(\Phi, \delta_1) = \sin\frac{\delta}{2}\begin{pmatrix} \cot\frac{\delta}{2} - {\rm i}\frac{\delta_1}{\delta}\cos 2\Psi & -\frac{2\Phi}{\delta} - {\rm i}\frac{\delta_1}{\delta}\sin 2\Psi \\ \frac{2\Phi}{\delta} - {\rm i}\frac{\delta_1}{\delta}\sin 2\Psi & \cot\frac{\delta}{2} + {\rm i}\frac{\delta_1}{\delta}\cos 2\Psi \end{pmatrix}$$

are the Jones matrices of a reciprocal quartz polarisation rotator and of the Faraday element, respectively, taking into account the linear birefringence induced additionally to the circular one [9, 18, 19]; $\delta_1$ and $\Psi$ are the phase difference and the slope of the direction of intrinsic polarisation (Fig. 2) of the thermally induced linear birefringence to the $x$ axis; $\Phi$ is the angle of rotation of polarisation plane for the Faraday element; $\Phi_0 = \pi/4 = \delta_c/2$; and $\delta^2 = \delta_l^2 + \delta_c^2$.

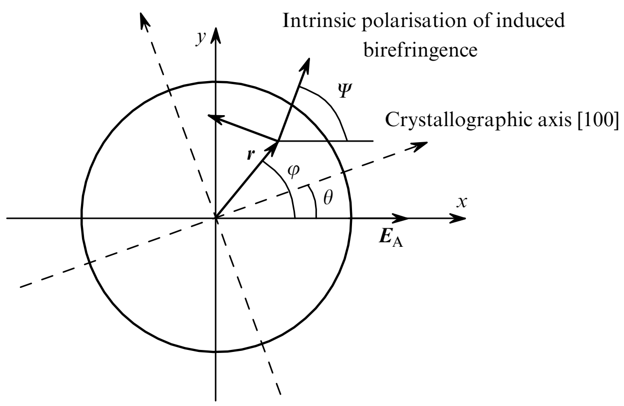


**Figure 2.** Cross section of a magneto-active crystal.

We assume that the magnetic field and the heat release (i.e., quantities $\Phi$ and $\delta_1$) are homogeneous over the entire length. One can see from (1) that for both these designs $\delta_1$ is the phase incursion per transit through the entire Faraday rotator.

Consider the case when the linear birefringence is weak, i.e.,

$$\delta_1 \ll 1. \tag{4}$$

In this approximation, by substituting (3) into (2), we obtain the following expressions for the elements of the matrix $M_{\text{new}}$:

$$M_{11\,\text{new}} = M^*_{22\text{new}} = \mathrm{O}(\delta_1^2) - \mathrm{i}\left[\frac{2\delta_1}{\pi} m_1 + \mathrm{O}(\delta_1^3)\right],$$

$$M_{21\,\text{new}} = -M^*_{12\,\text{new}} = 1 + \mathrm{O}(\delta_1^2) - \mathrm{i}\left[\frac{2\delta_1}{\pi} m_2 + \mathrm{O}(\delta_1^3)\right],$$

where

$$m_1 = \sin\frac{t\pi}{2}\cos\frac{(1-t)\pi}{2}\cos 2\Psi + \sin\frac{(1-t)\pi}{2} \times\left[\cos(2\Psi + 2\beta) - \left(1 - \cos\frac{t\pi}{2}\right)\cos 2\Psi\right];$$

$$m_2 = \sin\frac{t\pi}{2}\cos\frac{(1-t)\pi}{2}\sin 2\Psi + \sin\frac{(1-t)\pi}{2} \times\left[\sin(2\Psi + 2\beta) - \left(1 - \cos\frac{t\pi}{2}\right)\sin 2\Psi\right].$$

Equating $m_1$ and $m_2$ to zero, we obtain the conditions when the difference between the matrix of rotation through 90° and the matrix $M_{\text{new}}$ becomes of the order of $\delta_1^2$:

$$\beta = \frac{\pi}{2}, \quad t = \frac{2}{3}\,. \tag{5}$$

By substituting (3) into (1) and taking into account (4), we obtain that the difference between the matrix of rotation through 90° and the matrix $M_{\text{old}}$ is of the order of $\delta_1$. Thus, if the condition (5) is fulfilled, depolarisation substantially decreases in the new design of the Faraday rotator. One can see from (5), that to provide these conditions, one should place a 90° quartz rotator between two Faraday rotators, which rotate the polarisation plane through 30° and 15°, respectively. Below, we will study the new Faraday rotator at these optimal parameters.

## 3. Compensation for birefringence in the AE

In the absence of thermal effects in a magneto-active medium (at low radiation power), both these designs of the Faraday rotator provide the ideal compensation for birefringence in the AE. Let us determine the compensation efficiency at high powers. Let the field $\boldsymbol{E}_{\text{A}}$ at the point A have a Gaussian intensity distribution with the maximum $E_0$ and the radius $r_0$, and be polarised along the $x$ axis:

$$\boldsymbol{E}_{\text{A}} = \boldsymbol{x}_0 E_0 \exp\left(-\frac{r^2}{2r_0^2}\right), \tag{6}$$

where $\boldsymbol{x}_0$ is the unit vector along the $x$ axis. Then, the field $\boldsymbol{E}_{\text{D}}$ at the point D during the back passage is determined by the Jones matrices (1) and (2) of the active element $A$ and the Faraday mirror $M$, respectively:

$$\boldsymbol{E}_{\text{Dold,Dnew}} = AM_{\text{old,new}}A\boldsymbol{E}_{\text{A}}. \tag{7}$$

An inaccuracy of the compensation for birefringence in the AE is characterised by the depolarisation ratio $\gamma$, which is the ratio of the power passed backward through the polariser to the total laser radiation power:

$$\gamma_{\text{old,new}} = \int_0^{2\pi}\mathrm{d}\varphi\int_0^{\infty}|\boldsymbol{E}_{\text{old,new}}\boldsymbol{x}_0|^2 r\mathrm{d}r \times\left(\int_0^{2\pi}\mathrm{d}\varphi\int_0^{\infty}|\boldsymbol{E}_{\text{old,new}}|^2 r\mathrm{d}r\right)^{-1}, \tag{8}$$

where $r$ and $\varphi$ are the polar coordinates. We assume that the optical diameters of the Faraday rotator and AE are such that the aperture loss can be neglected and the integration over $r$ in expression (8) can be performed to infinity.

We assume that the pump power distribution and, hence, the heat release in the AE are homogeneous over the volume. Then, for an infinitely long cylindrical AE with the [111] crystal orientation, which has the field gain $K_0$, the matrix $A$ has the form [20]

$$A = K_0 \sin\frac{\delta_{\text{a}}}{2}\begin{pmatrix} \cot\frac{\delta_{\text{a}}}{2} - \mathrm{i}\cos 2\varphi & -\mathrm{i}\sin 2\varphi \\ -\mathrm{i}\sin 2\varphi & \cot\frac{\delta_{\text{a}}}{2} + \mathrm{i}\cos 2\varphi \end{pmatrix}, \tag{9}$$

where

$$\delta_{\text{a}} = \frac{r^2}{r_0^2}p_{\text{a}}; \quad \xi_{\text{a}} = \frac{2p_{\text{a}44}}{p_{\text{a}11} - p_{\text{a}12}};$$

$$Q_{\text{a}} = \left(\frac{1}{L_{\text{a}}}\frac{\mathrm{d}L_{\text{a}}}{\mathrm{d}T}\right)\frac{n_{\text{a}}^3}{4}\frac{1+\nu_{\text{a}}}{1-\nu_{\text{a}}}(p_{\text{a}11} - p_{\text{a}12});$$

$$p_{\text{a}} = \frac{1}{2\lambda}\frac{Q_{\text{a}}}{\varkappa_{\text{a}}}\frac{1+2\xi_{\text{a}}}{3}\frac{r_0^2}{R^2}P_{\text{a}}; \tag{10}$$

$\nu_{\text{a}}$, $\varkappa_{\text{a}}$, $n_{\text{a}}$, $p_{\text{a}ij}$, $L_{\text{a}}$, $R$ are the Poisson coefficient, the thermal conductivity, the refractive index, the photoelasticity coefficients written in the two-index notations of Nye [21], the length and radius of the AE, respectively; $T$ is the AE temperature; $\lambda$ is the wavelength; $P_{\text{a}}$ is the heat release power in the AE. These expressions are also valid for glass AEs, for which $\xi_{\text{a}} = 1$. Below, we will neglect the gain saturation in the AE and will assume that $K_0 = \text{const}$.

Now, to determine $\gamma_{\text{old,new}}$, it is necessary to find only $\delta_1$ and $\Psi$, which are specified by the orientation of a magneto-active crystal and the temperature gradient in the crystal. Let us restrict our consideration to orientations [001] and [111] of a cubic crystal (which are used more often compared to other orientations, see [14]), and also to a glass magneto-active element. For an infinitely long cylindrical element, we will use the expressions from Refs [10, 14, 20]:

$$\delta_1(r,\varphi) = 2p\left[r_0^2\frac{1 - \exp(-r^2/r_0^2)}{r^2} - 1\right] \times\left[\cos^2(2\varphi - 2\theta) + \xi^2\sin^2(2\varphi - 2\theta)\right]^{1/2} \text{ and}$$

$$\tan(2\Psi - 2\theta) = \xi\tan(2\varphi - 2\theta)$$

$$\text{for the [001] orientation,} \tag{11}$$

$$\delta_1(r,\varphi) = 2p\left[r_0^2\frac{1-\exp(-r^2/r_0^2)}{r^2} - 1\right]\frac{1+2\xi}{3} \quad \text{and}$$

$$\Psi = \varphi \quad \text{for the [111] orientation,}$$

where

$$\xi = \frac{2p_{44}}{p_{11}-p_{12}}; \quad Q = \left(\frac{1}{L}\frac{dL}{dT}\right)\frac{n_0^3}{4}\frac{1+\nu}{1-\nu}(p_{11}-p_{12}); \qquad (12)$$

$$p = \frac{L}{\lambda}\frac{\alpha Q}{\varkappa}P_0; \qquad (13)$$

$\nu$, $\varkappa$, $\alpha$, $n_0$, $p_{ij}$, $L$ are the Poisson coefficient, the thermal conductivity, the absorption coefficient, the refractive index, the photoelasticity coefficients and the length of a magneto-active medium, respectively; $\theta$ is the angle between the cristallographic axis and the $x$ axis (see Fig. 2); $P_0$ is the laser radiation power at the point B or C (we assume that the powers at these points are the same because absorption in the magneto-active medium is weak). Expressions (11) are also valid for glass Faraday elements, for which $\xi = 1$. The factor 2 in expressions for $\delta_1$ reflects the fact that the heat release is doubled because of two transits of radiation through the Faraday rotator.

By substituting (3) and (11) into (1) and (2), and the result of this substitution together with (6), (7) and (9) into (8), after integration, taking into account (4) and (5), we obtain the depolarisation ratio $\gamma_{\text{old,new}}$. For the [001] orientation of a magneto-active crystal, the depolarisation ratio depends on the angle $\theta$. By rotating the crystal around the axis, one can easily change the angle $\theta$, thereby minimising the depolarisation ratio. By analysing expression (8) after integration, one can easily show that for both of these designs of the Faraday rotator the optimal angle is $\theta_{\text{opt}} = 0$. For this angle, we obtain the final expression for the depolarisation ratio:

$$\gamma_{\text{old}} = \frac{p^2}{\pi^2}\left\{8A_1 + \int_0^\infty \left[4(\xi^2-1)\sin^2\frac{p_a y}{2} + (5\xi^2+2\xi+1)\right.\right.$$
$$\left.\left.\times\sin^2(p_a y)\right]\left[1-\frac{1-\exp(-y)}{y}\right]^2\exp(-y)dy\right\} \text{ for the}$$
$$\text{[001] orientation,} \qquad (14)$$

$$\gamma_{\text{old}} = \frac{8p^2}{\pi^2}\left(\frac{1+2\xi}{3}\right)^2\left\{A_1 + \int_0^\infty \sin^2(p_a y)\left[1-\frac{1-\exp(-y)}{y}\right]^2\right.$$
$$\left.\times\exp(-y)dy\right\} \text{ for the [111] orientation,}$$

$$\gamma_{\text{new}} = \frac{(2\sqrt{3}-\pi)^2p^4}{\pi^4}\left\{(6\xi^4+4\xi^2+6)A_2 - (5\xi^4+2\xi^2+1)\right.$$
$$\left.\times\int_0^\infty \sin^2(p_a y)\left[1-\frac{1-\exp(-y)}{y}\right]^4\exp(-y)dy\right\}$$

for the [001] orientation,

$$\gamma_{\text{new}} = \frac{8(2\sqrt{3}-\pi)^2p^4}{\pi^4}\left(\frac{1+2\xi}{3}\right)^4\left\{2A_2 - \int_0^\infty \sin^2(p_a y)\right. \qquad (15)$$

$$\left.\times\left[1-\frac{1-\exp(-y)}{y}\right]^4\exp(-y)dy\right\} \text{ for the [111] orientation,}$$

where

$$A_1 = \int_0^\infty\left[\frac{1}{y} - \frac{\exp(-y)}{y} - 1\right]^2\frac{dy}{\exp y} \simeq 0.137;$$

$$A_2 = \int_0^\infty\left[\frac{1}{y} - \frac{\exp(-y)}{y} - 1\right]^4\frac{dy}{\exp y} \simeq 0.042.$$

For $\xi = 1$, the expressions for orientations [001] and [111] are the same and correspond to a glass magneto-active element.

Recently, the authors of Ref. [17] proposed to use a $\lambda/4$-plate with optical axes parallel to the $x$ and $y$ axes, i.e., to the axes of polariser (Fig. 1), for compensation for birefringence in an AE instead of a Faraday rotator. By replacing the matrix $M$ in (7) by the square of the matrix of the $\lambda/4$-plate and performing similar calculations, we obtain the depolarisation ratio $\gamma_4$ for this case:

$$\gamma_4 = \frac{3}{4}\frac{p_a^4}{(1+p_a^2)(1+4p_a^2)}. \qquad (16)$$

Similarly, by replacing the matrix $M$ in (7) by the unit matrix, we obtain the depolarisation ratio $\gamma_a$ in the absence of compensation for birefringence in the AE:

$$\gamma_a = \frac{p_a^2}{1+4p_a^2}. \qquad (17)$$

Thus, in the last two cases, the depolarisation ratio is determined only by the parameter $p_a$, the normalised heat release power (10) in the AE. However, in the case of the Faraday rotator with the conventional [see Eqn (14)] or new [see Eq. (15)] design (Fig. 1), the depolarisation ratio is also determined by two additional parameters: the normalised laser radiation power $p$ (13) in the Faraday rotator and the parameter $\xi$ of a magneto-active crystal.

## 4. Discussion of results and conclusions

Let us discuss the results obtained. Consider two cases: a glass magneto-active element ($\xi = 1$) and a terbium gallium garnet (TGG) magneto-active crystall, which is widely used in in high-power laser systems. The parameter $\xi$ for TGG was recently measured to be 2.2 [14]. First of all, note that the depolarisation ratio (14) for the conventional design of a rotator is proportional to the square of the normalised radiation power ($p^2$), while the depolarisation ratio (15) in the new design is proportional to the fourth degree ($p^4$) of the power. First, this demonstrates a more efficient operation of the new design at $p < 1$, and second, allows one to easily calculate the depolarisation ratio for any $p$ using the plots constructed according to expressions (14), (15) at $p = 1$ and presented in Fig. 3.

In the absence of birefringence in the AE (at $p_a = 0$), the depolarisation ratio in the case of the Faraday rotator is nonzero. At $p_a = 0$, all the integrals in (14) and (15) are zero, and simple expressions obtained in this case describe the dependence of depolarisation in the rotator itself on the laser radiation power (the parameter $p$). These expressions can be useful in analysis of other applications of the Faraday rotator, which are not related to a compensation for birefringence in the AE. In particular, for the [001] orientation the depolarisation ratio $\gamma_{\text{old}}(p_a = 0) = 8A_1p^2/\pi^2$, which coincides with the expression for this case in Ref. [9].

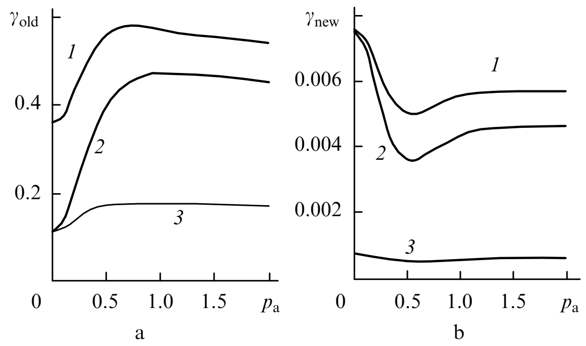


**Figure 3.** Dependences of the non-isolation on $p_a$ plotted by expressions (14) and (15) at $p = 1$ for the conventional (a) and new (b) schemes of the Faraday rotator using TGG with the orientations [111] (*1*) and [001] (*2*) and glass (*3*).

One can see from Fig. 3, that $\gamma_{new}(p_a > 0) < \gamma_{new}(p_a = 0)$, whereas $\gamma_{old}(p_a > 0) > \gamma_{old}(p_a = 0)$, i.e., the birefringence in the AE increases the depolarisation ratio of the conventional Faraday rotator and decreases the depolarisation ratio of the new rotator. This is valid both for crystal and glass magneto-active elements. The dependence of the depolarisation ratio on $p_a$ in the last case is very weak (see Fig. 3), and to estimate the depolarisation ratio at any $p_a$, one can use the value of $\gamma_{old,new}$ at $p_a = 0$, $\xi = 1$.

One can see from Fig. 3 that for both designs of the Faraday rotator for any $p_a$, the [001] orientation is preferable to the [111] orientation, although a substantial difference between them takes place only in the conventional rotator at small $p_a$. It also follows from this figure that the glass magneto-active element produces the depolarisation ratio that is lower than for the TGG element. It is important to note that this is valid only at the same values of $p$ (13).

Fig. 4 illustrates the efficiency of the compensation for birefringence in the AE for the new design of the Faraday rotator compared to those provided by the conventional design and the above-mentioned design with the $\lambda/4$-plate [17]. In the last case, unlike the new design of the rotator, the compensation is efficient only at very low $p_a$, i.e., at weak birefringence in the AE, in agreement with the results of Refs [22, 23]. However, from the practical point of view, the range of high $p_a$ is the most interesting. One can easyly see from Fig. 3 and 4, that in all cases, the depolarisation ratio is saturated rapidly with increasing $p_a$ and tends to a constant. Therefore, in the case of strong birefringence in the AE, all these compensation designs can be conveniently characterised by this constant value of the depolarisation ratio. By passing to the limit at $p_a \to \infty$ in (14)–(17), we obtain

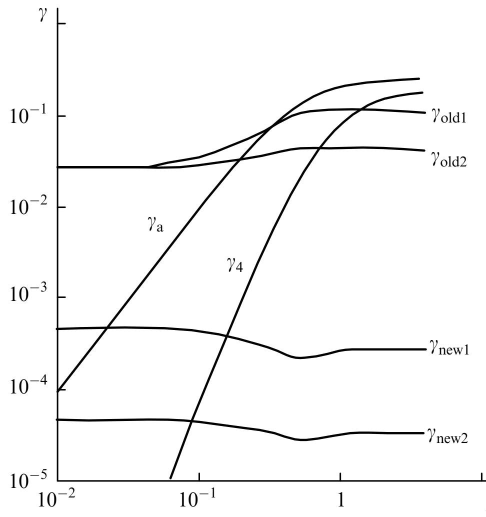


**Figure 4.** Dependences of the non-isolation $\gamma$ on $p_a$, plotted by expressions (14) and (15) at $p = 0.5$ for the conventional (TGG with the [001] orientation ($\gamma_{old1}$), glass ($\gamma_{old2}$)) and new (TGG with the [001] orientation ($\gamma_{new1}$), glass ($\gamma_{new2}$)) Faraday rotators, and also for the compensation scheme with the $\lambda/4$-plate [17] ($\gamma_4$), and without compensation ($\gamma_a$).

$$\gamma_{old}(p_a \to \infty) =$$

$$= \begin{cases} \dfrac{A_1 p^2}{2\pi^2}(9\xi^2 + 2\xi + 13) \text{ for the [001]orientation,} \\ \dfrac{4A_1 p^2}{3\pi^2}(1 + 2\xi)^2 \text{ for the [111] orientation,} \end{cases} \quad (18)$$

$$\gamma_{new}(p_a \to \infty) =$$

$$= \begin{cases} \dfrac{(2\sqrt{3}-\pi)^2 A_2 p^4}{2\pi^4}(7\xi^4 + 6\xi^2 + 11) \\ \qquad\qquad \text{for the [001] orientation,} \\ \dfrac{16(2\sqrt{3}-\pi)^2 A_2 p^4}{27\pi^4}(1 + 2\xi)^4 \\ \qquad\qquad \text{for the [111] orientation,} \end{cases} \quad (19)$$

$$\gamma_4(p_a \to \infty) = \frac{1}{4}, \qquad \gamma_a(p_a \to \infty) = \frac{3}{16}. \quad (20)$$

The formulas for the depolarisation ratio in the case of the Faraday rotator, which were obtained in this and previous sections, are valid if the condition (4) is fulfilled. The value of $\delta_1$ is difficult to measure in practice, and in addition, it depends on the transverse coordinates. For this reason, it is interesting to study the efficiency of the new design of the Faraday rotator, when the condition (4) is violated. In this case, it is impossible to obtain simple analytic expressions for the depolarisation ratio $\gamma_{old,new}$. However, the depolarisation ratio is determined by the same three parameters ($p_a$, $p$ and $\xi$), as for $\delta_1 \ll 1$. The numerical integration in (8) shows that the character of the dependence of the depolarisation ratio on $p_a$ and $\xi$ does not change substantially. Fig. 5 shows the dependences of the depolarisation ratio on $p$ obtained by the numerical integration of (8) at $p_a = 4$, together with the plots constructed by formulas (18)–(20). One can see that exact values of the depolarisation ratio at increasing $p$ are less than the values given by expressions (18) and (19).

Finally, let us estimate the radiation power $P_0$ incident on the Faraday rotator at which the depolarisation ratio is less than 1 %. One can see from Fig. 5 that in this case, expressions (18) and (19) can be used, which yield the values of the parameter $p_{old,new}$ equal to 0.15 and 1.2 for a TGG crystal with the [001] orientation, and 0.25 and 2.1 for glass. For magneto-active glass, by substituting the values $Q/\varkappa = 10^{-6}$ m W$^{-1}$ [24], $\alpha = 10^{-3}$ cm$^{-1}$ [25] and $L/\lambda = 6 \times 10^4$ into (13), we obtain that the depolarisation ratio is less than 1 % at powers lower than 40 W for the conventional design of the Faraday rotator and lower than 350 W

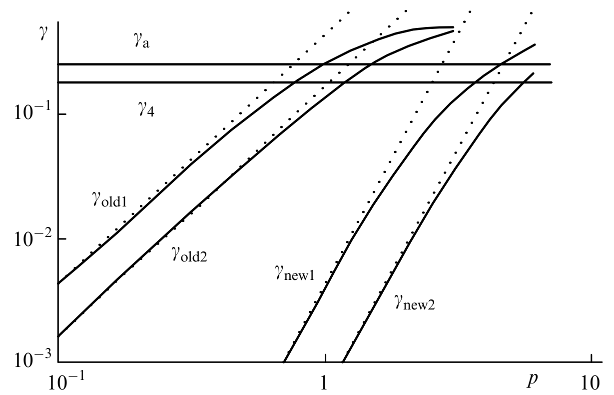


**Figure 5.** Dependences of the non-isolation $\gamma$ on $p$ at $p_a = 4$ for the conventional (TGG with the [001] orientation ($\gamma_{old1}$), glass ($\gamma_{old2}$)) and new (TGG with the [001] orientation ($\gamma_{new1}$), glass ($\gamma_{new2}$)) Faraday rotators, and also for the compensation scheme with the $\lambda/4$-plate [17] ($\gamma_4$), and without compensation ($\gamma_a$). Dotted curves show the corresponding dependences plotted by expressions (18) and (19).

for the new design. Similarly, by substituting the values $Q\alpha = 3.2 \times 10^{-7}$ K$^{-1}$ m$^{-1}$ [14], $\varkappa = 7.4$ W K$^{-1}$ m$^{-1}$, $L/\lambda = 3 \times 10^4$ into (13), we obtain for TGG with the [001] orientation that the depolarisation ratio is less 1 % at the power lower than 120 W for the conventional design of the Faraday rotator and less than 1000 W for the new design.

The above study of the conventional design (Fig. 1a) and the new design (Fig. 1b) of the Faraday rotator, which consists of the 30° Faraday element, the 90° reciprocal rotator, and the 15° Faraday element, allows one to make the following conclusions. The thermal polarisation distortions in the new design of the Faraday rotator are substantially weaker than in the conventional design. The inaccuracy of the compensation for birefringence in the AE by the Faraday mirror (the depolarisation ratio) is determined by the normalised heat release power $p_a$ [Eqn (10)] in the active element, the normalised laser radiation power $p$ [Eqn (13)] in the Faraday rotator, and the combination $\xi$ [Eqn (12)] of the photoelasticity coefficients of a magnetoactive material. The depolarisation ratio in the new design of the Faraday rotator is substantially less than in the conventional design for any values of these parameters. The obtained data show the possibility of creation of a Faraday mirror, which is capable to compensate thermally induced birefringence in the AE with the an error of 1 % at an average incident laser radiation power of 1 kW.

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## References


1. Andreev N F, Palashov O V, Pasmanik G A, Khazanov E A *Kvantovaya Elektron.* **24** 581 (1997) [*Quantum Electron.* **27** 565 (1997)]
2. Andreev N, Khazanov E, Kulagin O, Movshevich B, Palashov O, Pasmanik G, Rodchenkov V, Scott A, Soan P *IEEE J. Quantum Electron.* **35** 110 (1999)
3. Eichier H J, Mehl O, Eichier J *Proc. SPIE Int. Soc. Opt. Eng.* **3613** 166 (1999)
4. Lai K S, Wu R, Phua P B *Proc. SPIE Int. Soc. Opt. Eng.* **3928** 3451 (2000)
5. Hirano Y, Yamomoto S, Tajima T, Taniguichi H, Nakamura M *Proc. CLEO'2000* (San Francisco 2000), paper CPD7
6. Abramovici A, Althouse W E, Drever R W P, Gurvet Y, Kawamura S, Raab F J, Shoemaker D, Sievers L, Spero R E, Thorne K S, Vogt R E, Weiss R, Whitcomb S E, Zucker M E *Science* **256** 325 (1992)
7. Kanabe T, Kawashima T, Matsui H, Okada Y, Kawada Y, Eguchi T, Kandasamy R, Kato Y, Terada, Yamanaka M, Nakatsuka M, Izawa Y, Nakai S, Kanzaki T, Miyajima H *Proc. SPIE Int. Soc. Opt. Eng.* **3889** 190 (2000)
8. Khazanov E A, Kulagin O V, Yoshida S, Reitze D *Proc. CLEO'98* (San Francisco, 1998) paper CWF34
9. Khazanov E A, Kulagin O V, Yoshida S, Reitze D, Tanner D *IEEE J. Quantum Electron.* **35** 1116 (2000)
10. Khazanov E A Kvantovaya Elektron. 26 59 (1999) [*Quantum Electron.* **29** 59 (1999)]
11. Khazanov E A *Proc. SPIE Int. Soc. Opt. Eng.* **3609** 181 (1999)
12. Khazanov E, Andreev N, Babin A, Kiselev A, Palashov O, Reitze D *J. Opt. Soc. Am. B: Opt. Phys.* **17** 99 (2000)
13. Khazanov E A *Kvantovaya Elektron.* **30** 147 (2000) [*Quantum Electron.* **30** 147 (2000)]
14. Khazanov E, Andreev N, Palashov O, Potoemkin A, Sergeev A, Mehl O, Reitze D *Appl. Opt.* (to be published)
15. Andreev N F, Bondarenko N G, Eremina E V, Kuznetsov C V, Palashov O V, Pasmanik G A, Khazanov E A *Kvantovaya Elektron.* **18** 1154 (1991) [*Sov. J. Quantum Electron.* **21** 1045 (1991)]
16. Denmar C A, Libby S I In: *OSA TOPS* (Washington) **26** 608 (1999)
17. Clarkson W A, Feigate N S, Nanna D C *Opt. Lett.* **24** 820 (1999)
18. Tabor M J, Chen F S *J. Appl. Phys.* **40** 2760 (1969)
19. Jaecklin A A, Lietz M *Appl. Opt.* **11** 617 (1972)
20. Massey G A *Appl. Phys. Lett.* **17** 213 (1970)
21. Nye J E *Physical Properties of Crystals* (London, Oxford University Press, 1964; Moscow: Inostrannaya Literatura, 1960)
22. Hua R, Wada S, Tashiro H *Opt. Commun.* **175** 189 (2000)
23. Kandasamy R, Yamanaka M, Izawa Y, Nakui S *Opt. Rev.* **7** 149 (2000)
24. Andreev N F, Babin A A, Kiselev A M, Palashov O V, Khazanov E A, Zarubina T V, Tshavelev O S *Opt. Zh.* **67** (6) 66 (2000)
25. Zarubina T V, Petrovskii G T *Opt. Zh.* **59** (11) 48 (1992)