PACS numbers: 42.25.Fx; 05.40.-a; 78.20.Ci
DOI: 10.1070/QE2001v031n04ABEH001951

Backscattering amplification of laser radiation in a medium with fluctuations of the imaginary part of permittivity

R Kh Almaev, A A Suvorov

Abstract. **The effect of backscattering amplification of laser radiation with respect to the radiation intensity reflected from an ordinary mirror in a medium with fluctuations of the real (refractive index) and the imaginary (absorption or amplification coefficient) parts of the permittivity is considered. Formulas for the backscattering amplification coefficient and the variance of the intensity fluctuations of the reflected wave propagating in a random dissipative (amplifying) medium are derived. Asymptotic expressions derived for the saturation region of intensity fluctuations take into account the effect of fluctuations of the refractive index and absorption (amplification) coefficient, as well as their correlation. The contribution of fluctuations of the complex permittivity parts and the characteristic spatial scale of the problem to the backscattering amplification coefficient is analysed. It is shown that for uncorrelated fluctuations of the real and imaginary parts of the permittivity of a random medium, the backscattering amplification coefficient in the region of strong fluctuations is larger than in a transparent random medium. It is also found that the correlation of pulsations of the real and imaginary parts of the permittivity suppresses the backscattering amplification effect in an absorbing medium and increases this effect in an amplifying medium.**

Keywords: *laser radiation, permittivity, fluctuations*

1. Introduction

Fluctuations of the imaginary part of the permittivity leading to a random variation in the amplitude characteristics of the wave (random absorption or random amplification) as well as its phase characteristics play an important role in the interaction of radiation with matter considered in a number of problems of laser physics and statistical optics. For example, the active medium of a He – Cd laser pumped by nuclear fission fragments of uranium exhibits noticeable pulsations of the complex permittivity ε [1], leading to an increase in the divergence of the laser beam [2].

R Kh Almaev Institute of Experimental Meteorology, prosp. Lenina 82, 249020 Obninsk, Kaluzhskaya oblast, Russia;
e-mail: skr@iem.obninsk.ru
A A Suvorov Institute of Physics and Power Engineering, pl. Bondarenko 1, 249020 Obninsk, Kaluzhskaya oblast, Russia;
e-mail: suvorov@ippe.rssi.ru

Received 21 June 2000; revision received 26 December 2000
Kvantovaya Elektronika **31** (4) 357 – 362 (2001)
Translated by R Wadhwa

In an X-ray laser, random inhomogeneities of the permittivity ε caused by the small-scale instabilities of laser-induced plasma [3, 4] considerably affect the energy parameters and spatial coherence of amplified spontaneous emission. The fluctuations of the imaginary part of the permittivity of the medium should be also taken into account in problems of propagation of radio waves in atmosphere and of laser radiation in a bleached liquid-drop aerosol (see, for example, Refs [5, 6]).

An interesting feature of the interaction of radiation with a random absorbing (amplifying) medium is the impossibility to compensate, with the help of a phase-conjugate (PC) mirror, the phase fluctuations of the reflected wave, which are induced by pulsations of the imaginary part of the permittivity ε of the medium [7]. This circumstance should be taken into account while developing adaptive optical systems operating in media with fluctuations of the real and imaginary parts of ε. Note also that the disregard of fluctuations of the imaginary part of ε may lead to serious errors in an analysis of propagation of radiation with wavelengths close to the absorption lines in the atmosphere. Moreover, we showed earlier [8, 9] that even relatively small pulsations of the attenuation coefficient of a turbulent medium result in considerable changes in the behavior of the variance of radiation intensity fluctuations over long paths (in the region of saturation of intensity fluctuations).

Up to now, the main attention of researchers studying the propagation of waves in media with a random attenuation (amplification) was concentrated on the radiation transfer along straight paths [2 – 6, 8 – 11]. It is well known, however (see, for example, [12, 13] and references therein), that the propagation of waves over location paths has some peculiarities associated with double passage of a signal through correlated random inhomogeneities (on the forward and backward paths). For transparent turbulent media, in which the fluctuations of the real part of the permittivity ε are the only source of wave randomization, the effects accompanying the location signal propagation are studied in detail by using the classical theory of waves in random media [14 – 16]. However, in the case of absorbing (amplifying) random media, such investigations virtually have not been carried out; for this reason, it would be interesting to analyse the effect of pulsations of the imaginary part of ε and their correlations with the fluctuations of the real part of the permittivity on the characteristics of a location signal.

In this paper, we consider the features of the manifestation of an effect typical of location problems, the so-called backscattering amplification of radiation with respect to the radiation intensity reflected from an ordinary mirror during its propagation in a turbulent medium with fluctuation of the real ($\tilde{\varepsilon}_{\rm R}$) as well as imaginary ($\tilde{\varepsilon}_{\rm Im}$) parts of ε. For definiteness, we will henceforth assume that the medium in which radiation propagates is randomly dissipative and will obtain results specifically for this case. A generalization of the obtained results to a random amplifying medium will be carried out by reversing the sign of $\tilde{\varepsilon}_{\rm Im}$.

2. General concepts of the theory

We assume that a laser beam propagates in the positive direction of the z axis from a source located in the plane $z=0$. A mirror with a preset distribution of the reflection coefficient is located in the plane $z=L$, while the detector of the reflected signal is placed in the plane of the source ($z=0$). We will describe the propagation of the laser beam in an absorbing turbulent medium for the direct $[U_+(\boldsymbol{\rho},z)]$ as well as the reflected $[U_-(\boldsymbol{\rho},z)]$ wave with the help of the known parabolic equation of quasi-optics for the complex amplitude of the wave (see, for example [13])

$$\pm 2\mathrm{i}k\frac{\partial U_\pm}{\partial z}+\Delta_\perp U_\pm+k^2\Delta\varepsilon U_\pm=0, \tag{1}$$

with the following boundary conditions: $U_+(\boldsymbol{\rho},z)|_{z=0}=U_0(\boldsymbol{\rho})$ in the plane of the source and $U_-(\boldsymbol{\rho},z)|_{z=L}=f(\boldsymbol{\rho})U_+(\boldsymbol{\rho},L)$ in the reflection plane. Here, k is the wave number; $\boldsymbol{\rho}=\{x,y\}$ is the radius vector in the plane $z=$ const; $\Delta_\perp$ is the Laplace operator in variables x,y; $f(\boldsymbol{\rho})$ is the mirror reflection coefficient (which is complex in the general case); $\Delta\varepsilon=(\varepsilon-\varepsilon_{0\rm R})/\varepsilon_{0\rm R}=\mathrm{i}\bar{\varepsilon}_{\rm Im}+\tilde{\varepsilon}$ the relative change in the permittivity of the medium; $\varepsilon_{0\rm R}$ is the average real part of ε ($\varepsilon_{0\rm R}\approx 1$ for optical radiation in a gaseous medium); $\bar{\varepsilon}_{\rm Im}$ is the average imaginary part of ε; $\tilde{\varepsilon}=\tilde{\varepsilon}_{\rm R}+\mathrm{i}\tilde{\varepsilon}_{\rm Im}$ is the random component of the permittivity of the medium.

Using the Huygens–Kirchhoff principle, we can conveniently present the solution of Eqn (1) for problems of location transport in the integral form:

$$U_+(\boldsymbol{\rho},z)=\int \mathrm{d}^2\rho' U_0(\boldsymbol{\rho}')G_+(\boldsymbol{\rho},z;\boldsymbol{\rho}',0), \tag{2}$$

$$U_-(\boldsymbol{\rho},z)=\int\!\!\int \mathrm{d}^2\rho'\mathrm{d}^2\rho'' f(\boldsymbol{\rho}'')G_-(\boldsymbol{\rho},0;\boldsymbol{\rho}''L)$$
$$\times U_0(\boldsymbol{\rho}')G_+(\boldsymbol{\rho}'',L;\boldsymbol{\rho}',0), \tag{3}$$

where $G_\pm(\boldsymbol{\rho},z;\boldsymbol{\rho}',z')$ is the Green function of Eqn (1) for the forward (+) and reflected (–) waves.

The expression for the complex amplitude of the reflected wave written in form (3) allows us to construct various multiplicative combinations of complex amplitudes as well as (after averaging over the ensemble of realisations) various statistical moments of the location signal amplitude in the integral form.

To reveal the effects associated with the double passage of a location signal through a medium with correlated inhomogeneities on the forward and backward paths, one normally uses the quantities composed of the ratio of the chosen wave parameter for the location path to the same parameter for the forward path of the same length. In this work, we used the gain relative to the mean intensity to analyse the backscattering amplification in a random absorbing medium:

$$\bar{N}(L)=\frac{\langle I_-(\boldsymbol{\rho},0)\rangle}{\langle I_+(\boldsymbol{\rho},2L)\rangle}, \tag{4}$$

where $\langle I_-(\boldsymbol{\rho},0)\rangle$ is the mean intensity of the reflected laser beam traversing the path from the source ($z=0$) to the mirror ($z=L$) and from the mirror ($z=L$) to the detector ($z=0$) after the reflection; $\langle I_+(\boldsymbol{\rho},2L)\rangle$ is the mean intensity of radiation traversing the forward path of length $2L$ from the same source ($z=0$) to the detector in the plane $z=2L$; the angle brackets indicate the averaging over the ensemble of realizations of the random field of ε.

The quantities $\langle I_-\rangle$ and $\langle I_+\rangle$ appearing in Eqn (4) can be written in the integral form with the help of expressions (2) and (3) for U_- and U_+ as well as the complex conjugate expressions. The application of the reciprocity principle generalised to the case of absorbing media in this case makes it possible to eliminate the dependence of $\langle I_-\rangle$ on the Green function of the reflected wave. As a result, we obtain the following expressions for $\langle I_+\rangle$ and $\langle I_-\rangle$:

$$\langle I_+(\boldsymbol{\rho},2L)\rangle=\frac{c}{8\pi}\langle U_+(\boldsymbol{\rho},2L)U_+^*(\boldsymbol{\rho},2L)\rangle$$
$$=\frac{c}{8\pi}\int\!\!\int \mathrm{d}^2\rho'\mathrm{d}^2\rho''\Gamma_{20}(\boldsymbol{\rho}',\boldsymbol{\rho}'')\langle G_2(\boldsymbol{\rho},2L|\boldsymbol{\rho}',0;\boldsymbol{\rho}'',0)\rangle, \tag{5}$$

$$\langle I_-(\boldsymbol{\rho},0)\rangle=\frac{c}{8\pi}\langle U_-(\boldsymbol{\rho},0)U_-^*(\boldsymbol{\rho},0)\rangle$$
$$=\frac{c}{8\pi}\int\!\!\int\!\!\int\!\!\int \mathrm{d}^2\rho'\mathrm{d}^2\rho''\mathrm{d}^2t'\mathrm{d}^2t''\Gamma_{20}(\boldsymbol{\rho}',\boldsymbol{t}')f(\boldsymbol{\rho}'')f^*(\boldsymbol{t}'')$$
$$\times\langle G_4(\boldsymbol{\rho}'',L;\boldsymbol{t}'',L|\boldsymbol{\rho},0;\boldsymbol{\rho}',0;\boldsymbol{t}',0)\rangle, \tag{6}$$

where $\Gamma_{20}(\boldsymbol{\rho},\boldsymbol{\rho}'')=U_0(\boldsymbol{\rho}')U_0^*(\boldsymbol{\rho}'')$ is the coherence function of the source and

$$\langle G_2(\boldsymbol{\rho},2L|\boldsymbol{\rho}',0;\boldsymbol{\rho}'',0)\rangle=\langle G_+(\boldsymbol{\rho},2L;\boldsymbol{\rho}',0)$$
$$\times G^*{}_+(\boldsymbol{\rho},2L;\boldsymbol{\rho}'',0)\rangle; \tag{7}$$

$$\langle G_4(\boldsymbol{\rho}'',L;\boldsymbol{t}'',L|\boldsymbol{\rho},0;\boldsymbol{\rho}',0;\boldsymbol{t}',0)\rangle=\langle G_+(\boldsymbol{\rho}'',L;\boldsymbol{\rho},0)$$
$$\times G_+^*(\boldsymbol{\rho}'',L;\boldsymbol{\rho}',0)G_+^*(\boldsymbol{t}'',L;\boldsymbol{\rho},0)G_+(\boldsymbol{t}'',L;t',0)\rangle. \tag{8}$$

One can see from Eqns (4)–(8) that the gain written taking into account the above expressions depends on the distribution of the complex amplitude U_0 of the radiation source and its calculation requires the statistical moments of the Green function G_+.

As mentioned in the Introduction, the effect of even relatively small pulsations of absorption is especially noticeable on long paths over which the intensity fluctuations are saturated. This situation will be analysed by us here. The statistical moments of the wave (both forward and reflected) in the region of saturation of intensity fluctuations can be calculated by represanting the Green function G_+ in the form of the Feynman integral over trajectories:

$$G_+(\boldsymbol{\rho},z;\boldsymbol{\rho}',z')$$
$$=\int_{\rho'}^{\rho}\mathrm{D}^2v(\xi)\exp\left\{\frac{\mathrm{i}k}{2}\int_{z'}^{z}\mathrm{d}\xi\left[\dot{v}^2(\xi)+\Delta\varepsilon(v(\xi),\xi)\right]\right\}, \tag{9}$$

where $\int D^2 v(\xi)$ denotes the functional integration with respect to $v(\xi)$; $\dot{v} = dv/d\xi$; the integration is carried out over all trajectories beginning at the point $(\boldsymbol{\rho}', z')$ and terminating at the point $(\boldsymbol{\rho}, z)$. The above representation of the Green function will be used in the subsequent analysis for obtaining specific results.

3. Solution of the problem

Consider the peculiarities of double passage of a laser beam in a medium with correlated random inhomogeneities of the complex permittivity in the spherical wave mode, when reflection is performed by a 'point' mirror. Such a situation is realised in practice when a mirror of radius l is located in the far-field diffraction zone relative to the detector and the radiation source ($ka^2/L \ll 1$, $kl^2/L \ll 1$, where a is the aperture radius of the source) and also when the sizes of the mirror and the source aperture are smaller than the coherence radius ρ_c of the wave.

To eliminate the effect of reflection parameters of the mirror (to create identical conditions for the propagation of forward and reflected waves) on the amplification of backward scattering, we assume that the intensity $\langle I_+(\boldsymbol{\rho}, 2L)\rangle$ of radiation traversing the direct path length $2L$, which appears in formula (4), is formed as follows. The signal from the source approaches the screen in the plane $z = L$, which introduces amplitude–phase changes in it with a complex amplitude f_0, similar to those introduced by the mirror in the reflected signal, and then propagates to the direct signal detector located in the plane $z = 2L$. Assuming that random inhomogeneities of the medium on the path segments $(0, L)$ and $(L, 2L)$ are uncorrelated and using Eqns (5)–(8), we can write

$$\langle I_-(\boldsymbol{\rho}, 0)\rangle = \left(4\pi^2 a^2 l^2 |f_0|\right)^2 \times I_0 \langle |G_+(0, L; \boldsymbol{\rho}, 0)|^2 |G_+(0, L; 0, 0)|^2\rangle, \quad (10)$$

$$\langle I_+(\boldsymbol{\rho}, 2L)\rangle = \left(4\pi^2 a^2 l^2 |f_0|\right)^2 \times I_0 \langle |G_+(0, 2L; \boldsymbol{\rho}, L)|^2\rangle \langle |G_+(0, L; 0, 0)|^2\rangle, \quad (11)$$

where I_0 is the intensity of the radiation source. Expressions (10) and (11) show that the intensity of the reflected wave is proportional to the correlation function of the intensity of the spherical wave, i.e., the fourth statistical moment of the function G_+, while the intensity of the forward wave is proportional to the product of the mean intensities of the spherical wave, i.e., the product of second statistical moments of the function G_+.

We will assume that random inhomogeneities of the medium are statistically homogeneous and isotropic, are optically soft and have characteristic scales which, on the one hand, are much larger than the radiation wavelength, and on the other hand, much smaller than the path length. In this case, the approximation of the Markovian random process for ε fluctuations is applicable for calculating the static moments of the spherical wave intensity, which determine $\langle I_-\rangle$ and $\langle I_+\rangle$ (see, for example, Ref. [17]). Using the asymptotic method [18] and the 'cumulative' method of the solution of wave problems [7] for calculating the fourth moment of function G_+ and omitting intermediate calculations, we obtain the following expressions for $\langle I_-(\boldsymbol{\rho}, 0)\rangle$ in the saturation region under investigation:

$$\langle I_-(\boldsymbol{\rho}, 0)\rangle = 2\langle I_+(0, 2L)\rangle e^{\gamma^2(L)} \quad \text{at}\ \rho \ll \rho_c, \quad (12)$$

$$\langle I_-(\boldsymbol{\rho}, 0)\rangle = \langle I_+(0, 2L)\rangle \quad \text{at}\ \rho \gg \rho_c, \quad (13)$$

where

$$\gamma^2(L) = \pi k^2 L \int_0^1 d\xi \int d^2 q \Big\{ \Phi_-(q) \Big[1 - \cos\Big(\frac{q^2 L}{k}\xi(1-\xi)\Big)\Big] - 2\Phi_{\mathrm{RIm}}(q) \sin\Big[\frac{q^2 L}{k}\xi(1-\xi)\Big]\Big\} \times \exp\Big\{-\frac{k^2 L}{2}\Big[\int_0^{\xi} d\eta D_+\Big(\frac{\boldsymbol{q}L}{k}\eta(1-\xi)\Big) + \int_0^{1-\xi} d\eta D_+\Big(\frac{\boldsymbol{q}L}{k}\xi\eta\Big)\Big]\Big\} + 2\pi k^2 L \int_0^1 d\xi \int d^2 q \Phi_{\mathrm{Im}}(q) \times \exp\Big\{-\frac{k^2 L}{2}\Big[\int_0^{\xi} d\eta D_+\Big(\frac{\boldsymbol{q}L}{k}\eta(1-\xi)\Big) + \int_0^{1-\xi} d\eta D_+\Big(\frac{\boldsymbol{q}L}{k}\xi\eta\Big)\Big]\Big\}; \quad (14)$$

$\Phi_{\mathrm{R}}, \Phi_{\mathrm{Im}}, \Phi_{\mathrm{RIm}}$ are the fluctuation spectra of the real and imaginary parts of ε as well as their correlations; $\Phi_-(q) = \Phi_{\mathrm{R}}(q) - \Phi_{\mathrm{Im}}(q)$; $D_+(\boldsymbol{r}) = D_{\mathrm{R}}(\boldsymbol{r}) + D_{\mathrm{Im}}(\boldsymbol{r})$; $D_{\mathrm{R}}, D_{\mathrm{Im}}$ are the structural functions of fluctuations of the real and imaginary parts of the permittivity of the medium:

$$D_\alpha(\boldsymbol{r}) = 2\pi \int d^2 q \Phi_\alpha (1 - \cos \boldsymbol{qr}).$$

Expressions (12) and (13) make it possible to write the following relations for the backscattering amplification coefficient (4):

$$N(L) = 2e^{\gamma^2(L)} \quad \text{for}\ \rho \ll \rho_c, \quad (15)$$

$$\bar{N}(L) = 1 \quad \text{for}\ \rho \gg \rho_c. \quad (16)$$

It follows from these expressions that the intensity of the scattered wave exceeds the intensity of the wave that has traversed the double path without reflection in the axial region of the beam at a distance from its centre smaller than the coherence radius of the wave. Since the coherence radius for strong intensity fluctuations coincides with the smallest scale of the correlations of radiation intensity pulsations [18], the amplification of backscattering takes place in the region $\rho < \rho_c$ as a result of correlation of random variations of the intensities of the forward and reflected waves (see, for example, Ref. [12]).

The emergence of backscattering amplification for a location signal propagating in a random absorbing (amplifying) medium can be visually illustrated using the following simplified model. Let a spherical wave propagate from the point $\rho = 0$, $z = 0$ in the positive direction of the z axis and be isotropically scattered at the point $\rho = 0$, $z = L$ with the scattering amplitude f_0. We compare the mean intensity of the wave for its propagation along two paths: the location path along which the wave runs from the source to the scatterer, undergoes backscattering and is received at the point $\rho = 0$, $z = 0$, and the forward path along which the spherical wave approaches the scatterer, is directed forward from it and is received at the point $\rho = 0$, $z = 2L$.

We assume that random variations of the real and imaginary parts of ε are uncorrelated and disregard the effect of inhomogeneities of $\tilde{\varepsilon}_{\rm Im}$ in the transverse directions on the wave parameters. We also take into account the fact that in view of δ correlation on the path segments $(0, L)$ and $(L, 2L)$, the fluctuations of ε are statistically independent. In this case, the mean intensities of the backward ($\langle I_{\rm s-}\rangle$) and forward ($\langle I_{\rm s+}\rangle$) scattering of waves at the points of reception can be presented in the form

$$\langle I_{\rm s-}\rangle = |f_0|^2 \langle I_{\rm s}^2(L,0)\rangle \langle {\rm e}^{-2\tilde{\tau}(L,0)}\rangle {\rm e}^{-2\bar{\tau}},$$

$$\langle I_{\rm s+}\rangle = |f_0|^2 \langle I_{\rm s}(2L,L)\rangle \langle {\rm e}^{-\tilde{\tau}(2L,L)}\rangle \langle I_{\rm s}(L,0)\rangle \langle {\rm e}^{-\tilde{\tau}(L,0)}\rangle {\rm e}^{-2\bar{\tau}},$$

where $I_{\rm s}(z_1, z_0)$ is the intensity at the point $\rho = 0, z = z_1$ of a spherical wave whose source is at the point $\rho = 0$, $z = z_0$ in a medium without fluctuations of ${\rm Im}\varepsilon$; $\bar{\tau}$ is the mean optical thickness of the path of length L; $\tilde{\tau}(z_1, z_0)$ are the optical thickness fluctuations for the path on segment from z_1 to z_0. Assuming that the fluctuations of ε are statistically homogeneous and exhibit the same properties on both segments of the path, we obtain the following expression for the backscattering amplification coefficient $\bar{N}$ (4):

$$\bar{N} = \frac{\langle I_{\rm s}^2(L,0)\rangle}{\langle I_{\rm s}(L,0)\rangle^2} \frac{\langle {\rm e}^{-2\tilde{\tau}(L,0)}\rangle}{\langle {\rm e}^{-\tilde{\tau}(L,0)}\rangle^2} > 1. \qquad (17)$$

Note that the estimate $\bar{N} > 1$ is obtained from the well-known relation $\langle t^2\rangle > \langle t\rangle^2$ between the statistical moments of the random quantity. In the simplified model under study, the fluctuations of the real (factor $\langle I_{\rm s}^2\rangle / \langle I\rangle^2 > 1$) and imaginary (factor $\langle \exp(-2\tilde{\tau})\rangle / \langle \exp(-\tilde{\tau})\rangle^2 > 1$) parts of ε make independent contributions to the backscattering amplification.

4. Discussion of results

Let us now return to the initial problem and analyse expression (15), which can be written in an alternative form:

$$\bar{N}(L) = 1 + \sigma_I^2(L), \quad \rho \ll \rho_{\rm c}\,, \qquad (18)$$

where $\sigma_{\rm I}^2$ is the relative variance of the intensity fluctuations for a spherical wave propagating without reflection in a random absorbing medium along a path of length L in the saturation region. Relation (18) is obtain by using formulas (10)–(12) and the definition of the relative variance of a random quantity.

Let us analyse the propagation of a laser beam in a medium for which the components of the permittivity fluctuations are described by the spectrum of autocorrelations and correlation of the form

$$\Phi_\alpha(q) = 0.033 C_\alpha^2 \left(q_0^2 + q^2\right)^{-11/6} \exp\left(-q^2/q_{\rm m}^2\right), \qquad (19)$$

where C_α^2 is the structural constant for the fluctuations of the real part of ε (for $\alpha = {\rm R}$), pulsations of the imaginary part of ε (for $\alpha = {\rm Im}$), and their correlations (for $\alpha = {\rm RIm}$ or ImR; $C_{\rm ImR}^2 = C_{\rm RIm}^2$); $q_0 = 2\pi/L_0$; L_0 is the outer scale of turbulence; $q_{\rm m} = 5.92/l_{\rm m}$; $l_{\rm m}$ is the inner scale of turbulence. A spectrum of form (19) is typical of media with developed turbulence and is used, for example, for studying the propagation of waves in turbulent atmosphere [14, 15].

It follows from expressions (14), (15), and 18) that the backscattering amplification coefficient in a random dissipative medium is determined both by fluctuations of the real and imaginary parts of ε and also by their correlations. The dependences of the contributions of fluctuations of the permittivity parts and their correlations to the backscattering amplification coefficient on the parameters of the problem (length L, the modulus of ε pulsations, etc.) are different. An analysis of the integrals with respect to spatial frequencies in (14) indicates that the integrands are bounded both by the scales of permittivity fluctuations and by the scale $q_{\rm c} \sim k\rho_{\rm c}/L$.

As the path length (or the modulus of the permittivity fluctuations of the medium) increases, the coherence radius decreases, which in turn leads to a decrease in the scale $q_{\rm c}$ bounding the integrands on the high-frequency side (since $q_{\rm c} \ll q_{\rm m}$ for media with developed turbulence). This means that the relation between the scales q_0 and $q_{\rm c}$ determines to a certain extent the dependence of the backscattering amplification coefficient on the path length in the range of strong fluctuations.

Thus, in a transparent turbulent medium, the quantity $\bar{N}(L)$ tends monotonically to the value two with increasing in the path length (see, for example, [13]), and the relation between q_0 and $q_{\rm c}$ affects only the saturation rate of $\bar{N}(L)$, while in a dissipative random medium the dependence of the backscattering amplification coefficient on L is extremely nonmonotonic for $q_0 \leqslant q_{\rm c}$, and an asymptotically monotonic saturation $\bar{N} \to 2$ is observed only for $q_0 \geqslant q_{\rm c}$. Consequently, for $\varepsilon_{\rm Im} \neq 0$, the behavior of $\bar{N}(L)$ differs significantly from its behavior in a transparent random medium. The relative amplification of backscattering considerably depends in this case on the extent of correlation between the fluctuations of the imaginary and real parts of the permittivity as well as on the sign of $\varepsilon_{\rm Im}$.

In the case when $\tilde{\varepsilon}_{\rm R}$ and $\tilde{\varepsilon}_{\rm Im}$ are not correlated, the value of σ_I^2 in the saturation region for $q_0 \leqslant q_{\rm c}$ exceeds the relative variance of the intensity fluctuations for a transparent medium [9] , and it may be concluded in accordance with relation (18) that $\bar{N}(L) > \bar{N}(L)|_{\varepsilon_{\rm Im}=0}$ in this situation. The latter inequality holds for both absorbing and amplifying media. Thus, the backscattering enhancement in a random medium with uncorrelated pulsations $\varepsilon_{\rm R}$ and $\varepsilon_{\rm Im}$ is more significant than in a transparent medium.

The existence of a correlated coupling between $\tilde{\varepsilon}_{\rm R}$ and $\tilde{\varepsilon}_{\rm Im}$ is reflected in the behavior of $\bar{N}(L)$, and comparative amplification of backscattering for absorbing and amplifying media is different. Thus, the correlation term for absorbing media appears in γ^2 with the minus sign [see relation (14)]; in the case of positive $\tilde{\varepsilon}_{\rm R}$ and $\tilde{\varepsilon}_{\rm Im}$, this leads to a decrease in the coefficient $\bar{N}(L)$ with increasing path length or $\langle|\tilde{\varepsilon}_{\rm R}|\rangle$. As a result, the inequality $\bar{N}(L) < 2$ may be satis-fied for certain values of the parameters in the problem; i.e., the amplification of backscattering in an absorbing random medium will be smaller than in a transparent medium. A different situation is observed for an amplifying random medium. In this case, positive correlations between $\tilde{\varepsilon}_{\rm R}$ and $\tilde{\varepsilon}_{\rm Im}$ lead to an increase in $\bar{N}(L)$ (as compared to a transparent medium as well as a medium without correlations between $\tilde{\varepsilon}_{\rm R}$ and $\tilde{\varepsilon}_{\rm Im}$), which is the stronger the longer the path or the amplitude of permittivity fluctuations.

The effect of pulsations of the real and imaginary parts of the permittivity as well as of their correlation on the fluctuations of the wave intensity can be described qualitatively using the model of interaction between radiation and a random inhomogeneity, which was proposed by Tatarskii in monograph [14] for describing the propagation of a wave in a transparent turbulent atmosphere. In accordance with this model, a random inhomogeneity of ε_R (vortex) in a transparent medium is represented in the form of a spherical lens of radius l, whose permittivity differs from the mean value $\bar{\varepsilon}_R$ of the medium by $\pm\delta\varepsilon_R$: $\varepsilon_R = \bar{\varepsilon}_R \pm \delta\varepsilon_R$. In the absence of a random lens, the radiant flux through an area element of radius l is conserved, and the wave intensity within this area element (including its centre) is constant and equal, say, to I_0.

Let us now imagine that a random spherical lens of radius l with $\varepsilon_R = \bar{\varepsilon}_R \pm \delta\varepsilon_R$ appears on the path of the wave. Depending on the sign of $\delta\varepsilon_R$, this lens focuses (or defocuses) the radiation whose intensity behind the lens (on its axis) will be larger (smaller) than I_0: $I = I_0 \pm\delta I$. In this way, we can visualise the emergence of intensity fluctuations in a transparent random medium. This model, as applied to an ensemble of vortices, enabled the author of [14] to explain qualitatively the behavior of the wave intensity fluctuations in a turbulent atmosphere in the geometrical optics approximation as well as in the case when diffraction at vortices is taken into account and even in the range of strong fluctuations.

We will use now the approach described above to explain qualitatively the effect of pulsations of the complex permittivity of the medium on radiation propagating in a random absorbing or amplifying medium. We may assume that random lenses associated with pulsations of ε_R and suppressing (enhancing) spherical inhomogeneities (vortices) associated with pulsations of ε_{Im} emerge independently in a medium with uncorrelated pulsations of ε_R and ε_{Im}. In this case, the vortices associated with pulsations of ε_R cause, as before, the fluctuations of the wave due to focusing (defocusing). On the other hand, the vortices associated with pulsations of ε_{Im} lead to radiation intensity fluctuations as a result of additional absorption (amplification) in the inhomogeneities. In the case when the pulsations of ε_R and ε_{Im} are uncorrelated, their contributions to the fluctuations of radiation are independent and additive.

The situation is different when random variations of ε_R and ε_{Im} are correlated. Consider in greater detail the media which are encountered most often in real conditions, in which a random increase (decrease) in ε_R is accompanied by a simultaneous increase (decrease) in ε_{Im}. For instance, these are the media in which the pulsations of ε_R and ε_{Im} are associated with pulsations of the density of the medium. If the medium under investigation is absorbing, a simultaneous increase in ε_R and ε_{Im} leads, on the one hand, to an increase in the wave intensity behind a vortex as a result of focussing and, on the other hand, to a decrease in the intensity due to additional absorption in the vortex; i.e., these mechanisms operate in antiphase relative to the change in the intensity.

Thus, the correlated local variation of ε_R and ε_{Im} in absorbing media leads to a decrease in the wave intensity fluctuations as compared to the case when ε_R and ε_{Im} are uncorrelated. On the contrary, a simultaneous increase (decrease) in ε_R and ε_{Im} for an amplifying random medium causes an increase (decrease) in the radiation intensity behind an inhomogeneity; i.e., this leads to an increase in the randomization of radiation relative to the case of a medium with uncorrelated fluctuations of ε_R and ε_{Im}. Naturally, the quantitative expression describing these effects is determined by many factors such as the size distribution of vortices (ε_R and ε_{Im} fluctuation spectra), parameters of radiation, realisation of conditions for weak and strong fluctuations, and parameters of the reflector for location problems, etc.

To illustrate the above analysis, we present in Figs 1 and 2 the results of calculation of the dependence of $\bar{N}$ on β_R (where $\beta_R^2 = 0.31 C_R^2 k^{7/6} L^{11/6}$ is the relative variance of weak intensity fluctuations of a plane wave, which is a dimensionless parameter characterising the conditions of radiation propagation in a turbulent medium with a spectrum of form (19) (see, for example, [14])) for various values of parameter $\delta = \langle\tilde{\varepsilon}_{Im}^2\rangle/\langle\tilde{\varepsilon}_R^2\rangle$, as well as the coefficient $b_{RIm} = \langle\tilde{\varepsilon}_R\tilde{\varepsilon}_{Im}\rangle/\left(\langle\tilde{\varepsilon}_R^2\rangle\langle\tilde{\varepsilon}_{Im}^2\rangle\right)^{1/2}$ describing the correlation of pulsations of the real and imaginary parts of the permittivity of the medium. Figs 1 and 2 reflect the situation when the backscattering amplification coefficient in a medium with pulsations of the imaginary part of ε differs significantly from that for a transparent turbulent medium. In this case, expression (15) for $\bar{N}$ has the form

$$\bar{N}(L) = 2\exp\left[1.4\beta_R^{-4/5}(L) - 3.87 b_{RIm}\delta^{1/2}\beta_R^{8/5}(L) + 0.11\delta\beta_R^{10/11}(kL_0^2)\beta_R^{12/11}(L) - 0.62\delta\beta_R^4(L)\right]. \tag{20}$$

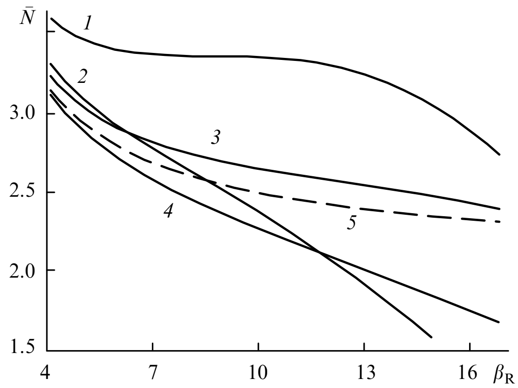

Figure 1. Backscattering amplification coefficient for the mean intensity of a reflected wave in a dissipative random medium as a function of β_R for $\delta = 5\times10^{-6}$ (*1, 2*), 10^{-6} (*3, 4*) and 0 (*5*), and also for $b_{RIm} = 0$ (*1, 3*) and 1.0 (*2, 4*).

It follows from this relation that the parameters simulating the conditions for the propagation of a location beam are the correlation coefficient b_{RIm}, the relative variance δ of fluctuations of the imaginary part of the permittivity, the parameter $\beta_R(L)$ on an arbitrary length L and the same parameter on the length kL_0^2. The results of calculations presented in the figures were obtained for $\beta_R(kL_0^2) = 1.56\times10^5$. In particular, such a situation is realised in a turbulent atmosphere for laser radiation in the visible range for $C_R^2 = 10^{-14}$ cm$^{-2/3}$ and $L_0 = 5$ m.

One can see from Figs 1 and 2 that in contrast to the case of a transparent random medium, when the value of $\bar{N}$ decreases monotonically with increasing β_R (curves *5* in both figures), in a medium with complex permittivity fluctuations, the variation of the backward scattering amplification coefficient with increasing β_R is ambiguous and strongly depends on the correlation coupling between $\tilde{\varepsilon}_R$ and $\tilde{\varepsilon}_{Im}$

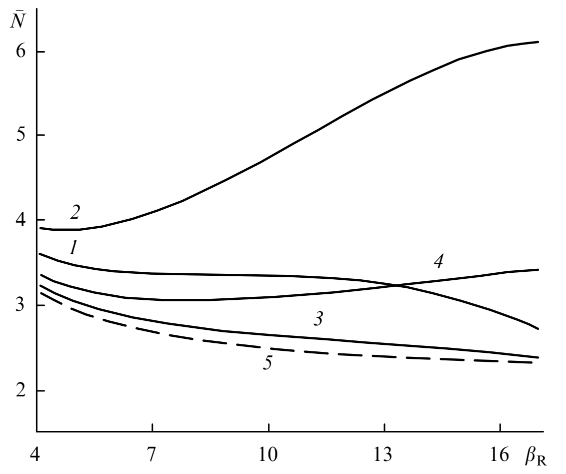

Figure 2. Backscattering amplification coefficient for the mean intensity of a reflected wave in an amplifying random medium as a function of β_R for $\delta = 5 \times 10^{-6}$ (*1, 2*), 10^{-6} (*3, 4*) and 0 (*5*) and also for $b_{RIm} = 0$ (*1, 3*) and 1.0 (*2, 4*).

[the correlation coefficient $b_{RIm} = \langle\tilde{\varepsilon}_R\tilde{\varepsilon}_{Im}\rangle\left(\langle\tilde{\varepsilon}_R^2\rangle\langle\tilde{\varepsilon}_{Im}^2\rangle\right)^{1/2})$] as well as on the sign of the imaginary part of ε. In the absence of correlations between $\tilde{\varepsilon}_R$ and $\tilde{\varepsilon}_{Im}$ (for $b_{RIm} = 0$), the quantity $\bar{N}$ exhibits the same behavior for an absorbing and an amplifying medium (curves *1* and *3* in Figs 1 and 2).

As mentioned above, the effect of the correlation term in expression (14) on the behavior of N depends on the sign of the imaginary part of ε. In the case of an absorbing medium, the value of N rapidly decreases with increasing β_R and becomes smaller then $\bar{N}(L)_{\varepsilon_{Im}=0}$ for certain values of β_R (curves *2* and *4* in Fig. 1). Conversely, for an amplifying medium, the coefficient N increases with β_R and may considerably exceed $\bar{N}(L)_{\varepsilon_{Im}=0}$ (curves *2* and *4* in Fig. 2).

Concluding the section, we write the expressions for the relative variance of intensity fluctuations for the reflected wave in a random medium with pulsations of the imaginary part of ε:

$$\sigma_{I\mathrm{ref}}^2(L)\big|_{\rho \ll \rho_c} = 6\mathrm{e}^{4\gamma^2(L)} - 1, \tag{21}$$

$$\sigma_{I\mathrm{ref}}^2(L)\big|_{\rho \gg \rho_c} = 4\mathrm{e}^{2\gamma^2(L)} - 1. \tag{22}$$

It follows from these expressions that as in the case of the gain, the value of $\sigma_{I\mathrm{ref}}^2$ in a random absorbing medium is determined by the fluctuations of the real and imaginary parts of ε as well as by their correlations. In addition, the radiation reflected strictly backward ($\rho = 0$) exhibits stronger intensity pulsations than the radiation reflected beyond the region ($\rho < \rho_c$). A comparison of expression (21) with the quantity $\sigma_I^2(2L) \geqslant 2$ on a straight path shows that the effect of the fluctuations of the imaginary part of the permittivity on the variance of fluctuations of a location signal is stronger than on $\sigma_I^2(2L)$.

5. Conclusions

It should be emphasized above all that the effect of even relatively weak fluctuations of the imaginary part of the permittivity on the propagation of laser radiation along the location path in the saturation region may be quite significant. (This conclusion is in good agreement with the results of the analysis of the effect of absorption pulsations on the statistical parameters of an electromagnetic wave propagating along a path without being reflected, which was carried out by us earlier [8, 9].) In an absorbing (amplifying) medium, backward scattering amplification takes place and may be stronger than in a transparent random medium. In the case when the pulsations of the imaginary and real parts of the permittivity are uncorrelated, the backscattering amplification is the same (for identical model parameters) for absorbing and amplifying media. However, the existence of a correlation coupling between ε_R and ε_{Im} reduces the backscattering enhancement in an absorbing random medium and increases it in an amplifying random medium

Acknowledgements. This work was supported by the Russian Foundation for Basic Research (Grant No. 98-02-17220).

References

1. Budnik A P, Dobrovol'skaya I V *Kvantovaya Elektron.* **24** 506 (1997) [*Quantum Electron.* **27** 492 (1997)]
2. Budnik A P, Dobrovol'skaya I V, Katsuro O N, Kozel S V, in: *Izbrannye trudy FEI 1996* (Selected Papers of Institute of Physics and Power Engineering) 1996 (Obninsk: National Science Center Institute of Physics and Power Engineering, 1997), p. 83
3. Starikov A F *Kvantovaya Elektron.* **21** 343 (1994) [*Quantum Electron.* **24** 320 (1994)]
4. Gurvich A S *Radiotekhn. Elektron.* **13** 1923 (1968)
5. Almaev R Kh, Semenov L P, Slesarev A G *Kvantovaya Elektron.* **9** 1565 (1982) [*Sov. J. Quantum Electron.* **12** 1003 (1982)]
6. Starikov A F *Kvantovaya Elektron.* **23** 205 (1996) [*Quantum Electron.* **26** 199 (1996)]
7. Almaev R Kh, Suvorov A A *Kvantovaya Elektron.* **20** 874 (1993) [*Quantum Electron.* **23** 758 (1993)]
8. Almaev R Kh, Suvorov A A Pis'ma *Zh. Eksp. Teor. Fiz.* **52** 718 (1990)
9. Almaev R Kh, Suvorov A A *Izv. Vissh. Uchebn. Zaved. Ser. Radiofiz.* **34** 671 (1991)
10. Almaev R Kh, Semenov L P, Suvorov A A *Waves in Random Media* **6** 87 (1996)
11. Almaev R Kh, Suvorov A A *Waves in Random Media* **6** 171 (1996)
12. Kravtsov Yu A, Saichev A I *Usp. Fiz. Nauk* **137** 501 (1982)
13. Banakh V A, Mironov V L *Lokatsionnoe rasprostranenie lazernogo izlucheniya v turbulentnoi atmosfere* (Laser Beam Propagation in Turbulent Atmosphere for Location Purposes) (Novosibirsk: Nauka, 1986)
14. Tatarskii V I *Rasprostranenie voln v turbulentnoi atmosfere* (Wave Propagation in a Turbulent Atmosphere) (Moscow: Nauka, 1967)
15. Gurvich A S, Kon A I, Mironov V L, Khmelevtsov S S *Lazernoe izluchenie v turbulentnoi atmosfere* (Laser Radiation in a Turbulent Atmosphere) (Moscow: Nauka, 1976)
16. Ishimaru A *Wave Propagation and Scattering Media* (New York: Academic Press, 1978)
17. Rytov S M, Kravtsov Yu A, Tatarskii V I *Vvedenie v statisticheskuyu radiofiziku. Sluchainye polya* (Introduction to Statistical Radio Physics: Random Fields) (Moscow: Nauka, 1978), Part II
18. Zavorotnyi V U, Klyatskin V I, Tatarskii V I *Zh. Eksp. Teor. Fiz.* **73** 481 (1977)