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Reduction of shot noise in an interference gravitational-wave detector

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Abstract. **The characteristics of a Michelson interferometer whose arms are formed by reflection Fabri – Perot interferometers (FPIs), which is designed for measurements of ultrasmall displacements, are studied. It is shown that the recent advances in the mirror coating technology along with the optimisation of the parameters of the FPI mirrors makes it possible to greatly improve the ratio of the signal to the shot noise. Optimal transmission of the front FPI mirror is approximately equal to the absorption coefficient of the mirrors.**

Keywords: *sensitivity, gravitational-wave detector, shot noise, Michelson interferometer, Fabri – Perot interferometer.*

1. Statement of problem

The creation of interferometric devices that would allow measurements of ultrasmall displacements caused by gravitational waves from extraterrestrial sources is an interesting problem of optics. The presently operating gravitational-wave detectors (GWDs) have not yet reached the required signal-to-noise ratio and therefore continue to be actively investigated and optimised (see, e.g., [1 – 3]). For the optimisation to be successful, one should pay special attention to those parameters of the system that are crucial for increasing the sensitivity – in particular, to losses in mirrors.

The advanced optical coating technology makes it possible to manufacture dielectric mirrors with losses of the order of 10^{-6}. If the properties of these mirrors could be used efficiently, the sensitivity would be enhanced greatly, in principle allowing one to solve the general problem as a whole. However, the parameters of the interferometers have to be chosen properly for this. This paper is devoted to the issues concerning the use of the advantages of mirrors with ultralow loss.

We will consider the simplest way of obtaining the useful signal from an interferometer, when the working point is located at the slope of the resonance curve of a Fabri – Perot interferometer (FPI) and the displacement of the mirrors is detected by directly measuring the optical signal at the output. We will consider only the static sensitivity and ignore the frequency characteristics. According to the published experimental data, shot noise is currently the main disturbance in a large part of the expected frequency range of gravitational waves. The elimination of this noise source could significantly influence the control of the remaining noise and frequency characteristics. Among the questions to be considered below, we especially note the question of whether the Michelson interferometer (MI) with Fabri – Perot arms (MIFP, see Fig. 1) – currently, almost the conventional optical scheme – offers any advantages over the scheme of Ref. [4], which consists of two optically independent FPIs that use the same light source and subtend an angle of 90°, as dictated by the quadrupole nature of gravitational waves.

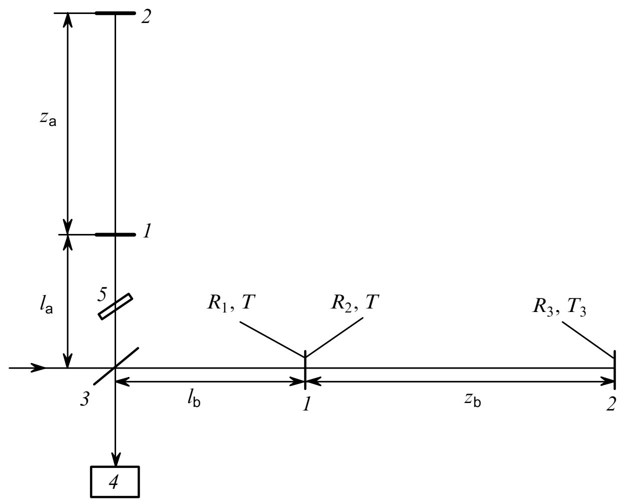

Figure 1. Scheme of the MIFP: *1* and *2* are the FPI mirrors; z_a and z_b are the distances between the FPI mirrors in arms a and b, respectively; l_a and l_b are the distances from FPI mirrors *1* to the beamsplitter *3*; *4* is the photodetector; *5* is the dispersion-compensating plate [5].

2. Scheme of an interference GWD

Fig. 1 shows the standard scheme of a MIFP. Laser light incident on beamsplitter *3* is directed to arms a and b. Each arm contains a reflection FPI consisting of mirrors *1* and *2* separated by the distance *z*. We assume that the FPIs in the two arms are identical. Suppose that under the action of a gravitational wave, arm a shortens by Δz and arm b lengthens, or vice versa. The output signal, proportional to Δz,

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can then be detected by a photodetector (PD) *4*. Since the two beams interfere in the plane of the PD, one can tune the MI to a dark interference fringe so as to decrease the constant component of the PD photocurrent and enhance the signal-to-shot noise ratio (SSNR). One can also add a plate to the scheme of the MI to compensate for the dispersion [5].

Let $R_1^{1/2}\exp(\mathrm{i}\Psi_1)$ and $R_2^{1/2}\exp(\mathrm{i}\Psi_2)$ be the complex reflection coefficients of mirrors *1* on the beamsplitter side and inside the FPI, respectively (we assume mirrors *1* in the two arms to be identical), and $T^{1/2}\exp(\mathrm{i}\Phi_1)$ be the transmission coefficient of the mirrors. Similarly, for mirrors *2* we have $R_3^{1/2}\exp(\mathrm{i}\Psi_3)$ and $T_3^{1/2}\exp(\mathrm{i}\Phi_3)$. For beamsplitter *3*, we have $(R_\mathrm{a}^\mathrm{bs})^{1/2}\exp(\mathrm{i}\Psi_\mathrm{a})$ and $(R_\mathrm{b}^\mathrm{bs})^{1/2}\exp(\mathrm{i}\Psi_\mathrm{b})$ for the light incident from the side of arm a and b, respectively, and $(T^\mathrm{bs})^{1/2}\exp(\mathrm{i}\Phi^\mathrm{bs})$.

Suppose that P_0 is the power of the laser and P_pd is the power of the light incident on the PD. The efficiency of the light energy transfer from the laser to the PD is then given by

$$R_\mathrm{MI} = \frac{P_\mathrm{pd}}{P_0}. \tag{1}$$

Denote the complex reflection coefficients of the FPIs in arms a and b by $\tilde{\rho}_\mathrm{a}$ and $\tilde{\rho}_\mathrm{b}$, respectively. Let l_a and l_b be the optical lengths of the intervals separating the FPIs from beamsplitter *3* (see Fig. 1). Assuming $R_\mathrm{a}^\mathrm{bs} = R_\mathrm{b}^\mathrm{bs} = R^\mathrm{bs}$, we can write

$$R_\mathrm{MI} = T^\mathrm{bs}R^\mathrm{bs}\left|\tilde{\rho}_\mathrm{a} + H^{1/2}\exp(\mathrm{i}\Phi_\mathrm{ab})\tilde{\rho}_\mathrm{b}\right|^2. \tag{2}$$

The parameter $\Phi_\mathrm{ab} = 2\omega(l_\mathrm{a} - l_\mathrm{b})/c + \Psi_\mathrm{b}^\mathrm{bs} - \Psi_\mathrm{a}^\mathrm{bs}$ characterises the tuning of the MI. Since it is difficult to make the two FPIs completely identical in practice, we introduced an asymmetry coefficient H into (2), which is equal to the ratio of the beam intensities coming from arms b and a.

3. Shot noise in a single reflection FPI

To calculate the SSNR α of a two-mirror reflection FPI, we can use the formula [6]:

$$\alpha = \frac{q}{A}\left(\frac{\eta P_0}{2\hbar\omega\Delta f}\right)^{1/2}|\Delta\varphi|, \tag{3}$$

where

$$q = \frac{KAS}{(K\tilde{R} + 1 - K)^{1/2}}; \tag{4}$$

Δf is the frequency band; η is the quantum efficiency of the PD; $\Delta\varphi$ is a small variation in the round-trip phase incursion $\varphi = \omega z/c - \Psi_2/2 - \Psi_3/2$ of the FPI, which is to be measured; $\tilde{R}$ is the reflection coefficient of the FPI in the working point; $S = |\mathrm{d}\tilde{R}/\mathrm{d}\varphi|$ is the slope of the FPI characteristic with respect to the phase in the working point; K is the coupling constant between the modes of the interferometer and the laser; $A = (A_2 + 1 - R_3)/2$ and $A_2 = 1 - T - R_2$.

One can see from formula (3) that the SSNR is inversely proportional to A; therefore, it is very important to have low losses in the mirrors. However, the sensitivity of the device also depends on parameter q, which takes the transmission of the mirrors into account. In the optimal case, q is of the order of unity, whereas for a wrong choice of the transmission coefficients of the mirrors, q can reduce by several orders of magnitude, resulting in a decrease in the SSNR.

To calculate quantities $\tilde{R}$ and S that appear in expression (4), we can use the formula [6–8]:

$$\tilde{R} = |\tilde{\rho}|^2 = 1 - \frac{8x}{(2+x)^2 + (16/A^2)(\varphi - \varphi_1)^2}, \tag{5}$$

where $x = T/A$. This formula is valid in the case of small detunings $\varphi - \varphi_1$ near the resonance at $\varphi = \varphi_1$. We assume that $A + T \ll 1$. In addition, due to the small absorption we can set $\Psi_1 + \Psi_2 - 2\Phi_1 = \pi$ [7, 8].

For each pair of quantities K and x, we first found the detuning $\varphi - \varphi_1$ that maximises q using formulas (3)–(5) and then calculated q, $\tilde{R}$, and S for the detuning found. Fig. 2 shows the results of these calculations for $K = 0.999$ in the interval $x = 10^{-1} - 10^4$.

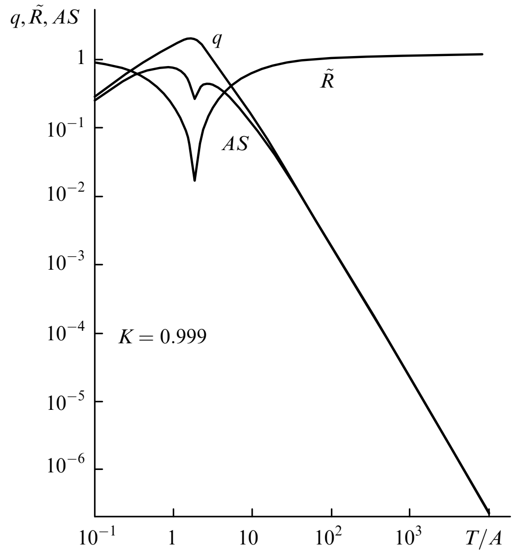

Figure 2. Coefficient q, proportional to the SSNR of a single FPI, reflection coefficient of the FPI, $\tilde{R}$, and the slope of the FPI characteristic in the working point, S, as functions of the transmission of the front FPI mirror, T.

One can see from Fig. 2 that the function $q(x)$ has a maximum of $q_\mathrm{max} = 1.91$ at $x = 1.9$. The curve $\tilde{R}(x)$ shows that q increases because of the rapid decrease in $\tilde{R}$. This regime corresponds to the 'matching' between the FPI and the light source, reducing $\tilde{R}$ down to 0.015. The FPI virtually ceases to reflect light, so that the shot noise becomes very small. At this regime, the slope S also reaches its minimum, $AS = 0.24$, but the coefficient q, which is proportional to $AS/\tilde{R}_{1/2}$, still remains relatively large.

If we need to increase the amplitude of the output signal, i.e., the slope, it is reasonable to deviate somewhat from the matching regime. For example, at $x = 1$ we obtain a slightly reduced value $q = 1.45$, but AS increases to 0.7 in return. Thus, we can recommend to choose the value of T between A and $3A$. If, on the contrary, one takes mirrors with a large transmission, the sensitivity will decrease noticeably. For example, consider $A = 2 \times 10^{-6}$ and $T = 2 \times 10^{-4}$, implying

$T/A = 100$. According to Fig. 2, we obtain $q = 2 \times 10^{-3}$, and formula (3) shows that the SSNR reduces by a factor of 1000 with respect to the maximum possible value for the given absorption in the mirrors. AS decreases to 2×10^{-3} as well.

4. Optimisation of MIFP

To calculate the signal-to-shot noise ratio of the MIFP, we can use formula (3) where q at $K = 1$ should be replaced by

$$q_{\rm MI} = \frac{AS_{\rm MI}}{R_{\rm MI}^{1/2}}, \qquad (6)$$

where $R_{\rm MI}$ is the efficiency of the power transfer from the laser to the output PD, defined by formula (2). We will consider the FPIs in the to arms to be completely identical except for fact that, upon the arrival of the measured signal, the length of one of them (for example, of arm b) acquires a small increment Δz. The quantity $S_{\rm MI}$ is the slope of the characteristic of the total MIFP system for the phase variation in one of the arms: $S_{\rm MI} = |{\rm d}R_{\rm MI}/{\rm d}\varphi_{\rm b}|$ where $\varphi_{\rm b} = \omega z_{\rm b}/c - \Psi_2/2 - \Psi_3/2$. The quantity $A = (A_2 + 1 - R_3)/2$ refers to each of the two FPIs.

We need to insert the expressions for the complex reflection coefficients of the FPIs into formula (2). Using the general formulas [7, 8] and making the same approximations as in derivation of formula (5), we obtain expressions of the form

$$\tilde{\rho}_{\rm a,b} = \frac{2A - T + {\rm i}4(\varphi_{\rm a,b} - \varphi_1)}{2A + T + {\rm i}4(\varphi_{\rm a,b} - \varphi_1)}. \qquad (7)$$

Using formulas (2) and (7) to calculate parameters $R_{\rm MI}$ and $S_{\rm MI}$, we obtain the dependence of $q_{\rm MI}$ on the parameter $x = T/A$ and the MI tuning parameter $\Phi_{\rm ab}$. Note that, again, in each case we should insert into (7) those FPI detunings $\varphi - \varphi_1$ that maximise $q_{\rm MI}$, in expression (7). In the calculations, we assumed $T^{\rm bs}R^{\rm bs} = 0.25$ and $H = 0.9$.

Fig. 3a shows the parameters of the MIFP as functions of the MI tuning parameter $\Phi_{\rm ab}$ for $x = T/A = 1$. If the MI is tuned to the centre of a bright fringe ($\Phi_{\rm ab} = 0$), then $q_{\rm MI} = 0.69$. With increasing $\Phi_{\rm ab}$, the coefficient $q_{\rm MI}$ gradually increases to 0.84 (at $\Phi_{\rm ab} = 2.8$). Thus, we obtain a large $q_{\rm MI}$ and a large sensitivity to displacements. Fig. 3a, as well as Fig. 3b, exhibits a sharp singularity at $\Phi_{\rm ab} \approx \pi$ (in the region of a dark fringe of the MI), where $R_{\rm MI}$ and $S_{\rm MI}$ are minimal and $q_{\rm MI}$ is slightly reduced. At the singularity point, these parameters depend on the arbitrarily specified asymmetry parameter H and therefore are essentially random.

Fig. 3b shows the same curves in the case when the front mirrors of the FPIs have a large transmission, far from the optimum: $T/A = 10^4$. In contrast to Fig. 3a, in this case we observe a strong dependence of $q_{\rm MI}$ on the MI tuning. When the MI is tuned to a bright fringe ($\Phi_{\rm ab} = 0$), $q_{\rm MI}$ reduces to 2.4×10^{-7}. Near a dark fringe, at $\Phi_{\rm ab} \approx 2.8$, $q_{\rm MI}$ increases sharply to 7.6×10^{-4}. Nevertheless, this value is still three orders of magnitude smaller than in Fig. 3. Thus, a properly tuned MI can greatly enhance the SSNR, but cannot compensate for the dramatic fall in this quantity caused by a wrong choice of the transmission of the FPI mirrors.

The parameter $\Phi_{\rm ab}$ can be easily controlled in experiments, allowing one to maximise $q_{\rm MI}$. It is therefore reasonable to change the computer program so as to find the working point for each $x = T/A$ that would be optimal not only with respect to φ but also with respect to $\Phi_{\rm ab}$, as determined by the maximisation of $q_{\rm MI}$.

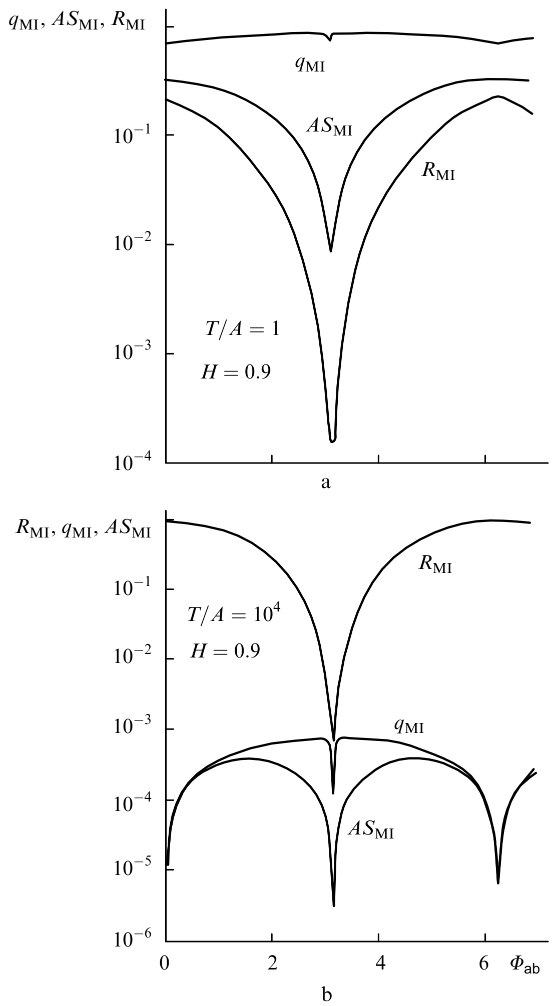

Figure 3. Coefficient $q_{\rm MI}$, proportional to the SSNR of the combined MIFP, coefficient $R_{\rm MI}$, and the slope, $S_{\rm MI}$, as functions of the detuning, $\Phi_{\rm ab}$, between the lengths, $l_{\rm a}$ and $l_{\rm b}$, of the MI arms. (a) The transmission of the front mirror of each FPI arm is close to the optimum, $T/A = 1$. (b) The transmission of the front FPI mirror is very large, being far from the optimum, $T/A = 10^4$.

Fig. 4 shows the results of these calculations in the optimal regimes. The maximum value of $q_{\rm MI}$ at $x \approx 2$ is 0.95. However, this regime features a relatively small slope $S_{\rm MI} = 0.012/A$, which furthermore can depend on H, field mismatch, aberrations, etc. Obviously, it is better to use FPI mirrors that are slightly different from the optimal ones with respect to $q_{\rm MI}$. For example, we can choose either a slightly smaller value of T (for $T/A = 1$ we have $q_{\rm MI} = 0.84$) or a slightly larger one (for $T/A = 4$, $q_{\rm MI} = 0.86$). However, the further increase in T is inadmissible: $q_{\rm MI} = 0.08$ at $x = 100$, $q_{\rm MI} = 0.006$ at $x = 1000$ and so on. The quantity $AS_{\rm MI}$ reduces as well. Therefore, Fig. 4 shows the maximum attainable parameters of the MIFP for various transmission coefficients of the FPI mirrors.

It is very interesting to compare Fig. 4 to the characteristics of a single FPI. Fig. 4 reproduces the $q(x)$ curve taken from Fig. 2. One can see that at small values of x (approximately, at $x \leqslant 4$), these points are close to the curve $q_{\rm MI}(x)$. The apparent difference by approximately a factor of 2 is caused by the fact that in our calculations of the MIFP only

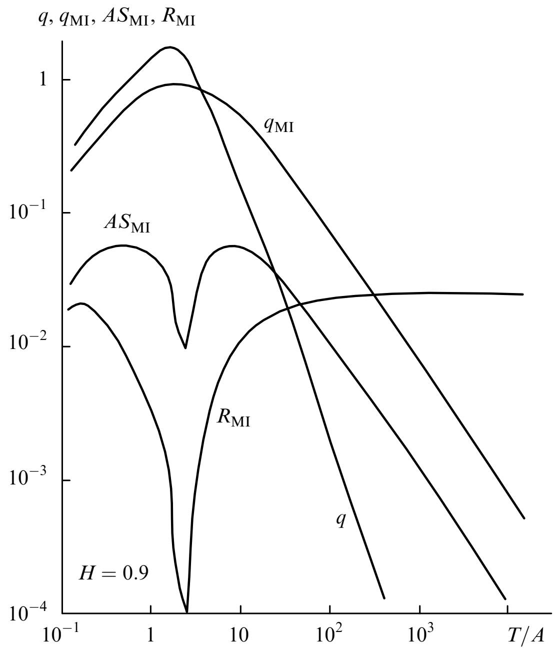

Figure 4. Dependence of quantities q_{MI}, AS_{MI}, and R_{MI} on T/A for the optimal tuning of both the MIFP and the MI, as determined by the maximisation of q_{MI} for each value of $x = T/A$, as well as the dependence $q(x)$ from Fig. 2.

arm b was considered active, whereas only half the P_0 power is coupled to this arm. However, as noted above, with the further increase in T, coefficient q falls off very steeply, much more steeply than q_{MI}. The reason for this is obvious. At small values of T, the FPI has a small proper reflection coefficient, which gives rise to large values of q and q_{MI}. With increasing T, the coefficient $\tilde{R}$ increases sharply, leading to a bright illumination of the PD and, consequently, strong shot noise. In contrast, if we have an MIFP at our disposal and have chosen a working point near a dark interference fringe of the MI, we can drastically reduce the intensity of the light incident on the PD and the shot noise. This leads to an important practical recommendation: when two independent FPIs are used [4], one should take mirrors with a very small transmission $T \approx (1-4)A$. If, on the contrary, one takes nonoptimal mirrors with a large T, one has to put up with a significant decrease in the SSNR and, in addition, necessarily use the MI.

5. Conclusions

The calculation of interferometric displacement meters for GWDs that was conducted here allows us to draw the following conclusions.

(1) The front mirror of the reflection FPI should have a transmission T approximately equal to $(1-4)A$. If this condition is violated, the noise increases considerably. However, one should avoid $T \approx 2A$ because at this value there is no reflection from the FPI (the 'matching' regime), which is related to a sharp decrease in the slope, $|d\tilde{R}/d\varphi|$. The latter recommendation refers exclusively to the case when the signal is decoupled directly from the FPI and one makes use of the slope of the resonant curve.

(2) When the value of T is optimal with respect to the shot noise, there is no difference between using two separate FPIs subtending an angle of 90° or a single MIFP. However, if for some reasons one uses a nonoptimal transmission T, the second variant is preferable as it allows one to considerably reduce the constant component of the output signal and, therefore, increase the SSNR.

(3) To estimate the presently attainable sensitivity of GWDs, limited by shot noise, we can set $\alpha = 1$ in formula (3) and calculate Δz similarly to the way it is done in Ref. [6]. Suppose, for example, that $A_2 = A_3 = 1.1 \times 10^{-6}$, $T = 2.2 \times 10^{-6}$, $T_3 = 0$, $P_0 = 1$ W, $q = 1.45$, $\eta = 0.8$, $\lambda = 8.5 \times 10^{-7}$ m, $z = 40$ m, and $\Delta f = 100$ Hz. The calculation then yields $\Delta z_{min}/z = 2 \times 10^{-23}$. This estimate allows us to hope that, using advanced mirrors, one can almost completely eliminate shot noise and significantly increase the sensitivity of GWDs, or shorten their base. Naturally, other sources of noise may then move to the forefront and will have to be minimised in due course.

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