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The possibility of demonstration of gamma-ray lasing in nuclei

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***Abstract*.** **Estimates are made of the parameters of a possible demonstration experiment on the generation of gamma rays in a single-pass laser with hidden population inversion in a free-nuclei amplifying medium.**

Keywords: *nuclear gamma-ray laser, demonstration experiment, hidden population inversion.*

1. Introduction

The idea of stimulated gamma radiation in a nuclear medium with hidden population inversion [1] is based on the positive effect of recoil of free nuclei in radiative processes. This phenomenon leads to a spectral shift of the emission and absorption gamma lines by doubled recoil energy. If this mutual shift of lines exceeds their width, there is no resonance absorption of the emitted photons by unexcited nuclei, which creates the prerequisites for the emergence of hidden population inversion, i. e., the nuclear medium becomes capable of quantum amplification of a flux of gamma photons without the requirement that the total population of the upper level in the laser transition be higher than that of the lower level.

On the other hand, since recoil manifests itself most vividly when free nuclei are involved, Doppler broadening of the emission lines of such nuclei is substantial. This has an extremely negative effect on the gain of the gamma-photon flux. To suppress Doppler broadening, the nuclear medium should be subjected to deep cooling to temperatures at which Doppler broadening becomes comparable to the natural linewidth. Today such cooling to the required submicrokelvin temperatures is made quite possible through the manipulation of neutral atoms using radiation from optical lasers [2].

In addition, the same shift of the nuclear absorption and emission lines makes it possible to optically pump a two-level nuclear structure by incoherent X-ray radiation following a scheme similar to the standard three-level scheme of optical lasers without, however, the additional third level. In this case the laser energy is supplied entirely by the pump source. In addition to such a 'two-level' scheme, it is possible, at least in principle, to build a gamma-ray laser using the so-called anti-Stokes scheme, where the laser energy is supplied from the energy stored in the metastable states of isomeric nuclei.

Thus, the general idea of a possible experimental demonstration of the quantum amplification of a gamma-photon flux in a nuclear medium with hidden population inversion (or building a mirrorless gamma-ray laser with the single-pass gain) is given by the schematic in Fig. 1. The main element of this schematic is a deeply cooled extended filamentary beam (*1*) of neutral atoms containing active nuclei. The beam is generated, cooled, shaped, and confined with the help of a source of atoms (*2*), optical lasers (*3*), a device (*4*) for filling a trap (*5*) by atoms (a magnetooptical trap, in particular), and an absorber (*6*) of atoms leaving the trap. Pumping is performed by a directed beam (*7*) of X-ray photons from a pump source (*8*) propagating through a system (*9*) of filtration and transport of radiation. The output gamma-photon flux (*10*) amplified by nuclei is emitted in the direction opposite to that of the pump radiation.

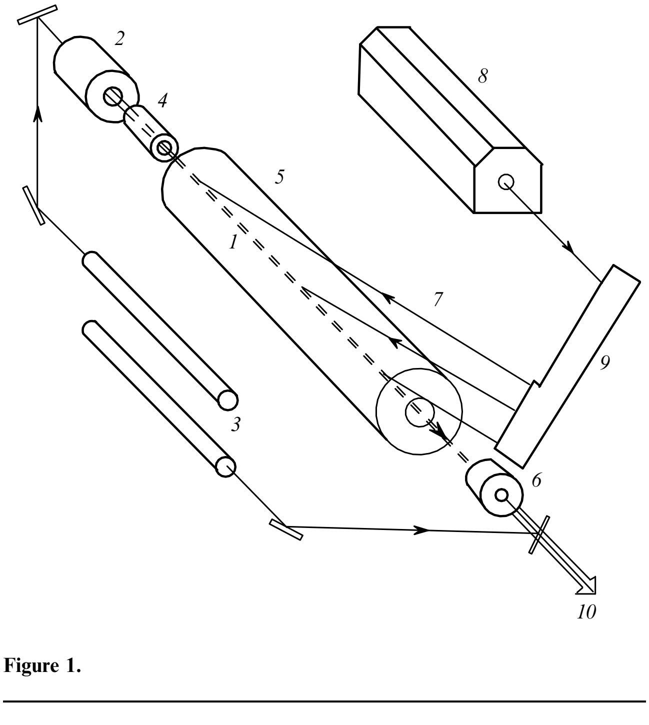

Figure 1.

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2. Selection of nuclides for the demonstration experiment

Optimising the selection of nuclides for the demonstration experiment is a problem that cannot be easily formalised due to the intricate interplay between many factors related both to the nuclear and atomic properties of the candidates. In view of this, planning at least a rough course in the selection of the proper candidates is crucial.

2.1. The 'two-level' scheme

1. The selection of the energy level diagram for this scheme is very simple since there are only two active levels in it: the ground level and the first excited level. It is desirable that the nucleus be stable or at least sufficiently long-lived. The problem of selecting the energy E_0 of the excited state is complicated, however. On the one hand, this energy must not be too high, since the stimulated emission cross section

$$\sigma_0 = \frac{\lambda^2}{2\pi}\frac{\Gamma_\gamma}{\Gamma_{\gamma\alpha}}\beta \tag{1}$$

and the cross section for absorption of the pump radiation is proportional to the square of the wavelength λ, while the frequency of the photons of the X-ray pump radiation, whose sources are usually limited in energy, actually coincides with the laser transition frequency. Here, Γ_γ is the natural radiative width; $\Gamma_{\gamma\alpha} = \Gamma_\gamma(1+\alpha)$ is the total natural transition width; α is the coefficient of internal electron conversion, and

$$\beta = \frac{\Gamma_{\gamma\alpha}}{\hbar\Delta\omega_{\mathrm{D}}} \tag{2}$$

is the ratio of the total natural transition width $\Gamma_{\gamma\alpha\chi}$ to the Doppler width $\hbar\Delta\omega_{\mathrm{D}}$.

On the other hand, the extremely low transition energy results in unacceptably large internal electron conversion coefficients α and rates of other nonradiative channels of excited-state decay, as well as cross sections χ of nonresonance losses of gamma photons because of the photoelectric effect involving atomic electrons, etc. It seems that reasonable energies E_0 of the laser transition lie in the range from several tens to several hundreds of kiloelectronvolts. In addition, it is obious that the difference between the angular momenta of the upper and lower levels should not be too great, i. e., the multipolarity order of the transition should be low.

2. The acceptable lifetime $\tau_{1/2}$ of the excited state is specified primarily by the available cooling of the atoms containing active nuclei, because one of the main conditions in the adopted concept of a free-nucleus gamma-ray laser is the most radical suppression of the Doppler broadening of the gamma line expressed by the linewidth ratio β. Naturally, the maximum value of this ratio, $\beta \to 1$, is desirable. This leads to the condition imposed on the 'longitudinal temperature' of the atomic beam:

$$T_{\mathrm{long}} \leqslant \frac{0.41A}{(E_0\beta\tau_{1/2})^2}, \tag{3}$$

where T_{long} is expressed in microkelvins; E_0 in kiloelectronvolts; the excited-state lifetime $\tau_{1/2}$ in nanoseconds; A is the number of nucleons in the nucleus. [Equation (3) should be used instead of incorrect equation (26) in Ref. [1], from which one must also delete the list of the temperature intervals characteristic of the various cooling methods that follows formula (26).]

If one does not use very sophisticated methods of modern laser cooling of atoms, the longitudinal temperature should be limited by the so-called recoil limit [2]: $T_{\mathrm{long}} > T_{\mathrm{rec}}$, where

$$T_{\mathrm{rec}} = \frac{(\hbar\omega_{\mathrm{opt}})^2}{k_{\mathrm{B}}Mc^2} \approx 12.5\frac{(\hbar\omega_{\mathrm{opt}})^2}{A}, \tag{4}$$

which is usually of about one microkelvin (here T_{rec} is expressed in microkelvins; $\hbar\omega_{\mathrm{opt}}$ is the energy of the photons from the cooling optical laser in electronvolts; M is the mass of the atom; c is the speed of light, and k_{B} is the Boltzmann constant). Comparison of (3) and (4) with allowance for $\beta \to 1$ and other reasonable values of the parameters shows that the desirable lifetimes $\tau_{1/2}$ of the laser level lie in the subnanosecond range.

3. The nucleus recoil energy

$$E_{\mathrm{rec}} \approx \frac{E_0^2}{2Mc^2} \approx 0.53\frac{E_0^2}{A} \tag{5}$$

at T_{long} [see Eqn (3)] is usually much higher than the energy needed for the emergence of hidden population inversion [see Eqns (10) in Ref. [1]; in equation (5) E_{rec} is measured in millielectronvolts if E_0 is measured in kiloelectronvolts].

4. The atomic optical transition used for laser cooling must have a sufficiently high probability w_{opt} and lie within a spectral range accessible by existing (or possible prototype) lasers.

5. The cooled atomic beam is emitted by a source, usually a heated cavity ('oven'). In view of this, the selected atoms must produce a substantial pressure p_{vap} of saturated vapour with a concentration n_{vap} at a moderate temperature T_{vap}.

6. The cross section χ for losses of gamma photons caused by a photoelectric effect involving atomic electrons, by Compton scattering, and by other processes is usually much smaller than the stimulated-emission cross section σ_0.

Table 1 lists the data obtained using Refs [3–7] (sometimes interpolation methods were employed) by the above scheme for four isotopes whose characteristics are, possibly, quite close to the desired compromise, although we do not claim that these characteristics are the optimal ones. The concentration n of the atoms in the beam and the pump power density j_{p} were estimated by equations (53) and (54) from Ref. [1] for the adopted relative concentration n_2/n, the length L of the atomic beam, and the single-pass gain G. However, some scepticism may arise due to the instability of the nuclide $^{54}_{25}$Mn, the necessity to use a short-wavelength UV laser for cooling $^{173}_{70}$Yb, and the too high temperature of the saturated vapour of the $^{173}_{65}$Tb atoms.

2.2. The anti-Stokes scheme

It may seem that the anti-Stokes scheme of a nuclear gamma-ray laser is more attractive than the 'two-level' one, because it is customary to assume that the former uses the intranuclear energy of metastable states for lasing, while the latter takes the energy solely from the pump source. However, this situation is partly an illusion, since most isomeric nuclides that can be considered as candidates for building a gamma-ray laser are of artificial origin, and the energy stored in their metastable states was spent (highly

Examples of 'two-level' nuclides (linewidth ratio $\beta = 1$, atomic-beam path is 10 m, the single-pass gain is 6).

Nuclides	$^{54}_{25}$Mn	$^{159}_{65}$Tb	$^{165}_{67}$Ho	$^{173}_{70}$Yb
Laser transition energy E_0/keV	54.4	58	94.7	78.65
Wavelength λ/nm	0.0228	0.0215	0.013	0.0158
Lifetime of the upper laser level $\tau_{1/2}/\mathrm{ns}$	0.049	0.0536	0.022	0.046
Natural width of the upper laser level $\Gamma_{\gamma\alpha}/\mu\mathrm{eV}$	9.3	8.5	21	10
Internal electron conversion coefficient α	0.212	11	3.13	7
Nucleus recoil energy $E_{\mathrm{rec}}/\mathrm{meV}$	29	11.2	28.3	19
Angular momentum and upper level parity J_2	2^+	$5/2^+$	$9/2^-$	$7/2^-$
Angular momentum and lower level parity J_1	3^+	$3/2^+$	$7/2^-$	$5/2^-$
Multipolarity of the laser transition	M1 + E2	M1 + E2	M1 + E2	M1 + E2
Nuclide lifetime /days	312.3	∞	∞	∞
Optical laser wavelength $\lambda_{\mathrm{opt}}/\mu\mathrm{m}$	0.403	0.433	0.41	0.399
Optical laser photon energy $\hbar\omega_{\mathrm{opt}}/\mathrm{eV}$	3.08	2.86	3.02	3.1
Probability of atomic optical transition multiplied by the statistical weight $w_{\mathrm{opt}}/10^8\,\mathrm{s}^{-1}$	1.4	25	7.2	1.6
Saturated vapour temperature $T_{\mathrm{vap}}/\mathrm{K}$	1491	2181	1720	994
Saturated vapour pressure $p_{\mathrm{vap}}/\mathrm{Pa}$	100	100	100	500
Saturated vapour concentration $n_{\mathrm{vap}}/10^{16}\,\mathrm{cm}^{-3}$	14.5	21.3	16.8	48.5
Stimulated emission cross section $\sigma_0/10^{-19}\,\mathrm{cm}^2$	7	0.6	0.7	0.52
'Longitudinal temperature' of an atomic beam $T/\mu\mathrm{K}$	3	6.9	15.7	5.5
Recoil temperature limit $T_{\mathrm{rec}}/\mu\mathrm{K}$	2.2	0.64	0.7	0.7
Photon loss cross section $\chi/10^{-19}\,\mathrm{cm}^2$	0.002	0.08	0.01	0.02
Concentration of atoms in a beam $n/10^{16}\,\mathrm{cm}^{-3}$	2.56	15.5	15	18
Relative concentration of excited nuclei n_2/n	0.1	0.2	0.2	0.2
Normalisation of spectral pump density $j_{\mathrm{h}}/10^{17}\,\mathrm{cm}^{-2}$	3.2	17	19	24
Pump brilliance $j_{\mathrm{p}}/10^{17}\,\mathrm{cm}^{-2}$	0.32	3.5	3.8	4.8

ineffectively, incidentally) in their production. The latter process may be thought of a sort of preliminary pumping with postponed realisation.

A severe internal contradiction is inherent in the anti-Stokes scheme. The substantial lifetime of the metastable state of an isomeric nucleus is caused by the large difference of the angular momenta of this state and the lower levels. At the same time, the transition from the metastable state to the upper trigger level and the laser transition from the latter to a lower level should be sufficiently rapid (and hence the difference in the angular momenta involved must be small) to ensure the efficient operation of the scheme as a whole.

It is difficult to combine these contradicting requirements in a simplest three-level scheme. The isomeric nucleus $^{242}_{95}$Am with the metastable state lifetime equal to 141 years and the angular momentum 5^- may serve as an example. The ground state has the angular momentum 1^-, so that the strongly forbidden downward transition has a high multipolarity E4. Both the trigger transition to the upper level with the angular momentum 3^- and the proposed laser transition to the ground level have unacceptably high multipolarities E2, which makes this nucleus (attractive in other respects, such as ease of production, long lifetime, and low energy (4.27 keV) of the trigger photon) an unlikely candidate.

It is not inconceivable that this contradiction will be eliminated in multilevel structures, where the trigger level decays via a cascade of transitions, so that the large initial difference of angular momenta is the sum of small differences in the angular momenta of several rapid transitions in the downward cascade. It is important that only the first stage in the cascade can be used as a laser transition, because all the following stages occur in nucleus beam that has lost its initial monoenergetic property due to recoil upon spontaneous emission of photons in the cascade, so that the emission line broadening becomes highly inhomogeneous. Unfortunately, no convincing examples of such isomers can be given at the present time.

3. Requirements concerning the atomic beam uniformity

The requirements concerning the uniformity of the atomic beam follow from the conditions needed to maintain resonance between the amplified gamma radiation and the nuclear transitions with narrow lines over the entire volume of the amplifying medium. The admissible spread Δv_{tr} of the transverse velocities of the atoms in the trap is limited by the inequality

$$\frac{1}{2}\left(\frac{\Delta v_{\mathrm{tr}}}{c}\right)^2 E_0 \ll \Gamma_{\gamma\alpha}, \tag{6}$$

which hinders the loss of resonance due to a second-order Doppler effect. This limitation proves to be not very stringent: $\Delta v_{\mathrm{tr}} < 300\ \mathrm{cm\ s^{-1}}$ if $E_0 = 100$ keV and $\Gamma_{\gamma\alpha} = 10^{-10}$ eV, i. e. the 'transverse temperature' of the atoms in the beam must not substantially exceed 0.1 K, which is sure to be higher than typical temperatures in atomic traps.

The spread Δv_{long} of the longitudinal velocities of atoms along the beam path is limited by a first-order Doppler effect:

$$\frac{\Delta v_{\mathrm{long}}}{c} E_0 \ll \Gamma_{\gamma\alpha}, \tag{7}$$

This yields, for example $\Delta v_{\mathrm{long}} \leqslant 0.3\ \mathrm{cm\,s^{-1}}$ and $\Delta T_{\mathrm{long}}/T_{\mathrm{long}} < 0.01$ if $E_0 = 100$ keV, $\Gamma_{\gamma\alpha} = 10^{-10}$eV and $T_{\mathrm{long}} = 1\ \mu$K.

An additional longitudinal inhomogeneity in atomic velocities may emerge due to the fall of the atoms in the gravitational field when the trap axis is deflected from the ideal horizontal position by an angle ψ. The fall of the atoms leads to a first-order Doppler shift of the resonance along the beam path length L and produces the difference in velocities along this path:

$$\Delta v_L = g\frac{L}{v_{\mathrm{long}}}\sin\psi = \frac{gh}{v_{\mathrm{long}}}, \tag{8}$$

where g is the acceleration of gravity; v_{long} is the longitudinal transport velocity of atoms in the beam; $h = L\sin\psi$ is the difference between the heights of the trap's ends. Taking into account the restrictions introduced by the first-order Doppler effect [Eqn (7)], we arrive at the following inequality:

$$h \leqslant \frac{c v_{\mathrm{long}} \Gamma_{\gamma\alpha}}{g E_0}. \tag{9}$$

which for the previous example yields $h \leqslant 30\ \mu\mathrm{m}$ if $v_{\mathrm{long}} = 10^5\ \mathrm{cm\ s^{-1}}$.

4. X-ray pumping

The required spectral density j_{p} of the X-ray pump radiation, expressed in units of $\mathrm{cm^{-2}\ s^{-1}\ Hz^{-1}} = \mathrm{cm^{-2}}$ and presented in the last line of Table 1, is extremely high – j_{p} reaches $\sim 10^{17}\ \mathrm{cm^{-2}}$. In addition, to provide pumping that does not perturb a cooled beam of nucleus-containing atoms, this radiation should be concentrated within the solid angle (equation (48) in Ref. [1])

$$(\Delta\varphi)^2 \leqslant 4\frac{Mc^2\Gamma_{\gamma\alpha}}{\beta E_0^2} \approx 4\frac{A\Gamma_{\gamma\alpha}}{\beta E_0^2}, \tag{10}$$

where $\Delta\varphi$ is expressed in milliradians, E_0 in kiloelectronvolts, $\Gamma_{\gamma\alpha}$ in nanoelectronvolts. For instance, $(\Delta\varphi)^2 \leqslant 0.004\ \mathrm{mrad^2}$ at $E_0 = 100$ keV, $A = 100$, $\Gamma_{\gamma\alpha} = 10^{-10}$ eV, with $\beta \to 1$.

To compare these requirements with the possibilities of real sources of X-ray radiation, it is convenient to express them in the commonly used units of photons $\mathrm{mm^{-2}\ s^{-1}\ mrad^{-2}}$ within a band whose width is 0.1 % of the frequency ($\sim 10^{19}$ Hz), where the surface area is the cross-sectional area of the luminous body of the source in square millimetres. In terms of these units, the required spectral density (brilliance) of the X-ray source is estimated as extremely high: of order 10^{33} (photons $\mathrm{mm^{-2}\ s^{-1}\ mrad^{-2}}$) within the specified band (0.1 % of the frequency). This value exceeds by several orders of magnitude the brilliances achieved with the best synchrotron sources; the first prototypes of the next generation of X-ray sources that would reach the required brilliance 'may be operational around the year 2010' [8].

However, this formidable high brilliance of X-ray sources in the above units is actually due to the arbitrarily wide band adopted as a standard. Such width (0.1 % of the frequency, i. e. approximately 10^{16} Hz) is absolutely superfluous for pumping. The integrated photon flux from the pump source can be reduced by many orders of magnitude by radically reducing the width of the band with the brilliance remaining unchanged. In this connection, of certain interest are sources of X-ray radiation using Thomson and Compton scattering of low-frequency photons (for example, laser photons) by relativistic electrons [9, 10].

Note that the subnanosecond lifetimes of the laser level of nuclei can require pumping by X-ray pulses travelling along an extended atomic beam, as is done in X-ray ion transition lasers.

Thus, the problem of X-ray pump has proved to be one of the most difficult ones in building nuclear gamma-ray lasers.

5. Expected output parameters of the demonstration experiment

The total flux of gamma photons emitted by the adopted mirrorless model upon a single-pass amplification of the spontaneous noise background can be estimated as

$$F = \frac{\pi D^2}{4}\frac{G-1}{\ln G} S_{\mathrm{sp}} L, \tag{11}$$

where

$$G = \exp[(\sigma_0 n_2 - \chi n)L] \tag{12}$$

is the gain achieved by a single pass of the nuclear medium of length L with a total concentration of the atoms n and a relative concentration of the excited atoms n_2/n; D is the diameter of the atomic beam, and

$$S_{\mathrm{sp}} = \frac{n_2}{\tau_{1/2}\ln 2}\frac{\Gamma_\gamma}{\Gamma_{\gamma\alpha}}\frac{\Delta\Omega}{4\pi}\beta \tag{13}$$

is the rate of spontaneous emission of gamma photons per unit volume of the nuclear medium into the gain band and into the solid angle

$$\Delta\Omega \approx \left(\frac{D}{L}\right)^2, \tag{14}$$

encompassing the selected modes. As a result,

$$F \approx \frac{\ln 2}{4\pi}\frac{n_2}{\tau_{1/2}(1+\alpha)}\frac{G-1}{\ln G}\left(\frac{D}{L}\right)^2 V\beta, \tag{15}$$

where V is the volume of the nuclear medium.

These expressions yield, for example, for ${}^{165}_{67}\mathrm{Ho}$ (see Table 1), the following estimates: the flux of gamma photons in the pulse is $F \approx 3\cdot 10^{17}\ \mathrm{s^{-1}}$ with a power of about 5 kW and energy flux density of 2.5 MW $\mathrm{cm^{-2}}$ within a solid angle of $\Delta\Omega \approx 0.0025\ \mathrm{mrad^2}$ if $D = 0.5$ mm. Finally, there is a very unexpected parameter: if coherence is sufficiently high, the electric field strength in the beam of gamma radiation may exceed 40 kV $\mathrm{cm^{-1}}$. Of course, the total energy of the ultrashort gamma pulse is low and does not exceed several microjoules.

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