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Generation of nonclassical light in parametric amplification of counterpropagating waves in consecutive interactions

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***Abstract*. A quantum theory of consecutive interactions of counterpropagating waves with multiple frequencies $\omega, 2\omega$, and 3ω is developed. Parametric processes of high- and low-frequency amplification of ω and 3ω waves are studied in the approximation of an undepleted 2ω pump field. Fluctuations of quadrature components, statistics of photons, and correlation between fluctuations of photon numbers are considered at these frequencies.**

Keywords: *quadrature-squeezed light, photon statistics, interaction of counterpropagating waves, consecutive interactions.*

1. Introduction

Degenerate parametric amplification with a high-frequency pump is currently the most widespread method of generation of nonclassical light (e.g., see Refs [1, 2]). In the case of type-I three-wave mixing (involving an extraordinary pump wave and an ordinary amplified wave, which corresponds to an e $\to$ oo interaction), a quadrature-squeezed light is produced. Type-II parametric interaction (an e $\to$ oe interaction) seems to hold much more promise in the context of applications of nonclassical light. Interactions of this type give rise to radiation consisting of orthogonal-polarised fields, which can reside in the so-called entangled states. Entangled polarisation states can be employed, for example, in quantum cryptography [3] and quantum teleportation [4] (see also the review [5]).

Parametric interactions are characterised by an especially high efficiency when phase-matching or quasi-phase-matching conditions are satisfied. The latter condition can be met by using periodically poled nonlinear crystals (PPNCs), e.g., crystals with a regular domain structure (RDS crystals). In quasi-phase-matched parametric interactions, the vector of the reciprocal 'nonlinear grating' (the grating of spatial modulation of the wave coupling coefficient) compensates for the mismatch of wave vectors. With an appropriate choice of the period of the nonlinear grating in a PPNC, two three-wave-mixing processes with multiple frequencies ω, 2ω, and 3ω can be simultaneously quasi-phase-matched [6, 7], which allows parametric amplification to be implemented in consecutive interactions with a low-frequency pump in both co- [6] and counterpropagating [8] waves. One of the specific features of consecutive interactions is that the energy of the pump wave (e.g., the 2ω wave) can be completely converted into the signal wave with the frequency 3ω.

Quantum statistical properties of light produced through a parametric interaction of copropagating waves in a low-frequency pump field have been analysed in Ref. [9]. It has been shown that interactions of this type give rise to a quadrature-squeezed light at both the low frequency ω and the high frequency 3ω (the pump frequency is 2ω).

The purpose of this paper is to investigate the quantum properties of light produced through a parametric interaction of counterpropagating waves in a low-frequency pump field. Our calculations were performed in the approximation of an undepleted field of a classical wave with the frequency 2ω. In this approximation, the interaction of waves in a geometry when the backward wave has a frequency ω or 3ω displays the features of interaction of counterpropagating waves. We will study the relation between quantum fluctuations of light at the frequencies ω and 3ω.

2. The basic relations

Consider a sequence of two three-wave mixing processes in a quadratic-nonlinear medium:

$$2\omega \to \omega + \omega, \qquad \omega + 2\omega \to 3\omega. \tag{1}$$

In the case under study, the 2ω wave is an intense pump wave. The first process in Eqn (1) is degenerate parametric amplification with a high-frequency pump, while the second process is optical frequency mixing. As mentioned above, such processes may efficiently occur simultaneously in the quasi-phase-matching regime. At the same time, from the energy viewpoint, the dynamics of processes (1) in PPNCs is similar to the dynamics of these processes in homogeneous nonlinear crystals if the characteristic length of nonlinear wave interaction $L_{\rm NL}$ (see below) is much greater than the modulation period Λ of the nonlinear susceptibility [10]. Therefore, the nonlinear medium can be considered homogeneous. Obviously, one should keep in mind that interactions of counterpropagating waves are possible, in principle, only in PPNCs.

For a quantum description of the evolution of interacting waves in space, it is convenient to employ an operator of field momentum [11]. In the interaction representation, the operator of field momentum for the considered process can be written as

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$$G_{\rm int} = \hbar(\gamma_2 a_1^{+2} a_2 + \gamma_3 a_3^{+} a_1 a_2 + \text{H.c.}), \qquad (2)$$

where $a_j\,(a_j^+)$ is the operator of annihilation (creation) of photon with frequency $j\omega$ and $\gamma_{2,3}$ are the coefficients of effective wave coupling, which are determined by crystal nonlinearities and the order of quasi-phase matching [6].

The spatial evolution of the operator a_j is governed by the equation [11]

$$-i\hbar\frac{da_j}{dz} = [a_j, G_{\rm int}]. \qquad (3)$$

We will analyse the behaviour of quantum fluctuations in the process of amplification of two weak waves with frequencies ω and 3ω in the wave field with a frequency 2ω. We will restrict our consideration to the approximation of a undepleted pump field. We also assume that the pump field is classical, and the operator $a_2(z)$ in Eqn (2) can be replaced by a number $C_2 = a_2(0)$. This procedure yields

$$G_{\rm int} = \hbar(k_2 a_1^{+2} + k_3 a_3^{+} a_1 + \text{H.c.}), \qquad (4)$$

where $k_j = \gamma_j C_2$.

Consider the 3ω wave propagating in the direction opposite of the propagation direction of the ω wave, which travels in the positive direction of the z axis. The quantum equations for the interaction of counterpropagating waves can be then represented as (cf. [9])

$$\frac{da_1}{dz} = -ik_3^* a_3 - i\,2k_2 a_1^+, \qquad \frac{da_3}{dz} = ik_3 a_1 \qquad (5)$$

The relevant boundary conditions are

$$a_1(0) = a_{10}, \qquad a_3(L) = a_{3L}, \qquad (6)$$

where L is the crystal length. Equations (5) are similar to the equations for a nonlinear coupler with an intense pump at the frequency of the second harmonic, considered in Ref. [12]. In the nonlinear section of the coupler, the fundamental wave is amplified in the field of the second harmonic. In the case under consideration, the backward 3ω wave plays the role of one of the main waves.

The solutions to Eqns (5) can be expressed in terms of the operators in the cross section $z = 0$ in the following form:

$$a_1(z) = u_2(z)a_{30} + v_2(z)a_{30}^+ + w_2(z)a_{10} + y_2(z)a_{10}^+, \qquad (7)$$

$$a_3(z) = u_1(z)a_{30} + v_1(z)a_{30}^+ + w_1(z)a_{10} + y_1(z)a_{10}^+, \qquad (8)$$

where

$$u_1(z) = \frac{2|k_2|}{\lambda_0^2}[\lambda_1\cosh(\lambda_2 z) - \lambda_2\cosh(\lambda_1 z)];$$

$$v_1(z) = -ie^{i3\varphi_2}F_1(z); \qquad w_1(z) = ie^{i\varphi_2}G_1(z);$$

$$y_1(z) = e^{i2\varphi_2}G_2(z); \qquad u_2(z) = -ie^{-i\varphi_2}G_1(z);$$

$$v_2(z) = y_1(z); \qquad y_2(z) = -ie^{i\varphi_2}F_2(z);$$

$$w_2(z) = \frac{2|k_2|}{\lambda_0^2}[\lambda_1\cosh(\lambda_1 z) - \lambda_2\cosh(\lambda_2 z)];$$

$$\varphi_2 = \arg C_2; \quad F_1(z) = \frac{2|k_2|}{\lambda_0^2}[\lambda_2\sinh(\lambda_1 z) - \lambda_1\sinh(\lambda_2 z)];$$

$$F_2(z) = \frac{2|k_2|}{\lambda_0^2}[\lambda_1\sinh(\lambda_1 z) - \lambda_2\sinh(\lambda_2 z)];$$

$$G_1(z) = \frac{2|k_2 k_3|}{\lambda_0^2}[\sinh(\lambda_1 z) - \sinh(\lambda_2 z)];$$

$$G_2(z) = \frac{2|k_2 k_3|}{\lambda_0^2}[\cosh(\lambda_1 z) - \cosh(\lambda_2 z)];$$

$$\lambda_{1,2} = |k_2| \pm \gamma; \ \lambda_0^2 = 4|k_2|\gamma; \ \gamma = \Big(|k_2|^2 + |k_3|^2\Big)^{1/2}.$$

However, conditions (6) are the true boundary conditions for the problem under consideration. Therefore, the sought-for solution to the set of Eqns (5) is given by

$$a_1(L) = U_2 a_{3L} + V_2 a_{3L}^+ + W_2 a_{10} + Y_2 a_{10}^+, \qquad a_3(0) = U_1 a_{3L} + V_1 a_{3L}^+ + W_1 a_{10} + Y_1 a_{10}^+, \qquad (9)$$

where

$$U_1 = \frac{u_1(L)}{J}; \qquad W_1 = \frac{1}{J}[v_1(L)y_1^*(L) - u_1(L)w_1(L)];$$

$$V_1 = -\frac{v_1(L)}{J}; \quad Y_1 = \frac{1}{J}[v_1(L)w_1^*(L) - u_1(L)y_1(L)];$$

$$J = |u_1(L)|^2 - |v_1(L)|^2;$$

$$U_2 = \frac{1}{J}[u_2(L)u_1(L) - v_2(L)v_1^*(L)];$$

$$V_2 = \frac{1}{J}[u_1(L)v_2(L) - u_2(L)v_1(L)]; \qquad (10)$$

$$W_2 = u_2(L)W_1 + v_2(L)U_1 + w_2(L);$$

$$Y_2 = u_2(L)Y_1 + v_2(L)W_1^* + y_2(L).$$

The following canonical commutative relations are valid under these conditions:

$$[a_1(0), a_1^+(0)] = [a_1(L), a_1^+(L)] = [a_3(L), a_3^+(L)] = [a_3(0), a_3^+(0)] = 1. \qquad (11)$$

The analysis of two coupled quasi-phase-matched processes performed in Ref. [10] (see also the review [13]) has demonstrated that pump photons with the frequency 2ω first decay into ω photons, which , in their turn, add up with pump photons to produce 3ω photons. As is well known, low-frequency parametric amplification involves the redistribution of vacuum fluctuations, resulting in the formation of quadrature-squeezed light at the frequency ω. Since some of such photons are involved in the formation of light quanta with the frequency 3ω, formation of nonclassical light can be expected at this frequency.

3. Fluctuations of quadrature components

Let us introduce the quadrature operators of the waves being amplified in the following way:

$$X_j(\theta_j) = a_j \exp(\mathrm{i}\theta_j) + a_j^+ \exp(-\mathrm{i}\theta_j),$$
$$Y_j(\theta_j) = \mathrm{i}[a_j \exp(\mathrm{i}\theta_j) - a_j^+ \exp(-\mathrm{i}\theta_j)] = X_j(\theta_j + \pi/2). \quad (12)$$

Here, θ_j are the phases of the reference wave in balance homodyne detection [2], $j = 1, 3$. Using the solution (7) and (8), we derive expressions for the quadrature components (12), which are determined by all the initial quadratures and which substantially depend on the phases θ_1 and θ_3 and the phase φ_2 of the pump wave. To simplify our analysis, we choose phase relations in such a way that the quadrature operators are independent of Hermitian-conjugated operators:

$$3\theta_1 = \theta_3, \qquad \varphi_2 + 2\theta_1 = -\pi/2. \quad (13)$$

The quadrature operators are then given by

$$X_1(L) = R_1(L)X_{10} - Q(L)X_{3L},$$
$$Y_1(L) = T_1(L)Y_{10} - P(L)Y_{3L},$$
$$X_3(0) = Q_1(L)X_{3L} + Q(L)X_{10},$$
$$Y_3(0) = P_1(L)Y_{3L} + P(L)Y_{10}, \quad (14)$$

where

$$Q(L) = \frac{G_1(L) - G_2(L)}{u_1(L) + F_1(L)}; \qquad P(L) = \frac{G_1(L) + G_2(L)}{u_1(L) - F_1(L)};$$
$$Q_1(L) = \frac{1}{u_1(L) + F_1(L)}; \qquad P_1(L) = \frac{1}{u_1(L) - F_1(L)};$$
$$R_1(L) = w_2(L) - F_2(L) - \frac{[G_1(L) - G_2(L)]^2}{u_1(L) + F_1(L)};$$
$$T_1(L) = w_2(L) + F_2(L) - \frac{[G_1(L) + G_2(L)]^2}{u_1(L) - F_1(L)}. \quad (15)$$

Expression (14) shows that fluctuations at the generated frequencies are coupled to each other, since the expressions for the quadratures with different frequencies involve the functions $Q(L)$ and $P(L)$, which include the mutual influence of these fluctuations.

Let us introduce the parameters

$$L_{\mathrm{NL}} = |k_2|^{-1}, \quad \beta = |k_3|/|k_2|, \quad \zeta = L/L_{\mathrm{NL}},$$

which simplify the analysis of our results and which denote the characteristic length of nonlinear interaction, the ratio of nonlinear coefficients of wave coupling, and the normalised crystal length respectively. The functions involved in Eqn (15) are then written as

$$u_1(\zeta,\beta) = A(\beta)\{\cosh[(1 - (1+\beta^2)^{1/2})\zeta] - \eta(\beta) \times \cosh[(1 + (1+\beta^2)^{1/2})\zeta]\},$$
$$F_1(\zeta,\beta) = A(\beta)\{\eta(\beta)\sinh[(1 + (1+\beta^2)^{1/2})\zeta] - \sinh[(1 - (1+\beta^2)^{1/2})\zeta]\},$$
$$F_2(\zeta,\beta) = A(\beta)\{\sinh[(1 + (1+\beta^2)^{1/2})\zeta] - \eta(\beta)\sinh[(1 - (1+\beta^2)^{1/2})\zeta]\}, \quad (16)$$
$$w_2(\zeta,\beta) = A(\beta)\{\cosh[(1 + (1+\beta^2)^{1/2})\zeta] - \eta(\beta)\cosh[(1 - (1+\beta^2)^{1/2})\zeta]\},$$
$$G_1(\zeta,\beta) = B(\beta)\{\sinh[(1 + (1+\beta^2)^{1/2})\zeta] - \sinh[(1 - (1+\beta^2)^{1/2})\zeta]\},$$
$$G_2(\zeta,\beta) = B(\beta)\{\cosh[(1 + (1+\beta^2)^{1/2})\zeta] - \cosh[(1 - (1+\beta^2)1/2)\zeta]\},$$

where

$$A(\beta) = \frac{1 + (1+\beta^2)^{1/2}}{2(1+\beta^2)^{1/2}}; \qquad B(\beta) = \frac{\beta}{2(1+\beta^2)^{1/2}};$$
$$\eta(\beta) = \frac{1 - (1+\beta^2)^{1/2}}{1 + (1+\beta^2)^{1/2}}.$$

We assume that the fields incident on the crystal are in the vacuum state. Taking into account Eqn (14), we derive the following expressions for quadrature variances $V_{jA} = \langle A_j^2 \rangle - \langle A_j \rangle^2$:

$$V_{1X} = [R_1^2(\beta,\zeta) + Q^2(\beta,\zeta)]V_0,$$
$$V_{1Y} = [T_1^2(\beta,\zeta) + P^2(\beta,\zeta)]V_0,$$
$$V_{3X} = [Q_1^2(\beta,\zeta) + Q^2(\beta,\zeta)]V_0,$$
$$V_{3Y} = [P_1^2(\beta,\zeta) + P^2(\beta,\zeta)]V_0, \quad (17)$$

where $V_0 = 1$ is the quadrature variance for the vacuum field.

Expressions (17) describe quadrature variances for the ω wave at the output of the nonlinear crystal (in the cross section $z = L$) and for the 3ω wave at the input of the nonlinear crystal (in the cross section $z = 0$).

With $k_3 = 0$, Eqns (16) and (17) yield

$$V_{3X} = V_{3Y} = V_0, \qquad V_{1X} = \mathrm{e}^{-2\zeta}V_0, \qquad V_{1Y} = \mathrm{e}^{2\zeta}V_0.$$

Quadrature variances of the 3ω field remain equal to the quadrature variances of vacuum fluctuations, while quadrature variances of the ω field change. One of these variances, namely, V_{1X}, decreases when the phase relation (13) is satisfied. This is due to the fact that only one of the quasi-phase-matched processes – parametric amplification with a high-frequency pump (the $2\omega \to \omega + \omega$ process) – occurs under these conditions.

With $k_2 = 0$, quadrature variances are equal to the variance of the vacuum field: $V_{3X} = V_{3Y} = V_{1X} = V_{1Y} = V_0$ since only the conditions for frequency up-conversion ($3\omega \to 2\omega + \omega$) are satisfied. This process cannot occur only in the presence of vacuum fluctuations at the frequency ω.

In the case when $k_2 \neq 0$ and $k_3 \neq 0$, quantum fluctuations are suppressed in both output modes. Provided that the initial phase conditions of the form (13) are chosen, fluctuations of the quadratures $X_3(0)$ and $X_1(L)$ are decreased, while fluctuations of the quadratures $Y_3(0)$ and $Y_1(L)$ are increased, which is consistent with the Heisenberg uncertainty relation. Note that quantum fluctuations at the frequency 3ω are always stronger than quantum fluctuations at the frequency ω (Fig. 1).

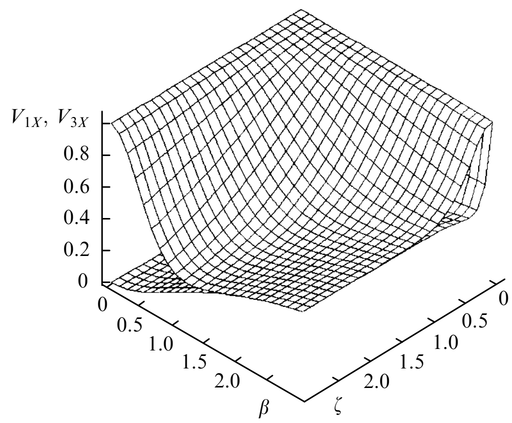

Figure 1. Variances V_{3X} (the upper surface) and V_{1X} (the lower surface) as functions of the normalised length of the nonlinear medium $\zeta = L/L_{\rm NL}$ and the ratio of coupling coefficients $\beta = |k_3|/|k_2|$.

In the above-considered process, a quadrature-squeezed light is produced in the following way. As mentioned above, the initial phase of the process involves parametric amplification with a high-frequency pump. This process is accompanied by the suppression of quantum fluctuations in one of the quadratures at the frequency of amplification (the X_1 quadrature in our case). Next, frequency up-conversion occurs with $k_3 \neq 0$. As a result, transformed fluctuations of the frequency ω are transferred to the 3ω wave. Redistribution of fluctuations at the frequency 3ω should be considered as parametric amplification with a low-frequency pump.

Fluctuation suppression in high-frequency parametric amplification is especially efficient in the absence of frequency mixing ($\beta = 0$), i.e., in the regime when high-frequency 3ω radiation is not generated. As β grows, fluctuation suppression at the frequency ω becomes less efficient, since 3ω quanta are now generated in a random way. A competition of these two factors may give rise to a minimum of the variance of the quadrature $X_3(0)$ as a function of β.

4. Correlation properties of output radiation

Consider now the statistics of the photon number of output radiation at amplification frequencies and examine correlation properties of these photon numbers. We introduce $\hat{n}_1$ and $\hat{n}_3$ to denote the operators of photon numbers at frequencies ω and 3ω, respectively. Then, we can write

$$\hat{n}_j = a_j^+ a_j = \frac{1}{4}\left(X_j^2 + Y_j^2 - 2\right), \quad j = 1, 3. \tag{18}$$

Using Eqns (17) and taking into consideration that $V_0 = 1$ for the vacuum field, we arrive at the following expressions for the mean photon numbers:

$$\begin{aligned} \bar{n}_1(\beta,\zeta) \equiv \langle \hat{n}_1 \rangle &= \frac{1}{4}\left[R_1^2(\beta,\zeta) + Q^2(\beta,\zeta) \right. \\ &\quad \left. + T_1^2(\beta,\zeta) + P^2(\beta,\zeta)\right], \\ \bar{n}_3(\beta,0) \equiv \langle \hat{n}_3 \rangle &= \frac{1}{4}\left[Q_1^2(\beta,\zeta) + Q^2(\beta,\zeta) \right. \\ &\quad \left. + P_1^2(\beta,\zeta) + P^2(\beta,\zeta)\right]. \end{aligned} \tag{19}$$

Formulas (19) describe the mean number of photons in output modes in the cross section $z = L$ for a mode with frequency ω and in the cross section $z = 0$ for a mode with frequency 3ω.

Now, let us determine the variances of photon numbers and the correlation function $\langle n_1, n_3 \rangle$ for the fluctuations of photon numbers:

$$\begin{aligned} \sigma_{n_1}^2 &= \langle \hat{n}_1^2(L) \rangle - \langle \hat{n}_1(L) \rangle^2, \\ \sigma_{n_3}^2 &= \langle \hat{n}_3^2(0) \rangle - \langle \hat{n}_3(0) \rangle^2, \\ \langle n_1, n_3 \rangle &= \langle n_1(L) n_3(0) \rangle - \langle n_1(L) \rangle \langle n_3(0) \rangle. \end{aligned} \tag{20}$$

Using Eqns (14) and (18), we derive the following expressions for the variances of photon numbers in output modes:

$$\begin{aligned} \sigma_{n_1}^2 &= \frac{1}{8}\left[\left(R_1^2 + Q^2\right)^2 + \left(T_1^2 + P^2\right)^2 - 2\left(R_1^2 T_1^2 + Q^2 P^2\right) \right. \\ &\quad \left. - 4(R_1 T_1 - 1)(QP - 1)\right], \\ \sigma_{n_3}^2 &= \frac{1}{8}\left[\left(Q_1^2 + Q^2\right)^2 + \left(P_1^2 + P^2\right)^2 \right. \\ &\quad \left. - 2\left(Q_1^2 P_1^2 + Q^2 P^2\right) - 4(Q_1 P_1 - 1)\left(QP - 1\right)\right]. \end{aligned} \tag{21}$$

The correlation function is then written as

$$\begin{aligned} \langle n_1, n_3 \rangle &= \frac{1}{8}\left[(R_1 - Q_1)^2 Q^2 + (T_1 - P_1)^2 P^2 \right. \\ &\quad \left. - (R_1 P - P_1 Q)^2 - (T_1 Q - Q_1 P)^2)\right]. \end{aligned} \tag{22}$$

Arguments appearing on the right-hand sides of functions are omitted in Eqns (21) and (22).

The correlation properties of output radiation can be conveniently described in terms of the correlation coefficient,

$$K(\beta,\zeta) = \frac{\langle n_1, n_3 \rangle}{\sigma_{n_1} \sigma_{n_3}}. \tag{23}$$

The results of calculations performed for $K(\beta,\zeta)$ are presented in Fig. 2. As can be seen from these data, the correlation coefficient of fluctuations of photons at frequencies ω and 3ω for $\beta < 1$ is higher than the correlation coefficient for $\beta > 1$, which is due to a more efficient sum-frequency generation in the latter case.

5. Quantum fluctuations in interactions of counterpropagating waves

The dependences of characteristics of output radiation on the parameters of the problem in the considered process have several distinguishing features relative to the interaction of copropagating waves [9]. For relatively large interaction lengths ($\zeta \to \infty$), the output operators of the field and quadratures are given by

$$a_1(L) \to -\left(1+\frac{1}{\beta^2}\right)^{1/2} a_{3L} + \frac{1}{\beta}a_{3L}^{+},$$

$$a_3(0) \to \left(1+\frac{1}{\beta^2}\right)^{1/2} a_{10} - \frac{1}{\beta}a_{10}^{+},$$

$$X_1(L) \to -\frac{\beta}{1+\left(1+\beta^2\right)^{1/2}} X_3(L),$$

$$X_3(0) \to \frac{\beta}{1+\left(1+\beta^2\right)^{1/2}} X_1(0), \qquad (24)$$

$$Y_1(L) \to -\frac{\beta}{\left(1+\beta^2\right)^{1/2}-1} Y_3(L),$$

$$Y_3(0) \to \frac{\beta}{\left(1+\beta^2\right)^{1/2}-1} Y_1(0).$$

Interestingly, the output operators are determined by input operators at a 'conjugate' frequency and the ratio of wave coupling coefficients β, and they are independent of the initial operators at their own frequencies.

Thus, as can be seen from Eqns (17) and (24), fluctuations of quadrature components in the interaction of counterpropagating waves cannot be made arbitrarily small. On the other hand, quantum fluctuations caused by initial fluctuations at the same frequency can be suppressed down to a zero level. The output noise under these conditions is determined by redistributed fluctuations arising in each mode due to a nonlinear interaction.

Fig. 2 displays the behaviour of the correlation coefficient $K(\beta,\zeta)$ for fluctuations of photon numbers at the frequencies ω and 3ω at the output of a nonlinear medium as a function of the normalised crystal length. Since, with $\zeta \to \infty$, the output values of field operators depend only on the initial, uncorrelated operators at the conjugate frequency, the correlation coefficient $K(\beta,\zeta)$ decreases with the growth in the crystal length. Correlations remain noticeable within the range of several nonlinear lengths.

Fig. 3 presents the output mean numbers of photons at generation frequencies as functions of the normalised length of the nonlinear medium.

Now, let us analyse the statistics of mode radiation by considering the Fano factor $\mathscr{F}_j = \sigma_{n_j}^2/\bar{n}_j$ $(j = 1, 3)$. The value $\mathscr{F}_j = 1$ corresponds to a coherent state of the field. The difference of the Fano factor from unity indicates the difference of the statistics of mode radiation from the Poissonian statistics. With $\mathscr{F}_j > 1$, we deal with a super-Poissonian statistics, while $\mathscr{F}_j < 1$ corresponds to a sub-Poissonian statistics. Exact expressions for $\mathscr{F}_j$ can be obtained with the use of Eqns (19) and (21). For relatively small crystal lengths ($\zeta \ll 1$), we have

$$\mathscr{F}_1 \approx 2 + (8 - \beta^2/4)\zeta^2 + \ldots,$$
$$\mathscr{F}_3 \approx 1 + \frac{4}{9}\beta^2\zeta^2 + \ldots . \qquad (25)$$

Hence, we find that, for small interaction lengths, the statistics of 3ω radiation is close to a Poissonian statistics. Biphotonic states, i.e., correlated pair of photons with a super-Poissonian statistics [1, 2], arise at the frequency ω.

In the case when $\zeta \to \infty$, we have

$$\mathscr{F}_j \to 2\frac{1+\beta^2}{\beta^2}, \quad j = 1, 3.$$

Thus, for relatively large interaction lengths, output radiation with the frequency ω has the same statistics as 3ω radiation.

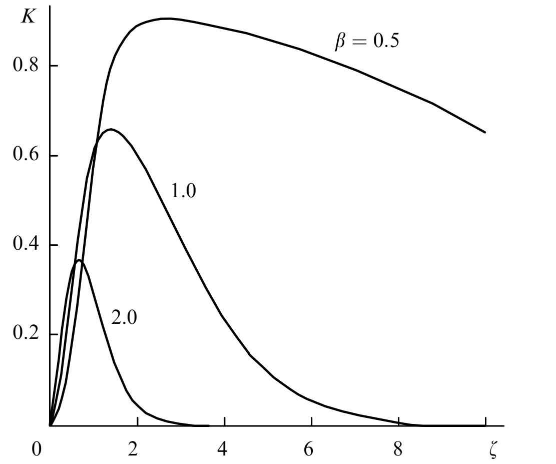

Figure 2. Correlation coefficient K for fluctuations of photon numbers as a function of the normalised length of nonlinear medium ζ for different ratios of coupling coefficients β.

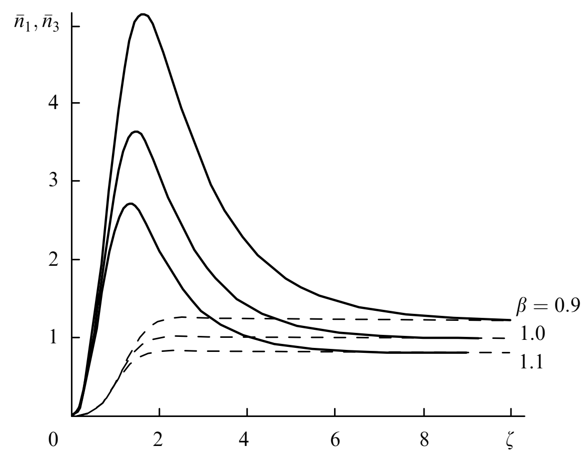

Figure 3. The mean numbers of photons with frequencies ω (solid curves) and 3ω (dashed curves) at the output of the nonlinear medium as functions of the normalised length of the nonlinear medium for different ratios of coupling coefficients β.

6. Conclusions

We have developed a quantum theory of a parametric interaction of counterpropagating waves with low and high frequencies (ω and 3ω) in a pump field with the frequency 2ω in the regime of quasi-phase-matched consecutive interaction of waves with multiple frequencies ω, 2ω, and 3ω. Our analysis has revealed the possibility of producing a quadrature-squeezed light in counterpropagating amplified waves with frequencies ω and 3ω. We have also demonstrated that the efficiency of suppression of quantum fluctuations depends on the length of the nonlinear medium and the ratio of the wave coupling coefficients. Photons at the frequencies ω and 3ω under these conditions have a super-Poissonian statistics.

We should emphasise the following specific feature of the considered process. Although our calculations were performed in the approximation of an undepleted pump field, as the interaction length grows, the mean number of photons in the waves being amplified tends to some stationary value on the order of unity (see Fig. 3). This result, on the one hand, justifies the assumptions used in our analysis, on the other hand, indicates a considerable influence of vacuum fluctuations on a parametric interaction of counterpropagating waves.

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