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Excitation of waveguide modes in a one-dimensional photonic crystal

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***Abstract*. The regimes of excitation of a four-wave mixing wave in a one-dimensional photonic crystal are studied and conditions are found at which the excited wave propagates along the structure in the waveguide regime. A set of waveguide eigenmodes of a multilayer structure is obtained for different polarisations of the field. The resonance character of excitation of waveguide modes is studied in the process of four-wave mixing. A manifold increase in the field amplitude inside the structure is found upon resonance excitation of waveguide modes.**

Keywords: *photonic crystals, waveguide modes, nonlinear-optical effects.*

1. Introduction

Studies of linear and nonlinear processes of the propagation and interaction of waves in photonic crystals attract great recent attention. The presence of regions of the strong spatial and time dispersion in such structures opens up new possibilities for controlling the pulse shape and the efficiency of nonlinear-optical transformations. In Refs [1, 2], the compression of femtosecond pulses in thin one-dimensional periodic structures has been demonstrated, and in papers [3 – 5], the control of the SHG efficiency and the generation of sum and difference frequencies in such structures have been achieved.

The nonlinear-optical response of a medium can be increased for two reasons. The first reason is related to the spatial and time dispersion, which allows one to control the conditions of synchronism of the interacting waves. The second reason is caused by the redistribution of the amplitude of an interference field in a crystal. By varying the angle of incidence, we can move the antinodes of the interference field from one material of the structure to another. Such a redistribution of the field amplitude results in the anomalous propagation of X-rays in crystals (the Borrmann effect [6]). It is well known that the wave amplitude increases no more than two times upon the constructive interference. It can increase much stronger upon excitation of waveguide modes.

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In a one-dimensional photonic crystal, an inhomogeneous wave can be excited, i.e., the wave propagating along the structure in the waveguide regime and exponentially decaying in vacuum. The inhomogeneous wave can be excited due to three-wave ($\omega_3 = \omega_1 - \omega_2$) or four-wave ($\omega_3 = \omega_1 + \omega_1 - \omega_2$) mixing. In this paper, we consider in detail the excitation of an inhomogeneous wave upon four-wave mixing. In the case of three-wave mixing, the results are analogous to those discussed below.

The choice of the mixing process (three- or four-wave mixing) providing the efficient excitation of an inhomogeneous wave depends on the specific parameters of a multilayer structure. Because upon interaction of the field with the multilayer structure the tangential component of the field momentum is conserved, we can easily imagine the situation when, at certain frequencies and angles of incidence of the pump wave on the multilayer structure, the tangential component of the wave vector both for three- and four-wave mixing can become larger that its wavelength in vacuum, resulting in the excitation of an inhomogeneous wave in vacuum. Therefore, the waveguide modes can be resonantly excited in the multilayer structure. In this paper, we determine the eigenmodes of the multilayer structure, which was earlier used in experiments [1 – 5], find the excitation conditions for waveguide modes, and calculate the field distribution in the structure at which the amplitude of the field of resonantly excited modes increases manifold.

2. Recurrent relations and the Green function

Consider the interaction of an electromagnetic field propagating in the plane yz with a layered medium, whose permittivity depends on the coordinate z. The dynamics of the s component of the electromagnetic field is described by the wave equation for the x component of the electric field strength

$$\Delta E_x - \frac{\varepsilon(z)}{c^2}\frac{\partial^2 E_x}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial J_x}{\partial t}, \qquad (1)$$

where $\boldsymbol{J}$ is the density of the atomic current; and $\varepsilon(z)$ is the permittivity of the medium. When a plane wave $E_x(y, z, t) = E_x(z)\exp(-\mathrm{i}\omega t + \mathrm{i}k_y y)$, interacts with the medium (where ω is the field frequency and k_y is the tangential projection of the wave vector), the wave equation (1) takes the form

$$\frac{\mathrm{d}^2 E_x}{\mathrm{d}z^2} + \left[k^2(z) - k_y^2\right]E_x = -\mathrm{i}\omega\frac{4\pi}{c^2}J_x, \qquad (2)$$

where $k(z)$ is the wave-vector modulus. The dynamics of the p component of the electromagnetic field can be conveniently considered using the x component of the magnetic field strength. The equation for this component, analogous to equation (2), has the form

$$\frac{d^2H_x}{dz^2}+\left[k^2(z)-k_y^2\right]H_x-\frac{1}{\varepsilon(z)}\frac{d\varepsilon(z)}{dz}\frac{dH_x}{dz}$$
$$=-\frac{4\pi}{c}\left[ik_yJ_z-\varepsilon(z)\frac{d}{dz}\left(\frac{J_y}{\varepsilon(z)}\right)\right]. \qquad (3)$$

If the efficiency of the nonlinear-optical conversion is small, the solutions of equations (2) and (3) for the nonlinear response of the medium can be represented using the Green function. For equation (2), the solution has the form

$$E_x(z)=-i\omega\frac{4\pi}{c^2}\frac{1}{w}\left[u(z)\int_0^z v(z')J_x(z')dz'\right.$$
$$\left.+v(z)\int_z^L u(z')J_x(z')dz'\right], \qquad (4)$$

where L is the length of the medium; $u(z)$ and $v(z)$ are linearly independent solutions of the homogeneous wave equation; w is the Wronskian. As two linearly independent solutions $u(z)$ and $v(z)$ of the homogeneous wave equation, we can choose the solutions corresponding to two plane waves, which are incident on the medium from opposite sides.

Consider a layer of length d in a multilayer structure, assuming that the linear refractive index within this layer is constant (Fig. 1a). We also assume that this layer is restricted by fragments *1* and *2* of the multilayer structure with the reflection and transmission coefficients R_1, R'_1, T_1, T'_1 and R_2, R'_2, T_2, T'_2, where coefficients without the prime correspond to the wave incident on the fragments of the medium from the left to the right, while coefficients with the prime correspond to the wave incident from the right to the left. Two linearly independent solutions $u(z')$ and $v(z')$ of the homogeneous wave equation in the layer under study have the form

$$u(z')=\frac{T_1[\exp(ik_zz')+R_2\exp(i2k_zd-ik_zz')]}{1-R_2R'_1\exp(i2k_zd)},$$
$$v(z')=\frac{T'_2[\exp(ik_zd-ik_zz')+R'_1\exp(ik_zd+ik_zz')]}{1-R_2R'_1\exp(i2k_zd)}, \qquad (5)$$

where k_z is the normal component of the wave vector and z' is the relative coordinate, which varies from zero to d.

The reflection and transmission coefficients of the layered structure can be calculated using the recurrent procedure. Consider a fragment of the layered structure consisting of two parts with known reflection and transmission coefficients R_1, R'_1, T_1, T'_1 and R_2, R'_2, T_2, T'_2. Then, the reflection and transmission coefficients R, R', T, T' of the entire fragment can be expressed in terms of the known coefficients of its parts as

$$R=R_1+\frac{T_1T'_1R_2}{1-R_2R'_1},\quad T=\frac{T_1T_2}{1-R_2R'_1},$$
$$R'=R'_2+\frac{T_2T'_2R'_1}{1-R_2R'_1},\quad T'=\frac{T'_2T'_1}{1-R_2R'_1}. \qquad (6)$$

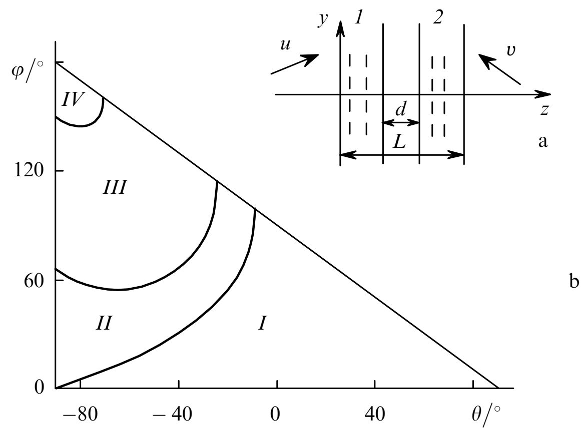

Figure 1. Scheme of the incidence of waves u and v on a multilayer structure (*1, 2* are fragments of this structure) (a) and regimes of excitation of the four-wave mixing wave $\omega_3=\omega_1+\omega_1-\omega_2$ in a multilayer structure as functions of the angle of incidence θ of the first pump wave and the angle φ between the pump waves for fixed pump wavelengths $\lambda_1=690$ nm and $\lambda_2=817$ nm (b). Region *I* is defined by the condition $|k_y(\omega_3)|<k_0(\omega_3)$, region *II* – by the condition $k_0(\omega_3)<|k_y(\omega_3)|<k_2(\omega_3)$, region *III* – by the condition – $k_2(\omega_3)<|k_y(\omega_3)|<k_1(\omega_3)$, and region *IV* – by the condition $k_1(\omega_3)<|k_y(\omega_3)|$.

As fragments of the medium, we can consider any objects, for example, a piece of the homogeneous medium or the interface between two layers. In the latter case, the reflection and transmission coefficients are determined by the Fresnel formulas. Note that the consecutive application of formulas (6) allows one to calculate the reflection and transmission coefficients for arbitrarily complex medium without using a direct numerical integration of the wave equation.

3. Conditions for excitation of an inhomogeneous wave

In a one-dimensional periodic structure, an inhomogeneous wave can be excited, i.e., the wave propagating along the layers in the waveguide regime. To implement this process, two or more pump waves are required, which are incident on the medium at different angles. In addition, the frequency of the excited inhomogeneous wave should be determined by the difference between at least two pump-wave frequencies. The inhomogeneous wave can be excited due to both three-wave ($\omega_3=\omega_1-\omega_2$) and four-wave ($\omega_3=\omega_1+\omega_1-\omega_2$) mixing. Here, we consider in detail the four-wave mixing.

Let an inhomogeneous wave be excited in the ZnS/SrF_2 periodic structure consisting of eight ZnS layers with the refractive index $n_1=2.31$ at 780 nm and of seven SrF_2 layers with the lower refractive index $n_2=1.43$ at 780 nm. We assume that the thickness of the layers was $3\lambda/4n_i$ at 780 nm. These structural parameters are typical for experiments [1–5], which we will compare with our calculations.

Consider the interaction of the above periodical structure with the field of two pump waves at $\lambda_1=690$ nm and $\lambda_2=817$ nm, which are incident on the medium at different angles. Different regimes of excitation of the four-wave mixing wave $\omega_3=\omega_1+\omega_1-\omega_2$ are determined by the tangential component of the wave vector $k_y(\omega_3)=2\sin\theta_1\omega_1/c-\sin\theta_2\omega_2/c$, where $\theta_{1,2}$ are angles of incidence of the pump waves. Let $k_0(\omega_3), k_1(\omega_3), k_2(\omega_3)$ be moduli of the wave vectors of the response field of the medium in vacuum, ZnS, and SrF_2, respectively, which obey the condition $k_0(\omega_3) < k_2(\omega_3) < k_1(\omega_3)$ for the chosen parameters of the medium and external field. Below, we will use angles $\theta = \theta_1$ и $\varphi = \theta_2 - \theta_1$ instead of angles $\theta_{1,2}$.

Fig. 1b shows the region of admissible angles θ and φ divided into four subregions. Subregion *I* is defined by the condition $k_y(\omega_3) < k_0(\omega_3)$, which corresponds to excitation of the four-wave mixing wave that is homogeneous both in vacuum and a photonic crystal. Subregion *II* is defined by the relation $k_0(\omega_3) < |k_y(\omega_3)| < k_2(\omega_3)$, which corresponds to excitation of the wave that is homogeneous in a photonic crystal but is inhomogeneous (exponentially decaying) in vacuum. Subregion *III* is defined by the relation $k_2(\omega_3) < |k_y(\omega_3)| < k_1(\omega_3)$, which corresponds to excitation of the wave that is homogeneous in ZnS layers and inhomogeneous in SrF_2 layers and vacuum. Subregion *IV* is defined by the relation $k_2(\omega_3) < |k_y(\omega_3)| < k_1(\omega_3)$, which corresponds to the case when the four-wave mixing wave cannot be excited. Thus, the inhomogeneous wave can be excited under the condition that a point specified by angles θ and φ lies in the suregion *II* or *III*.

4. Waveguide modes

In the analysis of excitation of an inhomogeneous wave in a layered medium, the solution of the homogeneous wave equation with the boundary conditions, which correspond to the exponential decay of the field outside a photonic crystal, plays an extremely important role. The solution of this boundary-value problem is a set of eigenvalues and modes corresponding to them. The eigenvalues can be conveniently characterised by the tangential component $k_y(\omega_3)$ of the wave vector of the response field of the medium. Fig. 2 shows the spectrum of eigenvalues for the s- and p-polarised field at the wavelength $\lambda_3 = 597$ nm. The tangential component of the wave vector varies from $k_0(\omega_3) = 10.52$ μm^{-1} to $k_2(\omega_3) = 15.13$ μm^{-1}, which corresponds to the angles θ and φ that specify a point in subregion *II* in Fig. 1b. The number of eigenvalues for s and p polarisation is eight and nine, respectively. Note that in the case of s polarisation, the eigenvalues are spaced on the scale $k_y(\omega_3)$ more densely than in the case of p polarisation.

As an example, Fig. 2c shows the averaged tangential component $P_y(\omega_3, z)$ of the Poynting vector, which corresponds to the s-polarised mode with the eigenvalue $k_y(\omega_3) = 13.16$ μm^{-1} (the fourth vertical bar on the scale $k_y(\omega_3)$ in Fig. 2a), as a function of the transverse coordinate z. The field energy flux along the structure in SrF_2 layers substantially exceeds that in ZnS layers. This feature is typical virtually for all s-polarised modes. In contrast to the s polarisation, the field energy flux along the structure in the case of p-polarised modes is distributed in a photonic crystal more 'uniformly'.

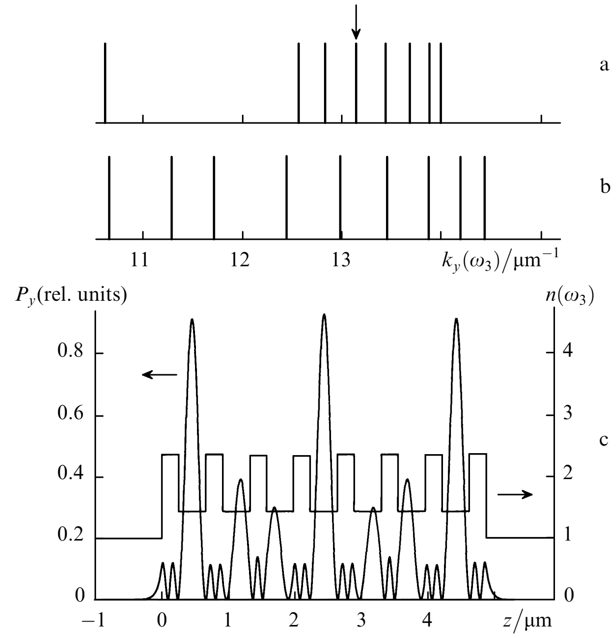

Figure 2. Eigenvalue spectrum of the tangential component of the wave vector of the s- (a) and p-polarised (b) field at the wavelength $\lambda_3 = 597$ nm and the dependence of the averaged tangential component of the Poynting vector of the s-polarised wave mode of the field, which corresponds to the eigenvalue of the tangential component of the wave vector $k_y(\omega_3) = 13.16$ μm^{-1} (shown by the arrow), on the transverse coordinate z.

5. Excitation of waveguide modes

Consider now excitation of the s-polarised four-wave mixing wave by the p-polarised ($\lambda_1 = 690$ nm) and s-polarised ($\lambda_2 = 817$ nm) pump waves. If the efficiency of nonlinear-optical conversion is small, we can assume that the density $\boldsymbol{J}$ of the atomic current depends only on the pump-field strength, whose depletion can be neglected.

In our numerical calculations, whose results are presented below, we assumed that only ZnS layers with a large refractive index were a nonlinear medium taking part in the generation of the field at the Raman frequency. We assumed that the density $\boldsymbol{J}$ of the atomic current induced in the medium was $\chi(\boldsymbol{E}\boldsymbol{E})\boldsymbol{E}$, where $\boldsymbol{E}$ is the pump-field strength and χ is the effective nonlinear susceptibility of ZnS layers. Such an approximation was rather course but quite acceptable for our purpose.

Fig. 3 shows the averaged tangential component $P_y(\omega_3, z)$ of the Poynting vector, i.e., the energy flux of the response field of the medium along the structure, as a function of the transverse coordinate z for different angles φ and θ. For a fixed angle $\varphi = 40°$ between the pump waves, the angle θ determines the tangential component of the wave vector of the four-wave mixing wave. If the tangential component of the wave vector coincides with one of the eigenvalues of the homogeneous wave equation (Figs 2a, b), we deal with the resonance excitation of the corresponding mode of the field.

Figs 3a and 3e show quasi-resonance cases, the exact resonance conditions being satisfied at angles $\theta = -49.35°$ and $-53.29°$ for the second and third s-polarised modes, respectively. Under quasi-resonance conditions, the field profile of the four-wave mixing wave coincides with that of the corresponding mode, while the field amplitude increases with decreasing detuning from the resonance. In non-resonance cases (Figs 3b–d), the field profile of the four-wave mixing wave is determined by two parameters (angles φ and θ) rather than by one (the tangential component of the wave vector), which takes place under resonance conditions.

To analyse resonance excitation of the waveguide modes, we consider the energy flux along the structure

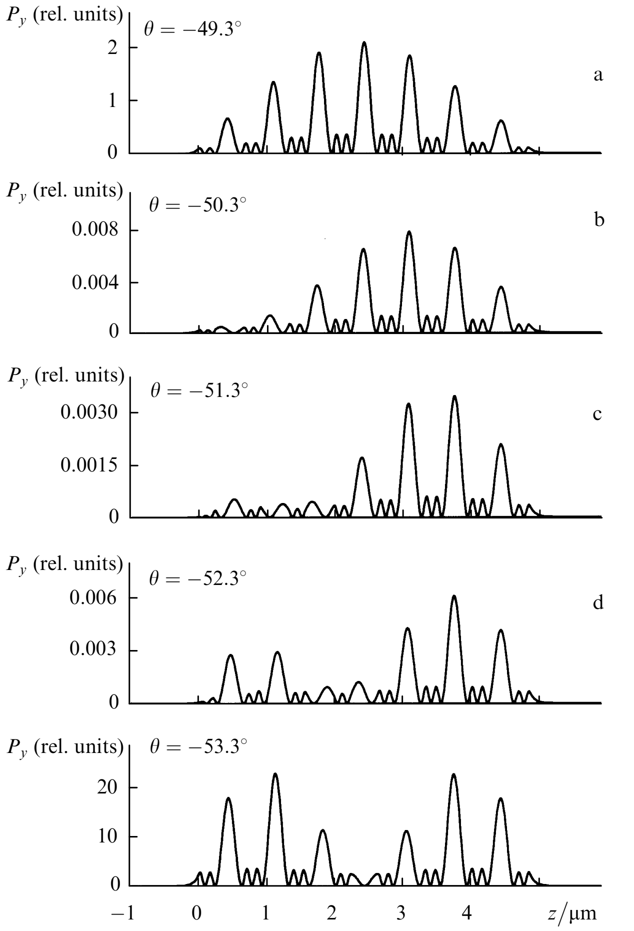

Figure 3. Dependences of the averaged tangential component of the Poynting vector of the s-polarised response field at the combination frequency on the transverse coordinate z for different angles of incidence θ of the first pump wave for the fixed angle $\omega_3 = \omega_1 + \omega_1 - \omega_2$ between p- and s-polarised pump waves.

$$S(\omega_3) = \int_0^L P_y(\omega_3, z)\mathrm{d}z,$$

where L is the length of a photonic crystal. Fig. 4 shows the dependences of the energy flux along the structure on the angle θ at the fixed angle $\varphi = 40°$ between the pump waves for two cases: excitation of the s-polarised four-wave mixing wave by the p-polarised ($\lambda_1 = 690$ nm) and s-polarised ($\lambda_2 = 817$ nm) pump waves and excitation of the p-polarised four-wave mixing wave by two p-polarised pump waves.

The positions of the intensity maxima of the excited wave are determined by the angle θ at which the tangential component of the wave vector coincides with one of the eigenvalues of the homogeneous wave equation (Figs 2a,b). The width of the resonance maxima depends on the magnitude of a change in the tangential component of the wave vector with variation of the angle θ in the vicinity of the resonance. The resonance enhancement of the field amplitude is caused by the interference between reflected and transmitted four-wave mixing waves. Within the framework of the model under study, the amplitude of resonance maxima tends to infinity because of the interaction of infinite plane waves with an infinite (along the y axis) photonic crystal.

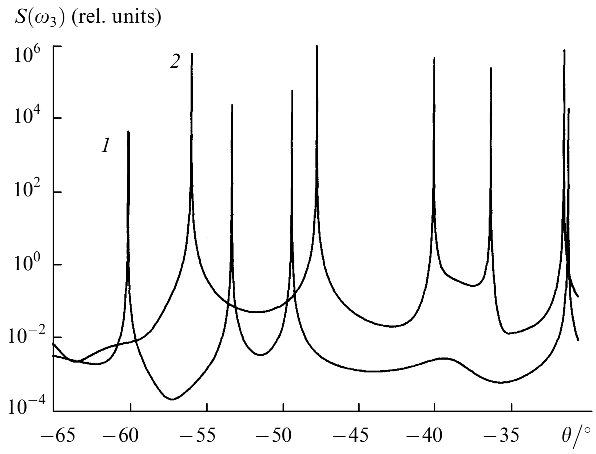

Figure 4. Dependences of the energy flux along a multilayer structure for the s- (*1*) and p-polarised (*2*) field of the four-wave mixing wave $\omega_3 = \omega_1 + \omega_1 - \omega_2$ on the angle of incidence of the first pump wave for the fixed angle $\varphi = 40°$ between the pump waves. In the first case, the medium interacts with p- and s-polarised pump waves; in the second case, the medium interacts with two p-polarised pump waves.

Consideration of the decay of the four-wave mixing wave restricts the increase of resonance maxima. We assumed in our numerical calculations that the four-wave mixing wave decayed on the length 1 cm in ZnS layers by a factor of e. A finite diameter of a laser beam under real experimental conditions and a finite time of the interaction of laser radiation with the structure reduce the amplitude of resonance maxima. The presence of different defects in a photonic crystal gives rise to the dependence of the eigenvalue spectrum on the longitudinal coordinate y, which can result in a strong change in the character of resonance excitation of the modes.

6. Conclusions

Therefore, to excite the four-wave mixing wave propagating along the structure in the waveguide regime in a one-dimensional photonic crystal, two or more pump waves are required, which are incident on the structure at different angles. The frequency of the excited wave should be determined by the difference between at least two frequencies of the pump waves. The conditions of excitation of waveguide modes are determined by the parameters of the pump field: wavelengths and the angles of incidence of the waves on the structure. Under quasi-resonance conditions, the distribution of the field of the four-wave mixing wave coincides with that for the corresponding mode, while the field amplitude increases manifold with decreasing detuning from the resonance. The results obtained clearly demonstrate the possibility to control the field distribution in the medium. The drastic increase in the field amplitude of an inhomogeneous wave under quasi-resonance conditions can be used to increase the efficiency of nonlinear-optical conversion or for diagnostics of various processes inside a multilayer structure.

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