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Nonreciprocal optical effects in moving periodic absorption and amplification gratings

O E Nanii, D D Shcherbatkin

***Abstract*. The nonreciprocal amplitude and phase effects emerging upon the passage of counterpropagating light waves through moving periodic absorption or amplification gratings are studied theoretically. Analytic expressions are derived for amplitude and phase nonreciprocity. It is shown that the maximum phase nonreciprocity corresponds to the incidence of light waves on a moving grating at the Bragg angle.**

Keywords: *absorption and amplification gratings, amplitude nonreciprocity, phase nonreciprocity.*

The interest in the investigations of nonreciprocal optical effects, which are being carried out extensively during the last years, is due to the possibility of their practical application in controlling the parameters of ring lasers, as well as the need to determine their origin for the purpose of identifying and eliminating possible sources of errors in laser gyroscopes [1]. In addition to the nonreciprocal devices based on the Faraday effect, which are being used in actual practice, the nonreciprocal effects have been also studied [2–7], which were predicted in [8] and are related to moving refractive-index gratings accompanying travelling acoustic waves, the so-called nonreciprocal acousto-optical effects. The results obtained in [2–7] suggest that nonreciprocal effects are present in all kinds of moving periodic gratings.

In this work, we study for the first time the nonreciprocal effects in moving periodic absorption and amplification gratings. Consider the diffraction of light waves from a moving periodic grating of amplitude inhomogeneities with the wave vector directed along the z axis. The absorption coefficient in such a grating is described by the expression

$$\gamma(z) = a + \frac{b}{2}\sin(\Omega t - Kz), \tag{1}$$

where a is the mean absorption coefficient (negative absorption coefficients correspond to amplification); b is the modulation depth of the periodic grating; K is the propagation constant; $\Omega = KV$; and V is the grating velocity.

O E Nanii, D D Shcherbatkin Department of Physics, M V Lomonosov Moscow State University, Vorob'evy gory, 119899 Moscow, Russia

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In the approximation of plane light waves (unbounded along the z axis), the variation of the amplitudes of incident and diffracted light waves is described by the coupled wave equations. In the case of diffraction from periodic absorption and amplification gratings, the coupled wave equations for the geometry shown in Fig. 1 have the form [9]

$$\frac{dA_1^+}{dx} = \zeta_{12}A_2^+\exp(i\Delta\alpha^+ x) - aA_1^+, \tag{2}$$

$$\frac{dA_2^+}{dx} = \zeta_{21}A_1^+\exp(-i\Delta\alpha^+ x) - aA_2^+, \tag{3}$$

where $A_{1,2}^+$ are the amplitudes of the incident and diffracted light waves, respectively, propagating in the positive direction for which the left to right direction is chosen in the present work; $\Delta\alpha^+ = \alpha_1^{x+} - \alpha_2^{x+}$ is the detuning between the projections of the wave vectors of the incident and diffracted waves; α_1^{x+} and α_2^{x+} are the projections of the propagation constants α_1^+, α_2^+ of the incident and diffracted waves on the x axis. The coupling constants ζ_{12} and ζ_{21} determine the diffraction efficiency and are complex conjugate ($\zeta_{12} = \zeta_{21}^*$) in the case of interaction with the absorption or amplification gratings in the general case.

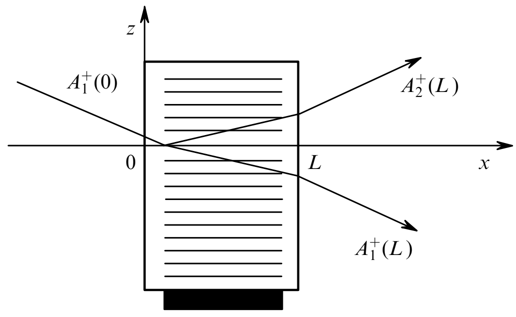

Figure 1. Schematic of the interaction of light waves upon diffraction from a moving periodic grating.

Let us introduce a real coefficient $\zeta = (\zeta_{12}\zeta_{21}^*)^{1/2} = |\zeta_{12}|$ characterizing the diffraction efficiency. The equality $\zeta_{12} = \zeta_{21} \equiv \zeta$ can be satisfied by an appropriate choice of the initial phases and position of the x axis. In this case, the solution of the system (2), (3) under the boundary condition $A_2^+(0) = 0$ and for length L of the medium (see Fig. 1) has the form

$$\frac{A_1^+(x)}{A_1^+(0)} = \exp\left[\left(\mathrm{i}\frac{\Delta\alpha^+}{2} - a\right)x\right]\left\{\cosh\left[\zeta^2L^2 - \left(\frac{\Delta\alpha^+x}{2}\right)^2\right]^{1/2}\right.$$

$$-\mathrm{i}\frac{\Delta\alpha^+x}{2}\left[\zeta^2L^2 - \left(\frac{\Delta\alpha^+x}{2}\right)^2\right]^{-1/2}$$

$$\left.\times\sinh\left[\zeta^2L^2 - \left(\frac{\Delta\alpha^+x}{2}\right)^2\right]^{1/2}\right\},$$

$$\frac{A_2^+(x)}{A_1^+(0)} = \exp\left(-\mathrm{i}\frac{\Delta\alpha^+x}{2} - ax\right)\zeta\sinh\left[\zeta^2L^2\right.$$

$$\left.-\left(\frac{\Delta\alpha^+x}{2}\right)^2\right]^{1/2}\left[\zeta^2L^2 - \left(\frac{\Delta\alpha^+x}{2}\right)^2\right]^{-1/2}$$

for $\Delta\alpha^+x/2 \leqslant \zeta L$, and

$$\frac{A_1^+(x)}{A_1^+(0)} = \exp\left[\left(\mathrm{i}\frac{\Delta\alpha^+}{2} - a\right)x\right]\left\{\cos\left[\left(\frac{\Delta\alpha^+x}{2}\right)^2 - \zeta^2L^2\right]^{1/2}\right.$$

$$-\mathrm{i}\frac{\Delta\alpha^+x}{2}\left[\left(\frac{\Delta\alpha^+x}{2}\right)^2 - \zeta^2L^2\right]^{-1/2}$$

$$\left.\times\sin\left[\left(\frac{\Delta\alpha^+x}{2}\right)^2 - \zeta^2L^2\right]^{1/2}\right\},$$

$$\frac{A_2^+(x)}{A_1^+(0)} = \exp\left(-\mathrm{i}\frac{\Delta\alpha^+x}{2} - ax\right)\zeta$$

$$\times\sin\left[\left(\frac{\Delta\alpha^+x}{2}\right)^2 - \zeta^2L^2\right]^{1/2}\left[\left(\frac{\Delta\alpha^+x}{2}\right)^2 - \zeta^2L^2\right]^{-1/2}$$

for $\Delta\alpha^+x/2 \geqslant \zeta L$.

The expression for the complex amplitude of the transmitted wave has the form

$$A_1^+(L) = \exp[\mathrm{i}\Phi^+(L)]|A_1^+(L)|,$$

where

$$\frac{|A_1^+(L)|}{A_1^+(0)} = \exp(-aL)\left\{\cosh^2\left[\zeta^2L^2 - \left(\frac{\Delta\alpha^+L}{2}\right)^2\right]^{1/2}\right.$$

$$+\left(\frac{\Delta\alpha^+L}{2}\right)^2\left[\zeta^2L^2 - \left(\frac{\Delta\alpha^+L}{2}\right)^2\right]^{-1/2}$$

$$\left.\times\sinh^2\left[\zeta^2L^2 - \left(\frac{\Delta\alpha^+L}{2}\right)^2\right]^{1/2}\right\}^{1/2}, \tag{4}$$

$$\Phi^+(x) = \frac{\Delta\alpha^+L}{2} - \arctan\left\{\frac{\Delta\alpha^+L}{2}\left[\zeta^2L^2 - \left(\frac{\Delta\alpha^+L}{2}\right)^2\right]^{-1/2}\right\}$$

$$\times\tanh\left[\zeta^2L^2 - \left(\frac{\Delta\alpha^+L}{2}\right)^2\right]^{1/2} \tag{5}$$

for $\Delta\alpha^+L/2 \leqslant \zeta L$;

$$\frac{|A_1^+(L)|}{A_1^+(0)} = \exp(-aL)\left\{1 - \zeta^2L^2\left[\left(\frac{\Delta\alpha^+L}{2}\right)^2 - \zeta^2L^2\right]^{-1/2}\right.$$

$$\left.\times\sin^2\left[\left(\frac{\Delta\alpha^+L}{2}\right)^2 - \zeta^2L^2\right]^{1/2}\right\}^{1/2}, \tag{6}$$

$$\Phi^+(x) = \frac{\Delta\alpha^+L}{2} - \arctan\left\{\frac{\Delta\alpha^+L}{2}\left[\left(\frac{\Delta\alpha^+L}{2}\right)^2 - \zeta^2L^2\right]^{-1/2}\right\}$$

$$\times\tan\left[\left(\frac{\Delta\alpha^+L}{2}\right)^2 - \zeta^2L2\right]^{1/2} \tag{7}$$

for $\Delta\alpha^+L/2 \geqslant \zeta L$.

Fig. 2 shows the dependences of the ratio $|A_1^+(L)|/A_1^+(0)$ of the transmitted wave amplitude modulus to the amplitude of the incident wave and of the phase incursion $\Phi^+(L)$ of the transmitted wave on the normalised detuning $\Delta\alpha^+L/2$. These dependences were calculated using formulas (4)–(7).

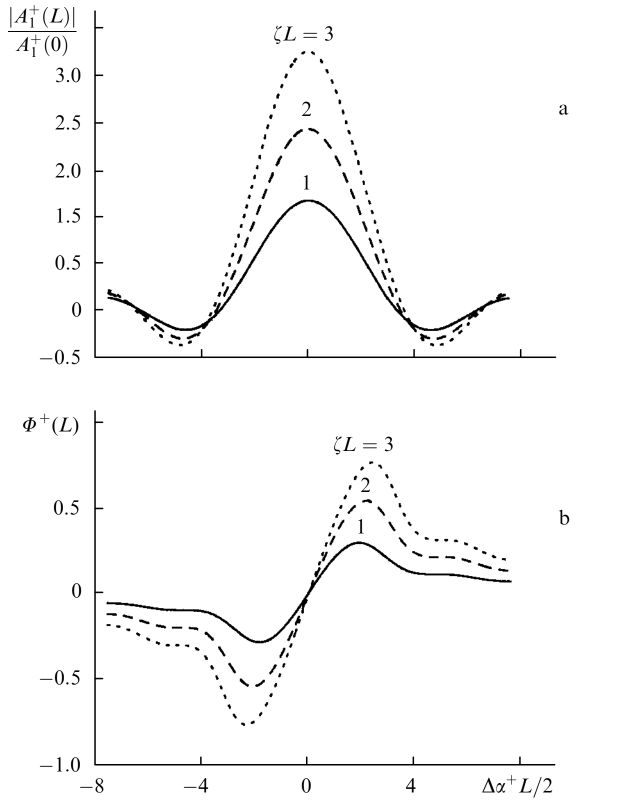

Figure 2. Dependences of the ratio $|A_1^+(L)|/A_1^+(0)$ of the modulus of the transmitted wave amplitude to the amplitude of the incident wave (a), and of the phase incursion $\Phi^+(L)$ of the transmitted wave (b), on normalised detuning $\Delta\alpha^+L/2$ for $\alpha = 0$ and for different ζL.

The propagation of a light wave in the opposite direction (from right to left) through the region of interaction with the periodic grating is described by the equations for the amplitudes $A_{1,2}^-$, which coincide with Eqns (2), (3) in which the direction of the x axis is reversed and the origin of coordinates is shifted to the right boundary of the interaction region. It is found that the detuning between the projections of the wave vectors of the counterpropagating incident and diffracted light waves along the corresponding x axis are different in the general case: $\Delta\alpha^+ \neq \Delta\alpha^-$. Indeed, although $\alpha_1^- = \alpha_1^+$, the wave vectors of the diffracted waves are not equal ($\alpha_2^- \neq \alpha_2^+$) because the waves diffracted in the opposite directions have different frequencies. It is this difference in the mismatchings of the wave vector projections of the counterpropagating incident and diffracted waves that is responsible for the emergence of nonreciprocal effects upon interaction of light with moving periodic gratings of all kinds.

The difference in the mismatchings of wave vectors in counterpropagating waves is proportional to the velocity of the moving grating:

$$|\Delta\alpha^{+} - \Delta\alpha^{-}| \simeq \frac{2\Omega n_2}{c} = \frac{4\pi(Vn_2)}{\Lambda c} = 2Kn_2\frac{V}{c},$$

where Λ is the grating period and n_2 is the refractive index of the medium [7]. The dependences of the normalised amplitude ($|A_1^{+}(L)|/A_1^{+}(0) - |A_1^{-}(L)|/A_1^{-}(0)$) and phase ($\Phi^{+}(L) - \Phi^{-}(L)$) nonreciprocities on normalised detuning are shown in Fig. 3 for various velocities of the moving diffraction grating. One can see from Fig. 3b that the maximum phase nonreciprocity corresponds to zero detuning (wave matching) between the projections of the wave vectors of the incident and diffracted waves, which takes place for the light wave incident at the Bragg angle.

Fig. 4 shows the schematic of a possible experimental realisation of the method proposed for creating amplitude-frequency nonreciprocity. Two light beams with slightly differing frequencies ω_1 and ω_2 ($\omega_1 - \omega_2 \ll \omega_1 + \omega_2$) are incident on the amplifying (absorbing) medium of a ring laser at angles $\pm\theta$ to the optical axis. In the absence of these waves, the oscillation frequency of the ring laser is ω_0. The frequencies ω_1 and ω_2 lie within the amplification (absorption) band of the corresponding element of the ring laser. The interference of these light beams produces a periodic intensity grating, which moves along the optical axis in a direction coinciding with the projection of the propagation direction of the higher-frequency beam. The moving periodic intensity grating creates a moving periodic amplification grating in the active medium.

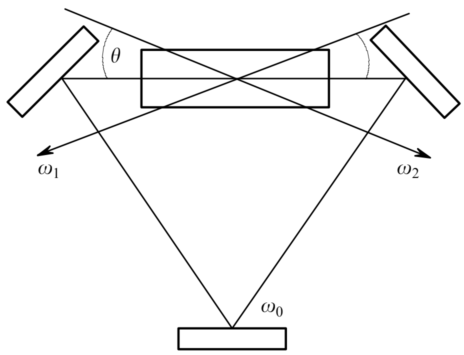

Figure 4. Principal schematic of the possible experimental realisation of the method proposed for creating an amplitude-frequency nonreciprocity

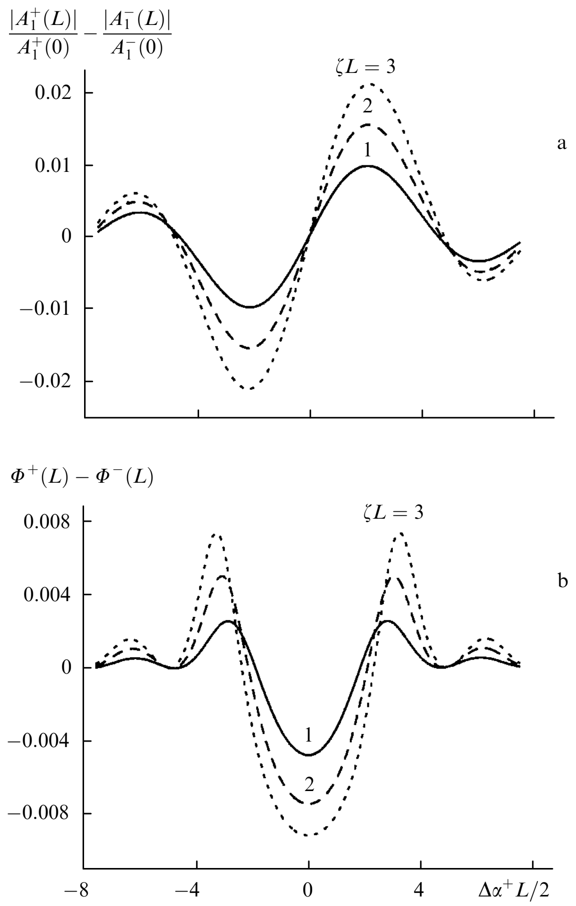

Figure 3. Dependences of the normalised amplitude ($|A_1^{+}(L)/A_1^{+}(0) - |A_1^{-}(L)|/A_1^{-}(0)$) (a) and phase ($\Phi^{+}(L) - \Phi^{-}(L)$) (b) nonreciprocities on normalised detuning $\Delta\alpha^{+}L/2$ for $\alpha = 0$ and for different ζL.

Thus, moving absorption or amplification gratings can be used for creating nonreciprocal phase or amplitude elements. The prospect of this technique stems from the possibility of creating such gratings upon the interference of counter-propagating waves of slightly different frequencies in resonantly absorbing or amplifying media.

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