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Mode separator for a beam with an off-axis optical vortex

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***Abstract*. The Gaussian beam diffraction by a thin amplitude grating with a dislocation shifted relative to the beam axis is considered. The first-order diffracted beam with an off-axis optical vortex is represented as a superposition of Laguerre – Gauss modes. The optical scheme permitting the spatial separation of modes with even and odd mode indices is proposed and realised experimentally. The method of separation is based on the difference in Gouy phase shifts for focused beams. The possibility of separating photons with zero and nonzero orbital angular momentum is discussed.**

Keywords: *optical vortices, Laguerre – Gauss modes, Gouy phase shift, photon orbital angular momentum*

Physical optics has been recently enriched by the concept of optical vortex (OV) [1] used for describing the structure of a wave field with phase defects (wave-front dislocations) [2]. The interest in OVs is due to their peculiar properties as well as possible applications in problems of manipulation with microparticles [3]. A distinguishing feature of an OV is that the field amplitude vanishes at its axis, while the phase becomes indefinite, or singular due to a jump by π or $m\pi$ in the case of an m-fold vortex (a review of the OV properties is given in [4, 5]). If the OV axis coincides with the beam axis (as, for example, for Laguerre – Gauss modes with a nonzero azimuthal index), the integer value of m is called the topological charge. The structure of beams with OVs can be determined from the expression for the Laguerre – Gauss modes $LG_p^{(l)}$ (in the cylindrical coordinates ρ, φ, z):

$$E(LG_p^{(l)}) = E_{LG}\frac{w_0}{w}\left(\frac{\sqrt{2}\rho}{w}\right)^{|l|}\exp\left(-\frac{\rho^2}{w^2}\right)L_p^{|l|}\left(\frac{2\rho^2}{w^2}\right)$$

$$\times\exp\left\{\mathrm{i}\left[kz+\frac{k\rho^2}{2R(z)}+l\varphi-Q\arctan\left(\frac{z}{z_\mathrm{R}}\right)\right]\right\}, \quad (1)$$

where E_{LG} is the amplitude parameter; w_0 is transverse size of the beam waist; $w = w_0(1 + z^2/z_\mathrm{R}^2)^{1/2}$ is the transverse size of the beam at a distance z from the waist; $R(z) = z(1 + z_\mathrm{R}^2/z^2)$ is the radius of curvature of the wave front at the beam axis; $z_\mathrm{R} = kw_0^2/2$ is the Rayleigh length; k is the wave number; $L_p^{|l|}$ is the associated Laguerre polynomial; l is the azimuthal mode index; p is the radial mode index [6]. For beams with a nonzero index l, the amplitude vanishes at the beam axis, while the phase is an exponential function of azimuth, $\exp(\mathrm{i}l\varphi)$, which corresponds to an OV with the topological charge $m = l$. The number $Q = 2p + |l| + 1$ is known as the mode index determining the affiliation to a family of modes with the same Q. The mode index determines the small correction to the phase velocity of the mode, associated with the Gouy phase shift $\arctan(z/z_\mathrm{R})$. For a Gaussian beam ($p, l = 0$) $Q = 1$ and the additional phase incursion (relative to a plane wave) over the distance between the waist and the far-field zone due to the Gouy phase shift is $-\pi/2$. The 'doughnut' mode $LG_0^{(1)}$ ($p = 0$, $l = 1$) has the index $Q = 2$, and the corresponding additional phase incursion is equal to $-\pi$.

The expression for the 3D equiphase surface (wave front) of the $LG_0^{(l)}$ mode at the waist can be obtained from Eqn (1) putting $R(z) \to \infty$ and $z \ll z_\mathrm{R}$:

$$kz + l\varphi = \text{const.} \quad (2)$$

The equation for the wave front (2) describes a helicoidal surface with a step equal to $l\lambda$ (λ is the wavelength). As the wave front leaves the waist, its shape remains helicoidal with a singularity (phase discontinuity of $l\pi$) at the axis. Such a wave-front structure leads to the interference fringe splitting (emergence of l new fringes) during the interference of the beam containing OVs with a plane wave, which is the main indication of the presence of OVs in interference detection [7].

Another important consequence is the presence of the orbital angular momentum of the beam [8] due to the circulation of the optical flux around the OV axis. The orbital angular momentum L_z for an axisymmetric OV beam with energy W, frequency ω, and topological charge m is defined as [8]

$$L_z = \frac{mW}{\omega}, \quad (3)$$

which gives the value $m\hbar$ when recalculated per photon.

The quantised value of the orbital angular momentum of a photon either can be random or may reflect the physical reality of the existence of the corresponding orbital quantum number. At the present time, there are arguments in favour of the quantum origin of the orbital angular momentum as well as of a purely classical description [9]. For instance, the coaxial interference of Laguerre – Gauss modes leads to a

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combined beam [10] for which the specific orbital angular momentum 'per photon' may be arbitrary. This can be interpreted as the result of superposition of photons with a nonzero orbital angular momentum and photons belonging to the $LG_p^{(0)}$ modes, which have no orbital angular momentum. The position of OV in a combined beam is displaced relative to the centre of the beam due to the presence of a mode with an axial peak [11].

In this work, an attempt is made to separate the vortex and the vortex-free components in a beam with an off-axis OV. The idea of the method is to make use of the difference in the Gouy phase shifts for modes with different values of Q.

It is often difficult to generate laser radiation at a required Laguerre – Gauss mode in laboratory conditions. For this reason, a few extracavity procedures have been worked out for obtaining OV beams, the simplest technique being diffraction by a synthesised hologram [12]. The first-order diffracted beam has a phase-singularity point in the cross section, i.e., contains an OV of an appropriate sign and charge.

Consider a hologram located at the waist of the readout Gaussian beam:

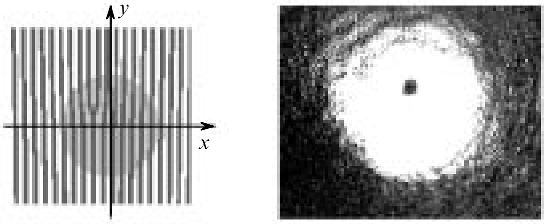
$$E_G \propto \exp\left(-\frac{\rho^2}{w_0^2}\right). \tag{4}$$

We will assume that the beam diameter is much smaller than the transverse size of the hologram, but much larger than its characteristic spatial period. For a hologram synthesised in the form of a fine amplitude grating with a dislocation, its transmission T has the form

$$T(\rho, \varphi) = T_0 + T_1(\rho)\cos(K\rho\cos\varphi + M\varphi), \tag{5}$$

where T_0 is the mean transmission; $T_1(\rho)$ is the band contrast; $K = 2\pi/\Lambda$; Λ being the grating period; M is the order of the dislocation in the grating [11]. The grating with $M = 1$ has the splitting of the central band into two bands (see Fig. 1a). Considering that $\rho\cos\varphi = x$, the diffraction field E_d in the case of grating readout by a plane wave $E_0\exp(ikz)$ can be represented as the sum of the fields of the order zero and ± 1:

$$E_d = E_0\left[T_0\exp(ikz) + T_1(\rho)\right.$$
$$\left.\times\frac{\exp(ikz + iKx + iM\varphi) + \exp(ikz - iKx - iM\varphi)}{2}\right], \tag{6}$$



a b

Figure 1. (a) Orientation of a Gaussian beam (grey circle) incident on a grating with phase singularity $M = 1$ (the centre of the grating is shifted relative to the beam axis) and (b) the cross section of the diffracted beam.

in the $+1$ order, the topological charge of the OV is $m = M$, while in the -1 order, $m = -M$. The Gaussian beam readout of the grating leads to the same result.

In actual practice, synthesised holograms may have a binary transmission distribution in the form

$$T(x, \varphi) = \begin{cases} 0, & \cos(Kx + M\varphi) \leqslant 0, \\ 1, & \cos(Kx + M\varphi) > 0. \end{cases} \tag{7}$$

For a binary grating with a dislocation described by formula (7), the derivation of the distribution function of the first-order diffracted field is quite cumbersome [13]. For this reason, while determining the basic properties of the diffracted beam, we will assume that $T_1(\rho)$ increases linearly with the distance from the grating centre and becomes equal to unity for $\rho = R_0$. (Note that the branching point of the band is shifted relative to the grating centre by $M\Lambda/2\pi$ [11].) In this case, the diffraction of the Gaussian beam ($w_0 \ll R_0$) by a grating with $M = 1$ leads to the following first-order diffraction field distribution in the plane behind the grating (in Cartesian coordinates):

$$E_1 \propto \frac{x + iy}{R_0}\exp\left(-\frac{x^2 + y^2}{w_0^2}\right), \tag{8}$$

which corresponds to the Laguerre – Gauss mode $LG_0^{(1)}$. Let us now suppose that the grating centre is slightly displaced relative to the centre of the readout beam, say, by a distance $x_0 < R_0$ along the x axis. Then the corresponding field distribution in the first diffracted order has the form

$$E_1 \propto \frac{x + x_0 + iy}{R_0}\exp\left(-\frac{x^2 + y^2}{w_0^2}\right). \tag{9}$$

This expression leads to a superposition of the $LG_0^{(1)}$ mode carrying OV and the 'admixture' in the form of the $LG_0^{(0)}$ mode (Gaussian beam) with a contribution proportional to x_0. As a result, a combined beam is formed, in which the OV shift relative to the beam axis is proportional to displacement x_0. Such a simplified approach leads to the following important conclusion: the field structure in the first diffraction order can be represented as a superposition of the axial OV and the vortex-free component. For a binary grating with the transmission function (7), the vortex component is a superposition of the $LG_p^{(M)}$ modes, while the vortex-free component is formed by the $LG_p^{(0)}$ modes. In this respect, we may speak of various types of photons in a beam with an off-axis OV, i.e., of photons with zero and nonzero orbital angular momentum.

The schematic of the experiment is shown in Fig. 2. The initial beam was obtained with the help of a syntesised binary hologram ($M = 1$) giving a beam with a single OV in the first diffraction order [11]. The grating centre is displaced relative to the centre of the beam incident on the grating (see Fig. 1a). The OV position in the diffracted beam did not coincide with the centre of the beam (see Fig. 1b) directed to the interferometer serving as a mode separator.

One of the arms of the interferometer formed by beamsplitters *3* and totally reflecting mirrors *4* contained two confocal lenses *5*. The propagation of the beam through the gap between the lenses leads to an addition phase incursion equal to the doubled Gouy shift due to beam focusing followed by expansion. Thus, the resultant phase incursion for the mode with index Q amounts to $-Q\pi$. If, for ex-

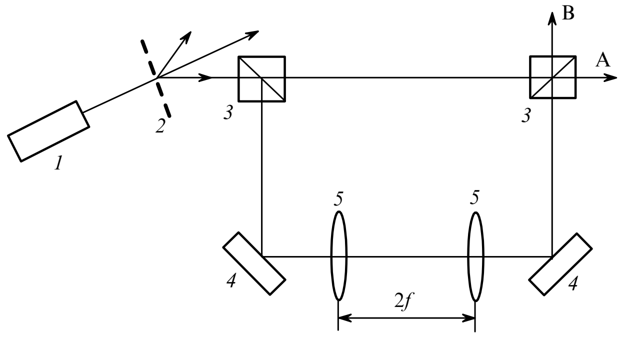


Figure 2. Experimental scheme: (*1*) He–Ne laser, (*2*) binary grating, (*3*) beamsplitters, (*4*) mirrors, (*5*) lenses.

ample, the path difference between the beam propagating to channel A without multiple reflections and the beam reflected to channel A from the second arm is equal to an even number of half-waves for a mode with even Q, the entire beam will be directed to channel A, whereas radiation in channel B will be absent due to interference quenching. For such a tuning of the interferometer, the path difference for a mode with an odd Q is equal to an odd number of half-waves, and the entire beam propagates through channel B. The interferometer was tuned in accordance with beam quenching at the exit of channel A for a Gaussian beam ($Q = 1$) at the separator entrance.

Because the beam with a single off-axis OV is a superposition of modes with even Q (vortex component) and odd Q (vortex-free component), the separator divides the vortex and vortex-free components into different channels. Note that the OV sign is reversed after reflection, but the beams interfering at the exit A have OV of the same sign as those in the initial beam due to an even number of reflections.

With an appropriate tuning, different patterns are observed at the exits A and B: an axial OV at the exit A and a beam with a smooth wave front at exit B (Fig. 3a, b), which was verified using the interference with an additional plane wave (Fig. 3c,d).

Thus, we experimentally divided the modes with an odd index Q (vortex-free component, $l = 0$) and even Q (vortex component, $l = 1$). This division may be interpreted as the possibility of spatial separation of photons, belonging to the 'vortex' $LG_0^{(1)}$ mode and to a Gaussian beam ($LG_0^{(0)}$ mode) and forming a combined beam as a result of coherent addition, using purely optical methods of spatial separation along different channels.

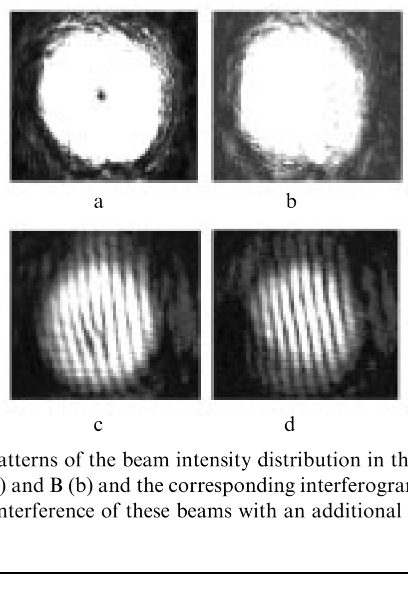


Figure 3. Patterns of the beam intensity distribution in the cross section at exits A (a) and B (b) and the corresponding interferograms obtained as a result of interference of these beams with an additional plane wave (c) and (d).

Note that this method is based on the difference in the Gouy phase shifts for modes with even and odd mode indices rather than on the existence of orbital angular momentum. For instance, the modes $LG_0^{(2)}$ ($Q = 3$) and $LG_1^{(0)}$ ($Q = 3$), one of which has an orbital angular momentum and the other has zero momentum, cannot be separated using our scheme.

In our opinion, the existence of an orbital angular momentum for a solitary photon can be established only in experiments with light absorption by a medium capable of detecting the angular momentum transfer. For this and other problems, the obtaining of a 'purified' OV is very important, and the technique proposed by us here may be used in such cases.

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References

1. Coullet P, Gil L, Rocca F *Opt. Commun.* **73** 403 (1989)
2. Nye J F, Berry M V *Proc Roy. Soc. Lond. A* **336** 165 (1974)
3. He H, Friese M E J, Heckenberg N R et al. *Phys. Rev. Letts.* **75** 826 (1995)
4. Soskin M, Vasnetsov M *Photonics Science News* **4** (4) 21 (1999)
5. Vasnetsov M, Staliunas K (Eds.) *Optical Vortices* (N.Y.: Nova Science, 1999)
6. Anan'ev Yu A *Opticheskie Rezonatory i Problema Raskhodimosti Lazernogo Izlucheniya* (Optical Cavities and Laser Beam Divergence Problem) (Moscow: Nauka, 1979)
7. Baranova N B, Zel'dovich B Ya, Mamaev A V et al. *Zh. Eksp. Teor. Fiz.* **83** 1702 (1982)
8. Allen L, Beijersbergen M W, Spreeuw R J C, Woerdman J P, *Phys. Rev. A* **45** 8185 (1992)
9. Tiwari S C *J. Mod. Opt.* **46** 1721 (1999)
10. Soskin M S, Gorshkov V N, Vasnetsov M V, et al. *Phys. Rev. A* **56** 4064 (1997)
11. Basistiy I V, Bazhenov V Yu, Soskin M S, Vasnetsov M V *Opt. Commun.* **103** 422 (1993)
12. Bazhenov V Yu, Vasnetsov M V, Soskin M S *Pis'ma Zh. Eksp. Teor. Fiz.* **52** 1037 (1990)
13. Sacks Z S, Rozas D, Swartzlander G A, Jr. *J. Opt. Soc. Am. B: Opt. Phys.* **15** 2226 (1998)