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The object image reconstruction from the speckle pattern of its field

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***Abstract*. It is shown that information on the speckle pattern allows one to form a kind of the resonator with a minimum-loss mode representing the required image. The image reconstruction process is simulated for a variety of objects. The outlook for the building of astronomical instruments, which are equivalent to modern telescopes and are based on the principle of detection and processing of information contained in the speckle field scattered by an object, are discussed.**

Keywords: *speckle pattern, image reconstruction, telescopic systems.*

The intensity distribution of light reflected from an object illuminated by coherent or quasi-coherent light with the coherence length exceeding the object depth appears, at some distance from the object, as a granular structure (speckle pattern). The characteristic size of a speckle is $d_c = \lambda/\theta$, where λ is the radiation wavelength and θ is the observation angle of the object. It is important that the speckle pattern does not change when the observation plane is moved in the object direction by the distance $l \leqslant l_f = d_c^2/\lambda = \lambda/\theta^2$ (where l_f is the Fresnel length), i.e., the speckle detection does not require very high accuracy of measurements (the accuracy required in optics is typically of the order of λ). Moreover, if the object is in the cosmos and is observed through a sufficiently thin layer of the turbulent atmosphere, which is located near the detection plane, the observed intensity distribution will coincide with that obtained in the absence of turbulence.

The information on a speckle pattern has long been used, probably beginning from the experiments of Brown and Twiss, for obtaining some notions of the shape and structure of an object being observed [1]. At the same time, the object image reconstruction from its speckle is a quite attractive problem. Indeed, the solution of this problem would mean the principal possibility of building astronomical instruments, which are equivalent to modern telescopes, but have a substantially greater angular resolution and are in fact insensitive to any distortions introduced by the atmospheric turbulence.

The problem of measurements would be reduced to the detection of the intensity distribution over some area followed by the computer-aided data processing with the help of a specially developed method. In this case, the angular resolution is $\theta_r \sim \lambda/D$, where D is the characteristic size of the detection area, which can be many times greater than the size of the largest modern telescopes along with the relatively low accuracy of fixation of detection elements. Probably, some other fields of applications also exist where such an approach to the imaging would be useful and possibly unique.

Note that the attempts to solve the problems of this type have been made in the last years [2, 3], however, rare successful results were, generally speaking, accidental and nonsystematic and were not distinctly substantiated from the methodological point of view. The aim of our paper is to develop a logical procedure for solving the problem of the object image reconstruction from its speckle field.

We assume, without loss of generality, that an object of a finite size is located in a plane, and the field distribution on its surface is described by the scalar complex function $f(\boldsymbol{\rho})$. Then, the speckle field $F(\boldsymbol{\rho}_1)$ in the detection plane, which is parallel (coplanar) to the object plane, is related to $f(\boldsymbol{\rho})$ at small observation angles by the integral Kirchhoff – Fresnel relationship

$$F(x_1, y_1) = A \int\int f(x, y) \times \exp\left\{ i\frac{k}{2L}\left[(x - x_1)^2 + (y - y_1)^2\right]\right\} dx dy, \quad (1)$$

where $k = 2\pi/\lambda$; L is the distance between the planes; and A is a constant, which is insignificant in this case.

The corresponding intensity distribution has the form

$$I(x_1, y_1) = |F(x_1, y_1)|^2 = F(x_1, y_1)F^*(x_1, y_1). \quad (2)$$

It can be shown that, when the distance L from the object is sufficiently large ($L \gg d$, where d is the characteristic object size), $I(x_1, y_1)$ only weakly depends on translational movements of the object during its detection. Below, we will consider rough scattering objects. This corresponds to real objects, because their surfaces are not processed with optical accuracy, they contain precipitated dust, etc. All this means that from the point of view of its phase

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characteristics, the field $f(\boldsymbol{\rho})$ represents a random statistical field and has the form

$$\langle f(\boldsymbol{\rho}_0) f(\boldsymbol{\rho}_0+\boldsymbol{\rho})\rangle \approx \begin{cases} 1, & \rho<\varepsilon, \\ 0, & \rho>\varepsilon, \end{cases}$$

where $\varepsilon \leqslant 10^{-3}$ m is the characteristic roughness size. The angle brackets denote both averaging over an ensemble and integration over the object plane for one realisation. For the case under study, we can show that, if we are interested only in the field distribution over the object and the distance to the detection plane is $L \geqslant \varepsilon d_{\text{ob}}/\lambda$ (where d_{ob} is the characteristic object size), then, without loss of generality, we have

$$F(x_1,y_1) = A\int\!\!\int f(x,y)\exp\left[-\mathrm{i}\frac{k}{L}(xx_1+yy_1)\right]\mathrm{d}x\mathrm{d}y, \tag{3}$$

i.e., the object field and its speckle are related by the Fourier transform. Therefore, the problem is reduced to the determination of the field intensity distribution $|f(\boldsymbol{\rho})|^2$ in the object plane of a rough plane object of finite size knowing the square of the modulus of the Fourier transform of the field scattered by the object.

We attempted to solve this problem by using the concepts and terminology that are accepted in the theory of open resonators. The essence of the approach is as follows. In each individual case, based on the available information on the speckle, a virtual resonator is formed in which the required distribution represents a minimum-loss mode or the most intense mode. Let us introduce the elements of the virtual resonator in the following way.

Because we are only interested in the field intensity distribution in the object plane, we have from the speckle field intensity $I(x_1,y_1)$, based on the known statistical relation between the average intensity $\bar I$ and the correlation function $K(\boldsymbol{\rho}_1)$, that $\langle I(\boldsymbol{r}_1+\boldsymbol{\rho}_1)I(\boldsymbol{r}_1)\rangle = \bar I^2 + |K(\boldsymbol{\rho}_1)|^2$. Here, the averaging means integration over the area for one specific realisation, which is admissible because the speckle has a random nature. Note that in the case of natural stationary objects, $|K(\boldsymbol{\rho}_1)|$ can be obtained by directly measuring the time dependence of the field intensity at different points followed by the averaging of their products, as for example, in experiments of Brown and Twiss.

According to the Van Cittert–Zernicke theorem,

$$K(x_1,y_1) = A\int\!\!\int |f(x,y)|^2\exp\left[-\mathrm{i}\frac{k}{L}(xx_1+yy_1)\right]\mathrm{d}x\mathrm{d}y. \tag{4}$$

Using the obvious relation

$$B(x,y) = \int\!\!\int |K(x_1,y_1)|^2\exp\left[\mathrm{i}\frac{k}{L}(xx_1+yy_1)\right]\mathrm{d}x_1\mathrm{d}y_1 =$$

$$= \int\!\!\int \tilde I[(x_2-x),(y_2-y)]\tilde I(x_2,y_2)\mathrm{d}x_2\mathrm{d}y_2, \tag{5}$$

where $\tilde I(x,y) = |f(x,y)|^2$ is the field intensity in the object plane, and taking into account that the object is finite, we can find a rectangle oriented in some way in the object plane, which contains the function $B(x,y)$ tangent to its sides. By dividing this rectangle into four equal parts by perpendiculars drawn from the middles of adjacent sides and separating one of them, we obtain a rectangle in which our object is located. The boundary points of the object touch all the four sides of the separated rectangle.

Below, we will use this rectangle as one of the mirrors of the 'resonator'. As another mirror, we will use a finite region in the speckle detection plane with the characteristic size D (in the limiting case, $D\to\infty$). We will use the following algorithm for determining the field formed in the object plane. Let us take, for example, the uniform distribution of the field on a mirror in the object plane as a zero-order approximation and obtain its Fourier transform in the speckle detection plane. Then, by keeping the phase invariable, we will take the distribution $|K(x_1,y_1)|$as an amplitude and perform the inverse Fourier transform. By keeping invariable the field that falls within the separated rectangle in the object plane and equating to zero the fields that fall outside it, we repeat this process once more, etc.

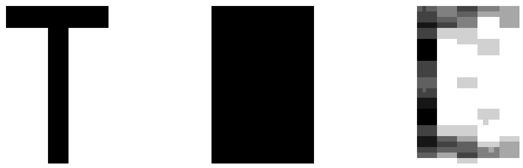
In this paper, we simulated such an iteration process with the help of a computer using the MathCAD-2000 package and the Paint Shop 7 graphic editor, which was applied for the synthesis of artificial objects and receiving scanned real images. Let us analyse the procedure proposed. The iteration method we used here resembles, on the one hand, the Fox–Lee method in the theory of open resonators and, on the other, the Gerchberg–Saxton iteration process [4, 5], which is applied in theoretical optics for reconstructing fields from their intensity distribution in two planes separated in space. Because the iteration procedure (the imposition of the speckle amplitude on one of the mirrors to the field) is substantially nonlinear, the concepts of the theory of linear open resonators can be only conditionally applied to this situation.

It seems plausible that the required intensity distribution in the object plane (image) is, in a sense, the most appropriate for the virtual resonator formed and, in this sense, represents its mode. This distribution completely fits into a rectangular mirror, its intensity distribution in the observation plane coincides with $|K(x_1,y_1)|^2$, and this distribution transfers in itself during the iteration process. Other possible images, which give the same distribution $|K(x_1,y_1)|^2$, but are displaced by an arbitrary vector $\boldsymbol{\rho}$ relative to the middle of the rectangular mirror, are vignetted by the mirror edges and are poorly 'survived' during the iteration process (modes with greater losses).

There is, however, the second image, which gives the same distribution $|K(x_1,y_1)|^2$ and fits into the mirror. This distribution corresponds to $K^*(x_1,y_1)$ in the speckle plane and to $I(-x,-y)$ in the object plane. In the case of symmetric starting conditions on the object mirror, both solutions are represented with the same weight. For a linear resonator, the distributions of equal intensities would be overlapped in the limit, and it would be difficult to separate the required image from this picture in the general case.

The case of $I(x,y) = I(-x,-y)$ is an obvious exception. Here, the situation is even more dramatic. Because the resonator is nonlinear, both distributions affect each other in a complicated way, so that the distribution obtained upon iterations can have nothing in common with the required distribution. Fig. 1c shows the results of the iteration procedure (2000 iterations) starting from symmetric initial conditions. Note that already after some 30 iterations, the image is obtained that virtually does not change upon further iterations.

This problem can be solved by two methods. In the first method, the iteration procedure starts from substantially



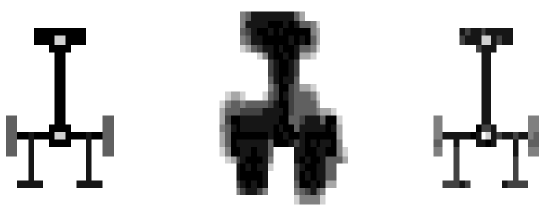
a b c

Figure 1. Results of computer simulation of the reconstruction of the image of the T-shaped object in the case of symmetric initial conditions: (a) actual image; (b) mirror in the object plane; (c) result of 2000 iterations.

asymmetric initial conditions in the object plane. As a result, in the presence of strong asymmetry in the required image, one of the distributions will be represented in the initial conditions with a noticeable lower weight than the other one. Then, the image obtained upon iterations will correspond to the required image with accuracy to the inversion. It is this case that is shown in Fig. 2. Here, the image of the T-shaped object was reconstructed after ~ 50 iterations. In practice, a distorted image obtained in some way can be used as initial conditions. Fig. 3 shows the reconstructed image of an artificially synthesised object.

a b c d

Figure 2. Effect of asymmetric initial conditions on the image reconstruction: (a) actual image; (b) start image; (c) result of 20 iterations; (d) result of 100 iterations.

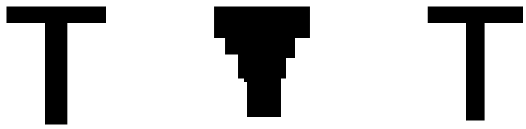


a b c

Figure 3. Same as in Fig. 2, but using the distorted object image (b) as initial conditions.

In the quasi-symmetric case, i.e., when the required and inverted images are strongly overlapped, the result is distorted by the inverted image and by the noise caused by the nonlinearity of the iteration procedure.

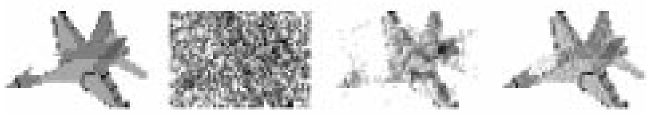
The second method, which seems to be more constructive, is based on the fact that by analysing the structure of the convolution $\iint \tilde{I}(\tilde{x} - x, \tilde{y} - y)\, I(\tilde{x}, \tilde{y}) \mathrm{d}\tilde{x}\, \mathrm{d}\tilde{y}$ in the object plane or the image distorted in the general case, for example, by atmosphere and obtained by a standard method (with the help of a telescope), we can correct the initial rectangular shape of a mirror in the object plane to vignette the inverted image. Then, only the mode that mainly corresponds to one image will survive in the 'resonator' during iterations (Fig. 4). In a given case, ~ 30 iterations are sufficient for obtaining the image without noise.



a b c

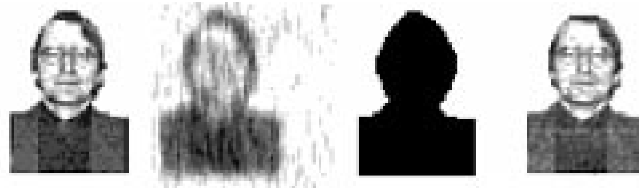
Figure 4. Effect of the corrected mirror geometry on the quality of the reconstructed image: (a) actual image; (b) mirror in the object plane; (c) result of 30 iterations.

For the quasi-symmetric case considered above, we can propose a combined method, which combines the first two methods and also includes the procedure of dynamic adaptation of the mirror contour in the object plane. The essence of the adaptation is as follows: after $10-50$ iteration cycles, the obtained object image is very noisy due to the overlap of the inverted image. In this case, the object contour can be determined much more accurately and, hence, the mirror geometry in the object plane can be additionally specified. Such a combined method can be used for the reconstruction of rather complicated images. The relevant results are presented in Figs 5 and 6. The strongly distorted images of an aircraft and a human face were taken as initial images. Then, the mirror geometry was corrected using the above method. In this case, a total number of iterations was $\sim 100-300$.



a b c d

Figure 5. Image reconstruction by the combined method: (a) actual image; (b) noisy image obtained; (c) result of 35 iterations; (d) image obtained after correction of the mirror geometry.



a b c d

Figure 6. Same as in Fig. 5; (c) corrected mirror geometry in the object plane.

Thus, we have shown that, by using information on the speckle, we can form a kind of the resonator whose minimum-loss mode represents the image we are interested in. We have obtained our results by computer simulations. However, in this case, numerical methods are completely adequate to analogue methods used in Fourier optics, which means that the method proposed by us can be applied in the real experimental situation. These results allow us to put forward the problem of the development of observation instruments, which do not require a precision manufacturing and are insensitive in many cases to atmospheric distortions whose angular resolution is determined by the aperture that considerably exceeds the aperture of modern telescopes.

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