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### Formation of a uniform intensity distribution in a cw CO<sub>2</sub> laser with a phase-stepped mirror

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Abstract. A method of forming non-Gaussian light beams with a uniform spatial intensity distribution is studied. It uses a generalised confocal laser cavity, with one of the mirrors made in the form of a mask with discretely arranged phase-shifting nonuniformities. The theoretical and experimental results of the study confirm the formation of such beams at the output of a CO<sub>2</sub> laser with a nonuniform phase-stepped mirror.

*Keywords:*  $CO_2$  laser, single-mode regime, spatial profile of a radiation beam.

### 1. Introduction

Many applications require a specified shape of the spatial distribution of a single-mode output beam that differs from the known mode distributions of quasi-optical cavities. For instance, a uniform distribution of the field amplitude on the output mirror or in a specified plane outside a cavity is often required. Such problems arise in the development of technological lasers, lasers for medicine and location, microwave semiconductor quasi-optical oscillators, and other devices operating in the millimetre range [1-4].

The phase of optical radiation is one of the parameters that can be used for controlling the spatial wavefront modulation for obtaining a specified intensity profile [5-8]. The formation of light beams with a specified field distribution is closely related to the problem of obtaining single-mode radiation. In Ref. [9], a method is described for selecting purely transverse laser modes by a phase-shifting mask located on one of the mirrors of an optical cavity along the node lines of a mode being selected. This method is capable of enhancing the mode discrimination compared to the absorption mask method or decreasing losses for the mode being selected, retaining the previous degree of discrimination. In Ref. [10], it was proposed to increase the output power of a single-mode laser by using stable cavities, which are equivalent to a confocal cavity and contain a positive lens between the mirrors, i.e., generalised confocal cavities. Among these cavities, symmetric cavities

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Received 15 September 2000; revision received 2 April 2001 *Kvantovaya Elektronika* **31** (6) 543–546 (2001) Translated by A N Kirkin satisfying the condition (1 - L/2R)(1 - L/F) + L/R = 0, where R is the radius of curvature of the mirrors, F is the focal distance of the lens, and L is the cavity length, possess the highest selectivity.

To obtain single-mode lasing with a specified field distribution, an intracavity method was proposed [11], which differs from the known methods by a simple cavity modification and small additional energy losses. The method is based on the fact that one of the reflectors contains absorbing or scattering discretely distributed inhomogeneities in the regions where the Fourier transform of the function characterising the desired field distribution vanishes. The theoretical and experimental results obtained in Ref. [12] confirmed the formation of non-Gaussian light beams with a uniform intensity distribution at the output of a  $CO_2$  laser with a nonuniform amplitude-stepped mirror. Modifications of this method were also proposed in Ref. [13] and experimentally realised for a Nd:YAG laser in Ref. [14].

In this paper, we determined configurations of a generalised confocal cavity and geometrical parameters of its phase-stepped mirror (PSM) that enable one to obtain a mode with a uniform intensity distribution at the output of a  $CO_2$  laser. The existence of such a mode was confirmed by computer-aided numerical solution of a system of integral equations for a generalised confocal cavity with a nonuniform PSM, whose geometrical parameters were varied, and by the experimental studies of a  $CO_2$  laser with a cavity of this kind.

### 2. Theoretical relations

The schematic diagram of a cavity under study is presented in Fig. 1. It is formed by two circular flat mirrors with radii  $a_1$  and  $a_2$  and reflectivities  $T_1$  and  $T_2$ , respectively, and an intracavity phase corrector in the form of a thin positive lens with the focal distance F. The distances from the phase corrector to the mirrors  $L_1$  and  $L_2$  are chosen to be approximately equal to F. The field distribution on the mirrors can be represented in the form

$$U_{nm}(r,\varphi) = E_{nm}(r) \exp(in\varphi),$$

where *n* and *m* are integer angular and radial indices; *r* and  $\varphi$  are polar coordinates.

The eigenmode problem for an asymmetric cavity is reduced to the system of integral equations

$$\int_{0}^{1} E_{mn}^{(p)}(\rho_p) T_p(\rho_p) K(\rho_p, \rho_l) \rho_p d\rho_p = \alpha_{nm}^{(l)} E_{nm}^{(l)}(\rho_l).$$
(1)



Figure 1. Schematic diagram of a generalised confocal cavity with a phase-stepped mirror.

Hereafter, p = 1, 2 is the reflector number;  $\rho_{p,l} = r_{p,l}/a_{p,l}$ . The constants  $\alpha_{nnn}^{(l)}$  symmetrise the equations. Of physical sense is their product, whose magnitude and argument are equal to the amplitude attenuation factor and the additional phase incursion of an electromagnetic wave during the round-trip transit time in the cavity. The kernels of integral equations (1) are written in the form

$$K(\rho_{p},\rho_{l}) = (-\mathrm{i})^{n+1} \eta_{pl} B_{pl} J_{n}(B_{pl}\rho_{p}\rho_{l})$$
  
× exp  $\left\{ \mathrm{i}\pi \left[ N_{p} \rho_{p}^{2} \left( 1 - \frac{N_{p}}{N_{0p} Z_{pl}} \right) + N_{l} \rho_{l}^{2} \left( 1 - \frac{N_{l} \eta_{pl}^{2}}{N_{0p} Z_{pl}} \right) \right] \right\},$ 

where

$$\begin{split} \eta_{pl} &= \frac{a_p}{a_l}; \quad B_{pl} = \frac{2\pi N_p N_l \eta_{pl}}{N_{0p} Z_{pl}}; \quad N_{p(l)} = \frac{a_{p(l)}^2}{\lambda L_{p(l)}}; \\ N_{0p} &= \frac{a_p^2}{\lambda F}; \quad Z_{pl} = \frac{1 - \gamma_p \gamma_l}{(1 - \gamma_p)(1 - \gamma_l)}; \quad \gamma_{p(l)} = 1 - \frac{L_{p(l)}}{F}; \end{split}$$

and  $J_n$  is the Bessel function of the first kind of order *n*. Let the amplitude field distribution on one of the mirrors be described by the circular function

$$\operatorname{circ} \rho = \begin{cases} 1, & \rho \leq 1, \\ 0, & \rho > 1. \end{cases}$$
(2)

Substituting (2) in (1) and using the terms of Fourier optics [15], we rewrite (1) in the form

$$\mathscr{F}\left\{(\operatorname{circ}\rho_p)T_p(\rho_p)K(\rho_p,\rho_l)\rho_p\right\} = \alpha_{nm}^{(l)}E_{nm}^{(l)}(\rho_l),$$
(3)

where the symbol  $\mathscr{F}$  denotes the Fourier transform for the function in braces. The Fourier transform of field distribution (2) in the case of an infinite phase correlator is described, within an insignificant constant factor, by the function

somb 
$$G = \frac{2J_1(\pi G)}{\pi G},$$
 (4)

where  $G = 2N_{pl}\rho_l$ ; and  $N_{pl} = a_p a_l / [\lambda F(1 - \gamma_p \gamma_l)]$  is the Fresnel number.

If phase-shifting regions are arranged on one of the mirrors so that  $\rho_l = v_{1g}/(2\pi N_{pl})$  ( $v_{1g}$  are the roots of the function  $J_1$ ; g = 1, 2, 3, ...) and the possibility of selection of transverse modes by phase-shifting masks [9] is taken into account, it is reasonable to expect that the solution of system (1) is given by the functions describing the field distributions on the mirrors in the analytic form close to

$$E_{nm}^{(p)}(\rho_p) = \operatorname{circ} \rho_p, \qquad E_{nm}^{(l)}(\rho_l) = \operatorname{somb} \rho_l. \tag{5}$$

Because  $E_{nm}^{(p)}$  is the Fourier transform of  $E_{nm}^{(l)}$  and vice versa, the oscillation type corresponding to such distributions may be defined as a Fourier mode.

Integral equations (1) were solved on a computer for symmetric cavities  $(a_1 = a_2 = a, \gamma_1 = \gamma_2 = \gamma)$  by the matrix method proposed in Ref. [16]. We studied the dependences of the amplitude field distributions and the mode energy loss on the Fresnel number  $N_{pl}$  of a cavity, the number and size of phase-shifting regions, and the phase shift in these regions. The main results are presented in Ref. [17].

# **3.** Experimental setup and geometrical parameters of a PSM

The experiments were made on the setup that is described in detail in Ref. [12]. However, in contrast to Ref. [12], a CO<sub>2</sub> laser was pumped by a dc longitudinal discharge. The voltage ( $\sim 3 \text{ kV}$ ) was applied to two ring electrodes made of stainless steel, which were 20 mm wide and located close to the inner wall of the laser tube. The discharge gap was 360 mm long. The CO<sub>2</sub>: N<sub>2</sub>: He: Xe = 1:1:4:0.25 gas mixture at a pressure of 8 Torr slowly circulated in the laser system.

The laser cavity was formed by two flat circular mirrors with radii of 4.9 mm and a spherical phase corrector, which represented a mirror with radius of 15 mm and a focal distance of 500 mm. Laser radiation was outcoupled through a flat semitransparent germanium mirror with transmittance of  $\sim 6.5$  %. A highly reflecting mirror represented either a flat uniform aluminium mirror on a glass substrate or a flat nonuniform PSM.

The PSM was produced by the photolithographic technique. An aluminium coating with a special profile was deposited on a substrate made of stainless steel. The working mirror surface represented a spatial phase filter, which consisted of a set of alternating rings chosen in a certain way. They had the same reflectivity throughout the mirror surface, but different piecewise constant phases. Phase-shifting mirror regions were made in the form of grooves with a prescribed depth. The geometry of the PSM and the groove dimensions were chosen on the basis of the calculations made earlier for  $\lambda = 10.6 \ \mu m [17]$ .

The profile of the reflector used in the experiments and the calculation model is shown in Fig. 1. The working surface of the PSM was 9.8 mm in diameter, and the central reflecting region was 1.32 mm in diameter. The width of the phase-shifting grooves and the reflecting rings (the ring and groove numbers are calculated from the mirror centre) are presented in Table 1. The depth of phase-shifting grooves was  $1.8 \pm 0.35 \ \mu m$  [i.e.  $(0.17 \pm 0.033)\lambda$ ], and the corresponding phase shift was  $122.26 \pm 23.8^{\circ}$ . The intensity and phase distributions on the output mirror of the laser cavity were calculated by Eqn (1), with the PSM parameters taken from Table 1.

Table 1.				
N₂	Groove width		Ring width	
	in λ	(mm)	in $\lambda$	(mm)
1	14.15	0.15	38.68	0.41
2	14.15	0.15	36.79	0.39
3	14.15	0.15	36.79	0.39
4	14.15	0.15	36.79	0.39
5	14.15	0.15	36.79	0.39
6	14.15	0.15	36.79	0.39
7	14.15	0.15	36.79	0.39
8	14.15	0.15	27.36	0.29

. . . .



**Figure 2.** Calculated distributions of field intensity *I* and phase  $\varphi$  on the output mirror of the CO<sub>2</sub> laser.

## 4. Comparison of experimental and numerical results

To lift frequency degeneration [18], we chose the laser cavity different from the confocal one ( $\gamma = -0.01$ ). The calculated field and phase distributions on the output mirror of the CO<sub>2</sub> laser having the given cavity configuration and containing the PSM whose parameters are presented in Table 1 are shown in Fig. 2. The normalised absolute measure of difference between the function  $\operatorname{circ} \rho$  and the intensity profile of the field  $U_1$  on the output mirror, which was defined [19] as

$$\prod = \frac{1}{M} \sum_{k=1}^{M} \left| 1 - |U_1(\rho_k)|^2 \right|$$

(*M* is the number of points where the function  $U_1$  is discretely set), did not exceed 15 %.

Because we were unable to measure the field intensity distribution directly on the output mirror due to specific design features of the CO<sub>2</sub> laser, the experimental field distributions in the output beam were recorded at the distances z = 105, 600, and 1370 mm from the uniform semitransparent mirror, which corresponded to the Fresnel region. The experimental intensity distributions and the corresponding calculated distributions in these sections are presented in Fig. 3.

These results allow us to analyse the dynamics of behaviour of the output beam and compare more reliably the experimental results with the numerical calculations. One can see that the experimental radial intensity profiles are close to the corresponding calculated profiles both in their shape and the beam radius (at a level of  $e^{-2}$ ).

To confirm realisation of single-mode operation of the CO<sub>2</sub> laser, with the radial intensity profile of output radiation close to the function circ  $\rho$ , we studied experimentally and numerically the transverse intensity distributions in the focal plane of phase correctors. The phase correctors represented spherical mirrors with radii of curvature of R = 1200 and 3000 mm. The mirrors were positioned at focal distances from the output laser mirror and oriented at a small (~ 5°) angle to the incident radiation.

In the focal region of these mirrors, the radial beam profiles were recorded. According to Fourier optics [15], in the case when the intensity distribution formed on the output laser mirror is approximately described by the function circ  $\rho$ , the field distribution observed in the focal plane of a phase corrector will be close to somb  $\rho$ . The experimental and calculated intensity distributions obtained for the given phase correctors are shown in Fig. 4.

Note a good focusing of the laser beam under study, which confirms the single-mode nature of laser radiation.



Figure 3. Calculated (solid curves) and experimental (dashed curves) radial intensity distributions for CO<sub>2</sub> laser radiation at different distances *z* from the uniform semitransparent mirror.



**Figure 4.** Calculated (solid curves) and experimental (dashed curves) intensity distributions in the focal region of different phase correctors.

The experimental beam radii at a level of 0.5 coincide with the calculated values. An increase in the experimental width of the focused beam at a level of  $e^{-2}$  for the phase corrector with R = 1200 mm is due to an imperfection of the measuring system (the sensitive region of the detector probing the beam was 0.5 mm in diameter).

Upon tuning the length of the CO<sub>2</sub> laser cavity by  $\lambda/2$ , we observed six lines in the emission spectrum. They corresponded to lasing on the 10*P* branch (14, 16, 18, 20, 22, and 24). The laser lines were identified using a spectrum analyser with an echelette in the Ebert mounting [20]. Note that the radial intensity profiles for all these laser lines, recorded at the same distances from the output mirror as before, were almost identical. This gives evidence of a wide spectral band of the PSM used by us. When tuning the laser cavity, we observed no other transverse modes in the laser emission spectrum in addition to the fundamental mode (Fourier mode).

The calculated diffraction losses per round trip  $(\Delta_{nm} = 1 - |\alpha_{nm}|^2)$  for the fundamental mode in the laser cavity under study are equal to 4.5 %. For the unoptimised transparency of the output mirror, the power of single-mode emission of the CO<sub>2</sub> laser operating on the 10*P*(20) line with the PSM under study was 0.63 W. For the cavity with uniform plane mirrors, the output power of multimode laser emission reached 1.5 W. Upon tuning the laser cavity formed by uniform flat mirrors, with the working aperture of the output mirror diaphragmed down to 2 mm, we observed two transverse modes. Their field distributions were typical of TEM<sub>00q</sub> and TEM<sub>01q</sub> modes of an open cavity. The output power of the laser operating on the TEM<sub>00q</sub> mode was 0.5 W.

Thus, the results of our experimental and theoretical study confirmed the potentiality of the intracavity phase method of phase filtering proposed by us for the development of single-mode lasers with a uniform distribution of output radiation. By varying geometrical dimensions of a cavity and configuration of its nonuniform mirror, one can reach extremely low diffraction losses.

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