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Time-delayed quantum interference and single-photon echo in coherent three-level systems

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***Abstract*. A new version of time-delayed quantum interference of photons is proposed, which may be manifested in the emergence of a single-photon echo signal in coherent three-level macroscopic media. Nonclassical properties of this type of interference, reflected in peculiarities of the interaction between the field and the medium, are analysed along with the properties of the emitted echo signal.**

Keywords: *time-delayed quantum interference, photon, single-photon echo, entangled quantum states, nonclassical dynamics*

1. Introduction

Considerable advances in laser technology stimulated the interest in experiments in which the quantum-mechanical nature of light becomes a decisive factor. In such investigations, the experiments devoted to extremely weak light fields 'containing' a countable number of photons (one-, two-, three-, etc. photon fields) and to their interaction with atoms, molecules, and macroscopic bodies occupy an important place. Special interest towards these fields is due to the fact that their nonclassical properties may determine the basic features of the experimentally observed effects. A wide range of problems in the physics of weak light fields is covered in monographs [1 – 3] dealing with fundamental problems in quantum optics.

This work is devoted to the interference of photons in macroscopic coherent media. The formation of an interference pattern by a flux of single photons was detected experimentally for the first time in Refs [4, 5]. Such a phenomenon is referred to as quantum interference (in the case of light, it is called optical, or amplitude interference) [6]. A detailed analysis of the features of quantum interference was carried out by Feynman [7] for an electron flux. In Refs [4, 5], the flux of single photons was created by extreme attenuation of light. The methods of preparing nonclassical states of light (in particular, for weak light fields) have been considerably improved in recent years [6], which is very important, in particular, for solving problems in quantum informatics [8, 9].

Note that the interpretation of quantum interference properties is directly connected with the interpretation of the basic concepts of the quantum theory. This is reflected in the fact that a number of important physical problems in the quantum theory of measurements [10, 11] and quantum informatics [12, 13] proved to be connected to a certain extent with the study of interference properties of photons in various states. While analysing these problems, it is apparently worth noting that single-photon interference in macroscopic media may be formed under more general physical conditions than in the above-mentioned experiments [4, 5]. Namely, in the presence of phase memory in a substance, interference of light (which will be considered below in the Fresnel – Young scheme) becomes possible even when the path difference L_{21} for a photon propagating along two spatially separated trajectories in the interferometer is considerably larger than the length of a photon wave packet $\delta l_{\rm ph} = c\delta t_{\rm ph}$ ($L_{21} \gg \delta l_{\rm ph}$), where $\delta t_{\rm ph} \approx (\delta\omega_{\rm ph})^{-1}$ is the duration of the wave packet; $\delta\omega_{\rm ph} = c\delta k$ is the spectral width of the wave packet [the definition of δk will be given below after formulas (1) – (3)]; $L_{21} = L_2 - L_1$; $L_{2,1}$ is the distances propagated by a photon along each of the two possible optical paths [14 – 16]. Such an interference will be referred to as time-delayed single-photon (TDSP) interference [14].

It should be emphasised that in the formation of the TDSP interference in a substance, the direct interference of a photon with itself is ruled out completely. Note also that the inclusion of the effect of phase memory of a substance on the formation of quantum interference is a problem on the transient stage of formation of a classical response – the event occurring in the medium itself as a result of its interaction with a photon or in a detector external relative to the medium. The transient stage of the response itself includes the formation, evolution, and decay of macroscopic coherence of the medium, which is induced by the interaction with a photon or any other microparticle being measured (see, for example, Ref. [17]).

The existence of phase memory in a medium permits the implementation of various versions of the TDSP interference differing in the type of interaction of light with atoms; some of such versions were proposed in papers [14 – 16, 18, 19]. In Ref. [20], the TDSP interference is considered from the point of view of the quantum theory of continuous measurements applied to a macroscopic body. Note that dynamic mechanisms of the TDSP interference under study may be applied in the search for new methods of generating

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new nonclassical states of transient light [15]. For this purpose, a number of schemes (including those in which squeezed states of light are used) were proposed in the recent paper [19], in which the methods suggested in Refs [14, 15] are developed.

In this paper, a new version of the TDSP interference is studied (Fig. 1). It is based on the interaction of photons and laser radiation with a system of three-level atoms (three-level medium) having the Λ configuration of allowed transitions. This version of the TDSP interference may display both classical and quantum properties of light depending on the method of its observation. Quantum effects can be observed due to the fact that, in contrast to the traditional amplitude interference, the TDSP interference in a three-level coherent medium may be realised as a dynamic [15, 16] interference pattern rather than as the static pattern observed in Refs [4, 5]. To be more precise, the TDSP interference may be formed only in the polarisation of the medium, determining the nonclassical dynamics of this polarisation. The probability of spatial excitation of atoms in the medium is not modulated in this case.

Considerable attention will also be paid to the discussion of the 'nonlocal' (in time) nature of the photon interaction with macroscopic media, which is responsible for an interesting realisation of the 'Schrödinger cat' state, whose physical nature remains an object of current interest. In the given case, the existence of the Schrödinger cat state is a necessary condition for the formation of the TDSP interference which, in turn, may be manifested as a single-photon echo signal.

2. Formation of TDSP interference

We will assume that light in the form of the flux of individual photons propagates through a system of slits and mirrors and then is incident on a coherent medium containing a system of three-level atoms, the features of interaction with this medium being of interest to us. The schematic of the proposed experiment is shown in Fig. 1. We denote by $|\Psi_{\rm in}\rangle$ the state of each photon before its incidence on the mirror. Behind the semitransparent and 100 % mirrors, a new state $|\Psi_{\rm out}\rangle$ is formed in medium (*3*) representing a superposition of two macroscopically different states of the type $|\Psi_{\rm in}\rangle$:

$$|\Psi_{\rm in}\rangle = |\Psi_{\rm ph1}\rangle, \tag{1}$$

$$|\Psi_{\rm out}\rangle = \frac{1}{\sqrt{2}}\big(|\Psi_{\rm ph1}\rangle + |\Psi_{\rm ph2}\rangle\big), \tag{2}$$

$$|\Psi_{\rm ph1,2}\rangle = \int_{-\infty}^{\infty} {\rm d}^3\boldsymbol{k}_{1,2} F_{1,2}(\omega_{\boldsymbol{k}_{1,2}}; \boldsymbol{k}_{1,2} - \boldsymbol{K}_{1,2}; t=-\infty) \times[\exp{\rm i}(\omega_{\boldsymbol{k}_{1,2}} t_{1,2})] a^+_{\boldsymbol{k}_{1,2}} |0\rangle |g\rangle. \tag{3}$$

Both states $|\Psi_{\rm ph1,2}\rangle$ are written in the interaction representation using the output modes of the field (behind all the mirrors) (see, for example, Refs [1–3; 6]); $|g\rangle = \prod_{j=1}^{N} |1_j\rangle$ is the ground state of medium (*3*); $|1_j\rangle$ is the lower state of the *j*th atom of medium (*3*); $|0\rangle$ is the vacuum state of light; $a^+_{\boldsymbol{k}}(a_{\boldsymbol{k}})$ are the creation (annihilation) operators for a photon of mode $\boldsymbol{k}$; $F_{1,2}(\omega_{\boldsymbol{k}_{1,2}}; \boldsymbol{k}_{1,2} - \boldsymbol{K}_{1,2}; t=-\infty)$ are quadratically normalised wave functions of a photon in the $\boldsymbol{k}$ space, which have maximum values for $\boldsymbol{k}_1 \approx \boldsymbol{K}_1$ or $\boldsymbol{k}_2 \approx \boldsymbol{K}_2$ and for frequency $\omega_{\boldsymbol{k}_{1,2}} = \omega_{\rm ph}$ and correspond to the wave packets propagating along the direction $\boldsymbol{K}_1$ or $\boldsymbol{K}_2$ (the deviations in the lengths of wave vectors $\boldsymbol{k}_{1,2}$ from $\boldsymbol{K}_{1,2}$ for

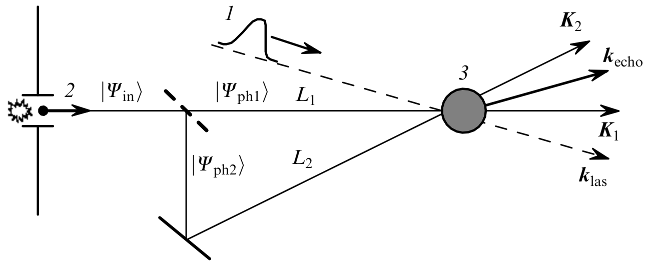

Figure 1. Schematic of formation of TDSP interference and single-photon echo for $L_2 - L_1 \gg c\delta t_{\rm ph}$, $\omega_{\rm ph} \simeq \omega_{31}$: (*1*) laser pulse ($k_{\rm las}, \omega_{\rm las} \simeq \omega_{32}$); (*2*) photons ($\delta t_{\rm ph}, \omega_{\rm ph}$); (*3*) coherent medium.

which the functions $F_{1,2}(\omega_{\boldsymbol{k}_{1,2}}; \boldsymbol{k}_{1,2} - \boldsymbol{K}_{1,2}; t=-\infty)$ are close to maximum are within the range δk; $t_{1,2}$ is the time of emergence of each of the two photon wave packets in medium (*3*); and $\tau = t_2 - t_1 = L_{21}/c$ is the relative delay; henceforth, we assume that $t_1 = 0$ and $\tau = t_2$.

Note that the presence of a macroscopic distance between the regions of localisation of the two spatially separated photon states $|\Psi_{\rm ph1}\rangle$ and $|\Psi_{\rm ph2}\rangle$ considerably extends experimental potentialities of studying nonclassical properties caused by the spatial delocalisation of photon states. Various experiments of this kind are being made and analysed at present (see , for example, Refs [10, 11, 13]; a detailed analysis of some of such experiments based on quantum kinematics is given in recent paper [21]). A distinguishing feature of the problem considered below is that it is oriented on the properties of nonclassical dynamics accompanying the formation of the TDSP interference.

Taking into account the initial state (1)–(3), consider the interaction of a photon with the system of three-level atoms of medium (*3*) (Fig. 1), when the photon frequency $\omega_{\rm ph}$ is in resonance with the atomic transition $|1\rangle \leftrightarrow |3\rangle$ ($\omega_{\rm ph} = cK_1 = \omega_{31}$). Let a short laser pulse whose frequency $\omega_{\rm las}$ coincides with the frequency of the second allowed atomic transition $|2\rangle \leftrightarrow |3\rangle$ ($\omega_{\rm las} = \omega_{32}$) be also incident on medium (*3*). We assume that the time $t_{\rm las}$ of action of the laser pulse satisfies the condition $t_{\rm las} > \delta t_{\rm ph}$; i.e., the laser pulse enters the medium after the first wave packet (the time of arrival of the first wave packet $t_1 = 0$).

The second wave packet of the photon enters the medium after the action of the laser pulse at instant $t_2 = \tau > t_{\rm las}$. Therefore, the interaction of the photon with the macroscopic medium, which is 'split' into two trajectories with a time interval τ between them, is additionally perturbed by the action of the intense short laser pulse on the medium. Note for comparison that in the schemes of single-photon interference studied earlier [14–16, 18, 19], it was assumed that a laser pulse acts on medium (*3*) only after the arrival of both photon wave packets in it, when the single-photon interference pattern has already been formed in the substance.

In a real experiment proposed by us, the scheme of interaction of a photon and a laser pulse with medium (*3*) depicted in Fig. 1 should be multiply repeated and the results of measurements should be accumulated. We consider as measurable quantities the amplitude $E_{\omega_{32}}(t, \boldsymbol{r})$ and intensity $I_{\omega_{32}}(t, \boldsymbol{r})$ of the field reemitted by the medium at the frequency $\omega \approx \omega_{32}$. Other details of such experiments are also considered in Refs [14–16].

We will solve the Schrödinger equation using the Hamiltonian

$$H = \int_{-\infty}^{\infty} \mathrm{d}^3\boldsymbol{k}\hbar\omega_{\boldsymbol{k}} a^+_{\boldsymbol{k}} a_{\boldsymbol{k}} + \sum_{j=1}^{N}\sum_{l=1}^{3} E_l^{(j)} P_{ll}^{(j)} - \sum_{j=1}^{N}\Big[dE(t;\boldsymbol{r},\boldsymbol{k}_{\text{las}})P_{32}^{(j)} + \text{h. c.}\Big] + \hbar\sum_{j=1,}^{N}\sum_{\substack{l,\xi=1\\(l>\xi)}}^{3}\int_{-\infty}^{\infty}\mathrm{d}^3\boldsymbol{k}\Big[g_{l\xi}(\boldsymbol{k})\mathrm{e}^{\mathrm{i}\boldsymbol{k}\boldsymbol{r}_j} a_{\boldsymbol{k}} P_{l\xi}^{(j)} + g_{l\xi}^*(\boldsymbol{k})\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\boldsymbol{r}_j} a^+_{\boldsymbol{k}} P_{\xi l}^{(j)}\Big]. \quad (4)$$

Here, $P_{mn}^{(j)} = |m_j\rangle\langle n_j|$ are the operators of the transition of the jth atom from the state $|n_j\rangle$ to the state $|m_j\rangle\,(m,n = 1-3)$; $d = d_{32} = d_{23}$ is the dipole moment of the atomic transition; N is the number of atoms. The first two terms in (4) are the energy operator for the field and the system of three-level atoms, the third term is the operator of the energy of interaction of atoms with the laser field, and the fourth term is the operator of the energy of interaction of atoms with photons.

We will describe the action of a laser pulse on atoms [the third term in Eqn (4)] using the classical expression for the electric field $\vec{\mathscr{E}}(t;\boldsymbol{r},\boldsymbol{k}_{\text{las}})$ of the pulse with preset parameters:

$$\vec{\mathscr{E}}(t;\boldsymbol{r},\boldsymbol{k}_{\text{las}}) = \boldsymbol{E}(t;\boldsymbol{r},\boldsymbol{k}_{\text{las}}) + \boldsymbol{E}^*(t;\boldsymbol{r},\boldsymbol{k}_{\text{las}}), \quad (5)$$

$$\boldsymbol{E}(t;\boldsymbol{r},\boldsymbol{k}_{\text{las}}) = \frac{1}{2}\boldsymbol{e}_{\text{las}}E_0(t-t_{\text{las}})\exp\left[-\mathrm{i}(\omega_{32}t - \boldsymbol{k}_{\text{las}}\boldsymbol{r} - \varphi_{\text{las}})\right]. \quad (6)$$

The interaction of radiation with atoms in Eqn (4) is written in the rotating wave approximation. We assume that the frequencies of transitions between states $|m_j\rangle$ and $|n_j\rangle$ of different atoms are described by the expression $\omega_{mn}^{(j)} = \omega_{mn}(1+\xi_j)$, where ξ_j is the parameter determining the correlated frequency shift $\delta\omega_{mn}^{(j)} = \xi_j\omega_{mn}$ between different pairs of energy levels of the jth atom. We will describe the set of frequencies in the system of atoms by introducing conventionally the distribution function $f(\xi)$ of the Gaussian type with a maximum at $\xi = 0$. For the Λ diagram of allowed transitions, the interaction constants satisfy the condition $g_{21}(\boldsymbol{k}) \ll g_{32}(\boldsymbol{k}),\ g_{31}(\boldsymbol{k})$. The remaining notation in Hamiltonian (4) is standard (see, for example, Refs [1–3, 22–24]).

We will seek an approximate solution for the wave function considering individual stages of the problem.

2.1 First stage (time interval $0 < t < t_{\text{las}}$)

For $t < t_{\text{las}}$, the second wave packet does not manage to reach the medium, so that its interaction with the photon is limited only to the first wave packet and determines the evolution of the first term $|\Psi_{\text{ph1}}\rangle$ in superposition (2). Thus, the solution for the wave function assumes the form

$$|\Psi(t)\rangle\Big|_{0<t<\tau} = |\Psi_{\text{tr}}(t)\rangle = \frac{1}{\sqrt{2}}\big(|\Psi_1(t)\rangle + |\Psi_{\text{ph2}}(-\infty)\rangle\big), \quad (7)$$

where, in accordance with the initial conditions determined by relations (1)–(3) and the form of the interaction Hamiltonian, the term $|\Psi_1(t)\rangle$ is described by the expression

$$|\Psi_1(t)\rangle = |\Psi_{\text{ph1}}(t)\rangle + \big|\Psi_{al}^{(3)}(t)\big\rangle = \int_{-\infty}^{\infty}\mathrm{d}^3\boldsymbol{k}F_1(\omega_{\boldsymbol{k}};\boldsymbol{k}-\boldsymbol{K}_1;t) \times a^+_{\boldsymbol{k}}|0\rangle|g\rangle + \sum_{j=1}^{N}\beta_1^{(j)}(t)\mathrm{e}^{\mathrm{i}\omega_{31}^{(j)}t_1}P_{31}^{(j)}|0\rangle|g\rangle, \quad (8)$$

where

$$\beta_1^{(j)}(t) = -\mathrm{i}\int_{-\infty}^{t}\mathrm{d}t\int_{-\infty}^{\infty}\mathrm{d}^3\boldsymbol{k}g_{31}(\boldsymbol{k}) \times \exp\left[\mathrm{i}\boldsymbol{k}\boldsymbol{r}_j - \mathrm{i}\big(\omega_{\boldsymbol{k}} - \omega_{31}^{(j)}\big)(t-t_1)\right]F_1(\omega_{\boldsymbol{k}};\boldsymbol{k}-\boldsymbol{K}_1;t). \quad (9)$$

In relations (7)–(9), the interaction representation is used; the first subscript in the wave function $|\Psi_{\text{a}1}^{(3)}\rangle$ indicates excitation of the atomic subsystem, the second subscript indicates the number of the wave packet of the photon, and the superscript shows the number of the atomic level.

While calculating function $|\Psi_1(t)\rangle$, we confine our analysis to the special case of an optically thick medium, which is the case when the size L of the medium satisfies the condition $\alpha L \gg 1$, where α^{-1} is the Beer depth of the resonant absorption of light [24]. For this purpose, we assume that the spectral width of the resonance transition $|1\rangle \to |3\rangle$ considerably exceeds the spectral width of the photon wave function: $\delta\omega_{31} \gg \delta\omega_{\text{ph}}$. Under such conditions of light propagation in a substance, the wave packet of a photon is absorbed almost completely by a system of resonance atoms during the time $\delta t \approx \delta t_{\text{ph}} + \alpha^{-1}/c$, so that the wave function in expression (8) acquires a simpler form:

$$|\Psi_1(t)\rangle\Big|_{t>\delta t} \approx \big|\Psi_{\text{a}1}^{(3)}(\delta t)\big\rangle.$$

The problems on the propagation of a light pulse in a resonance medium have been thoroughly investigated [24]. For example, a well-known analytic solutions was obtained for the problem of light propagation in the form of a single-photon wave packet [25]. This type of solutions is used, in particular, in problems on the propagation of γ-quanta through a system of Mössbauer resonant nuclei (see, for example, Ref. [26] and references therein). For the chosen conditions of resonant interaction, the envelopes of light pulses and, hence, of the photon wave packet do not change during propagation to the bulk of the medium, but only experience resonance decay with a constant α if the photon frequency is tuned to the centre of the atomic line. Under these conditions, the components of the wave function $|\Psi_{\text{a}1}^{(3)}|(\delta t)\rangle$ corresponding to the excitation of atoms in the bulk of the medium ($\sim \beta_1^{(j)}(t)P_{31}^{(j)}$) can be determined using expression (9) and taking into account the decay of the field amplitude inside the medium in accordance with the law $\exp[-\alpha(\boldsymbol{n}_1\boldsymbol{r}_j)/2]$ (where $\boldsymbol{n}_1 = \boldsymbol{K}_1/K_1$), as well as the emergence of an additional phase shift $\exp(\mathrm{i}\boldsymbol{K}_1\boldsymbol{r}_j)$ caused by the delay.

Thus, restricting ourselves to the plane-wave approximation and disregarding spontaneous decay into the side modes of the field during the time δt of interaction between the wave packet and the medium, we obtain

$$|\Psi_1(t>\delta t)\rangle \simeq \big|\Psi_{\text{a}1}^{(3)}(\delta t)\big\rangle = \sum_{j=1}^{N}\alpha_1\big(\omega_{31}^{(j)} - \omega_{31}\big)\chi_1(\boldsymbol{r}_j) \times \exp\Big[-\frac{\alpha(\boldsymbol{n}_1\boldsymbol{r}_j)}{2} + \mathrm{i}\big(\omega_{31}^{(j)}t_1 + \boldsymbol{K}_1\boldsymbol{r}_j\big)\Big]P_{31}^{(j)}|0\rangle|g\rangle, \quad (10)$$

where $\chi_1(\boldsymbol{r}_j)$ is the function determined by the field distribution in a plane perpendicular to direction $\boldsymbol{n}_1$, and

$$\alpha_1\big(\omega_{31}^{(j)} - \omega_{31}\big)\chi_1(\boldsymbol{r}_j) = -\mathrm{i}C_{\text{n}}\int_{-\infty}^{\infty}\mathrm{d}t\int_{-\infty}^{\infty}\mathrm{d}^3\boldsymbol{k}g_{32}(\boldsymbol{k}) \times F_1(\omega_{\boldsymbol{k}};\boldsymbol{k}-\boldsymbol{K}_1;t=-\infty)\exp[\mathrm{i}(\omega_{31}^{(j)} - \omega_{\boldsymbol{k}})t + \mathrm{i}(\boldsymbol{k}-\boldsymbol{K}_1)\boldsymbol{r}_j] \quad (11)$$

(we assume that $t_1 = 0$). The constant C_n can be determined from the normalisation condition for the state $|\Psi_{a1}^{(3)}(t>\delta t)\rangle$, which corresponds to complete transition of excitation from the wave packet to the atomic subsystem:

$$\langle\Psi_{a1}^{(3)}|\Psi_{a1}^{(3)}\rangle=\sum_{j=1}^{N}\left|\alpha_1(\omega_{31}^{(j)}-\omega_{31})\right|^2\chi_1^2(\boldsymbol{r}_j)\exp[-\alpha(\boldsymbol{n}_1\boldsymbol{r}_j)]=1.\quad(12)$$

Thus, in the case of prevailing inhomogeneous line broadening, the dipole moments of different atoms in the state $|\Psi_{a1}^{(3)}(t>\delta t)\rangle$ become dephased for $t>\delta t$. As a result, the macroscopic polarisation of the medium decreases strongly, weakening the coupling between the excited states $|\Psi_{a1}^{(3)}(t)\rangle$ of atoms and decreasing the field in the state $|\Psi_{ph1}(t)\rangle$. Before the second wave packet enters the medium, the wave function associated with atomic excitation varies only due to spontaneous noncollective (single-atom) processes. In subsequent analysis, we will take into account the role of spontaneous noncollective processes by introducing the exponential coefficients $\exp[-t(\gamma_{32}+\gamma_{31})/2$ and $\exp(-t\gamma_{21}/2)$ (where γ_{mn} are the constants of a spontaneous transition of an atom from level m to level n) of the wave functions for the excited atomic states [see formulas (16) and (17) below]. However, dephasing of the atomic dipoles in the medium immediately after the arrival of a photon in the medium is not necessarily irreversible, and this fact will be especially important for the subsequent analysis.

The following circumstance is also worth noting. The state $|\Psi_{tr}(t)\rangle$ formed for $t>\delta t$ is a quantum superposition of two macroscopically separated states, which are also different from the physical point of view. The first state $|\Psi_{a1}^{(3)}(t)\rangle$ is a superposition of a macroscopic number of excited atomic states. The second state $|\Psi_{ph2}(t)\rangle$ describes the second wave packet of the photon, which is at a macroscopic distance ($\sim L_{21}$) from the medium. Thus, the state $|\Psi_{tr}(t)\rangle$ is an example of entangled states of the two subsystems (of the photon and macroscopic medium) which are also known as hybrid states [27] because the states of the microsystem (photon) and macrosystem [medium (*3*)] are entangled in them. These states are sometimes referred to as the Schrödinger cat states (see Ref. [28]). Obviously, the coherence of atoms in the state $|\Psi_{tr}(t)\rangle$ should ultimately decay under the effect of local field fluctuations in the macroscopic medium.

Unless the second wave packet reaches the medium ($t\leqslant\tau\approx L_{21}/c$, see Fig. 1) or irreversible processes on the medium destroy the atomic coherence of the $|\Psi_{a1}^{(3)}(t)\rangle$state which has already been excited, the complete system formed by atoms and a photon can be described by the quantum superposition $|\Psi_{tr}(t)\rangle$. The lifetime t_{tr} of the intermediate state $|\Psi_{tr}(t)\rangle$ is bounded from above by the phase memory time T_2 of the medium ($t_{tr}\leqslant T_2$). Because the time T_2 in some media may be as long as a few microseconds, the $|\Psi_{tr}(t)\rangle$ state may be an interesting object for studying long-term properties of the behaviour and decay of hybrid states. In this connection, the possibility of restoration (rephasing) of the macroscopic coherence induced by a single photon and rapidly decaying immediately after the photon enters the medium may become of special importance.

2.2 Second stage ($t<\tau$)

Let us find the wave function after the action of a laser pulse incident on the medium at instant $t=t_{las}\gg\delta t_{ph}$. We assume that the duration of the pulse is small enough for describing its action by the unitary operator $U(\vartheta,\boldsymbol{k}_{las})$:

$$U(\vartheta,\boldsymbol{k}_{las})=\prod_{j=1}^{N}U_j,$$

$$U_j=P_{11}^{(j)}+\cos(\vartheta/2)\left(P_{22}^{(j)}+P_{33}^{(j)}\right)+\mathrm{i}\sin(\vartheta/2)\quad(13)$$

$$\times\left[P_{32}^{(j)}\exp(\mathrm{i}\zeta_j)+P_{23}^{(j)}\exp(-\mathrm{i}\zeta_j)\right],$$

where the effect of the difference in the transition frequencies of atoms during the action of the pulse is disregarded, the interaction representation is used, and the following parameters of the pulse are introduced: $\boldsymbol{k}_{las}$ is the wave vector; $\omega_{las}=\omega_{32}$ is the frequency; $\vartheta=\hbar^{-1}dE_0\delta t_{las}$ is the area; δt_{las} is the duration ($\delta t_{las}\gamma_{mn}\ll1$); $\zeta_j=\boldsymbol{k}_{las}\boldsymbol{r}_j+\omega_{32}^{(j)}t_{las}+\varphi_{las}$ is the phase in which the phase of the field and of the jth atom during the action of the laser pulse are taken into account (see also Ref. [16]).

As a result of evolution, the wave function takes the form

$$|\Psi(t_{las}+\delta t_{las})\rangle=U(\vartheta,\boldsymbol{k}_{las})|\Psi_{tr}(t_{las})\rangle$$

$$=\frac{1}{\sqrt{2}}\left[U(\vartheta,\boldsymbol{k}_{las},t_{las})|\Psi_{a1}^{(3)}(t_{las})\rangle+|\Psi_{ph2}(t_{las}+\delta t_{las})\rangle\right]$$

$$=\frac{1}{\sqrt{2}}\left[\cos(\vartheta/2)\left|\Psi_{a1}^{(3)}(t_{las})\right\rangle+\mathrm{i}\sin(\vartheta/2)\left|\varphi_{a1}^{(2)}(t_{las})\right\rangle\right.$$

$$\left.+|\Psi_{ph2}(t_{las}+\delta t_{las})\rangle\right],\quad(14)$$

where

$$\left|\varphi_{a1}^{(2)}(t_{las})\right\rangle=\sum_{j=1}^{N}\alpha_1\left(\omega_{31}^{(j)}-\omega_{31}\right)\chi_1(\boldsymbol{r}_j)\exp\left[-(\boldsymbol{n}_1\boldsymbol{r}_j)/2\right.$$

$$\left.+\mathrm{i}(\boldsymbol{K}_1-\boldsymbol{k}_{las})\boldsymbol{r}_j-\mathrm{i}\left(\omega_{32}^{(j)}t_{las}+\varphi_{las}\right)\right]P_{21}^{(j)}|0\rangle|g\rangle\quad(15)$$

is the state of the atoms excited to the second level, which is therefore connected with the first trajectory of the photon.

2.3 Third stage ($t>\tau$)

Let us find the wave function after the interaction of the medium with the second wave packet of the photon (at instant $t=t_2$; see Fig. 2). In this case, the interaction with the medium affects the evolution of the second term $|\Psi_{ph2}\rangle$ in expression (7). In analogy with the derivation of formulas (8) and (9), we obtain the following solution for this part of the wave function (7):

$$|\Psi_{ph2}(t)\rangle\Big|_{t>t_2+\delta t_{ph}}\to\left|\Psi_{a2}^{(3)}(t)\right\rangle+|\Psi_{ph2}(t)\rangle\Big|_{\alpha L\gg1}$$

$$\simeq\left|\Psi_{a2}^{(3)}(t)\right\rangle,\quad(16)$$

where we again use the condition that the medium is optically thick for the photon. The wave function $|\Psi_{a2}^{(3)}(t)\rangle$ can be described by an expression of type (10). The evolution of $|\Psi_{a2}^{(3)}(t)\rangle$ after the cessation of the coherent interaction between the photon and the medium is again determined only by the effect of noncollective spontaneous processes. Taking into account spontaneous processes, we can write the function $|\Psi_{a2}^{(3)}(t)\rangle$ in the form

$$\left|\Psi_{a2}^{(3)}(t)\right\rangle\simeq\exp[-(\gamma_{31}+\gamma_{32})(t-\tau)/2]\left|\Psi_{a2}^{(3)}(\tau+\delta t)\right\rangle.\quad(17)$$

Taking this expression into account, we can write the following expression for the wave function describing the excitation of the atomic subsystem immediately after the arrival of the second wave packet in the medium:

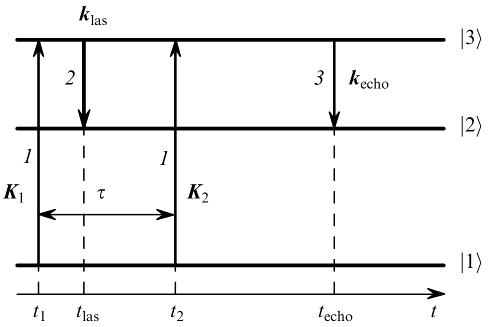

Figure 2. Scheme illustrating the emergence of single-photon echo: (*1*) photons; (*2*) laser pulse; (*3*) echo signal, $t_{\text{echo}} = t_2 + (\tau - t_{\text{las}} + t_1)\omega_{31}/\omega_{32}$, $\boldsymbol{k}_{\text{echo}} = \boldsymbol{k}_{\text{las}} + \boldsymbol{K}_2 - \boldsymbol{K}_1$.

$$|\Psi_{\text{a}}(t)\rangle \approx \exp[-(\gamma_{31} + \gamma_{32})(t - \tau)/2]\big|\Psi^{(3)}_{\text{a}2}(\tau)\big\rangle + \cos(\vartheta/2)$$
$$\times \exp[-(\gamma_{31} + \gamma_{32})t/2]\big|\Psi^{(3)}_{\text{a}1}(\delta t)\big\rangle + \text{i}\sin(\vartheta/2)$$
$$\times \exp[-(\gamma_{31} + \gamma_{32})\tau/2]\big|\varphi^{(2)}_{\text{a}1}(\tau)\big\rangle. \qquad (18)$$

This solution is valid for $t > \tau$ and will be used for determining the parameters of the field emitted by the medium.

3. Single-photon echo as a manifestation of TDSP interference

Let us analyse the behaviour of averaged atomic operators $\langle P^{\pm}_{\boldsymbol{k}}(t)\rangle$ and $\langle P^{-}_{\boldsymbol{k}}(t)P^{+}_{\boldsymbol{k}}(t)\rangle$ since the amplitude $\langle E^{\pm}_{\boldsymbol{k}}(\boldsymbol{r}, t)\rangle$ and intensity $\langle I_{\boldsymbol{k}}(r, t)\rangle = \langle E^{+}_{\boldsymbol{k}}(\boldsymbol{r}, t)E^{-}_{\boldsymbol{k}}(\boldsymbol{r}, t)\rangle$ of the field emitted in the direction $\boldsymbol{k}$ can be determined in terms of these atomic operators [22–24] averaged over the atomic state (18). For the emitted field operator $E^{\pm}_{\boldsymbol{k}}(\boldsymbol{r}, t + r/c)$ at point $\boldsymbol{r}$, we have

$$E^{\pm}_{\boldsymbol{k}}(\boldsymbol{r}, t + r/c) \sim \frac{\omega^2_{32}}{r}\frac{[\boldsymbol{k}[\boldsymbol{k}\boldsymbol{d}_{32}]]}{c^2k^2}P^{\pm}_{\boldsymbol{k}}(t), \qquad (19)$$

where

$$P^{-}_{\boldsymbol{k}}(t) = \sum_{j=1}^{N} P^{(j)}_{23}\exp\big[-\text{i}\big(\boldsymbol{k}\boldsymbol{r}_j + \omega^{(j)}_{32}t\big)\big];$$
$$P^{+}_{\boldsymbol{k}}(t) = \big(P^{-}_{\boldsymbol{k}}(t)\big)^{+}. \qquad (20)$$

In subsequent analysis, while using (19) and (20), we will restrict ourselves to the frequency range close to the atomic frequency ω_{32}, because the detection of the TDSP interference is possible just in this range.

4. Field amplitude $\langle E_{\boldsymbol{k}}(\boldsymbol{t}, \boldsymbol{r})\rangle$

Using relations (16)–(18), we can calculate the electric component of the field $\langle E_{\boldsymbol{k}}(\boldsymbol{t}, r)\rangle$ emitted in the direction $\boldsymbol{k}$:

$$\langle E_{\boldsymbol{k}}(t, \boldsymbol{r})\rangle = \langle E^{+}_{\boldsymbol{k}}(t, \boldsymbol{r})\rangle + \langle E^{-}_{\boldsymbol{k}}(t, \boldsymbol{r})\rangle, \qquad (21)$$

where

$$\langle E^{-}_{\boldsymbol{k}}(t, \boldsymbol{r})\rangle = \langle E^{+}_{\boldsymbol{k}}(t, \boldsymbol{r})\rangle^{*};\ \langle E^{+}_{\boldsymbol{k}}(t, \boldsymbol{r})\rangle \propto \langle P^{-}_{\boldsymbol{k}}(t, \boldsymbol{r})\rangle$$
$$= \langle\Psi_{\text{a}}(t)|P^{-}_{\boldsymbol{k}}|\Psi_{\text{a}}(t)\rangle = -\text{i}\frac{d_{23}}{2}\sin(\vartheta/2)$$
$$\times \exp[-(\gamma_{31} + \gamma_{32})(t + t_{\text{las}} - \tau)/2]A(t - t_{\text{echo}}; \boldsymbol{k}, \boldsymbol{k}_{\text{echo}})$$
$$\times \exp[-\text{i}\omega_{32}(t - t_{\text{echo}}) + \text{i}\boldsymbol{k}_{\text{echo}}\boldsymbol{r}]; \qquad (22)$$

$$t_{\text{echo}} = \tau + (\tau - t_{\text{las}})\omega_{31}/\omega_{32};\ \boldsymbol{k}_{\text{echo}} = \boldsymbol{k}_{\text{las}} + \boldsymbol{K}_2 - \boldsymbol{K}_1;$$

$$A(t - t_{\text{echo}}; \boldsymbol{k}, \boldsymbol{k}_{\text{echo}})\exp[-\text{i}\omega_{32}(t - t_{\text{echo}}) + \text{i}\boldsymbol{k}_{\text{echo}}\boldsymbol{r}]$$
$$= \big\langle\varphi^{(2)}_{\text{a}1}(t)\big|\sum_{j=1}^{N}P^{(j)}_{23}\exp\big[-\text{i}(\omega^{(j)}_{32}t + \boldsymbol{k}\boldsymbol{r}_j)\big]\big|\Psi^{(3)}_{\text{a}2}(t)\big\rangle. \qquad (23)$$

The function $A(t - t_{\text{echo}}; \boldsymbol{k}, \boldsymbol{k}_{\text{echo}})$ can be derived by going over to the continuous distribution of atoms in the medium of volume V and carrying out independent integration over space and the spectral distribution of atoms:

$$A(t - t_{\text{echo}}; \boldsymbol{k}, \boldsymbol{k}_{\text{echo}}) = \text{e}^{-\text{i}\varphi_{\text{las}}}\sum_{j=1}^{N}\alpha^{*}_{1}(\omega^{(j)}_{31} - \omega_{31})\alpha_2(\omega^{(j)}_{31} - \omega_{31})$$
$$\times \chi^{*}_{1}(\boldsymbol{r}_j)\chi_2(\boldsymbol{r}_j)\exp[-\alpha(\boldsymbol{n}_1\boldsymbol{r}_j + \boldsymbol{n}_2\boldsymbol{r}_j)/2 + \text{i}\xi_j\omega_{32}(t - t_{\text{echo}})$$
$$-\text{i}(\boldsymbol{k} - \boldsymbol{k}_{\text{echo}})\boldsymbol{r}_j] = \gamma_{\boldsymbol{k}-\boldsymbol{k}_{\text{echo}}}T_{12}(t - t_{\text{echo}})\text{e}^{-\text{i}\varphi_{\text{las}}}, \qquad (24)$$

where the functions

$$\gamma_{\boldsymbol{k}-\boldsymbol{k}_{\text{echo}}} = \frac{N}{V}\int_V \text{d}\boldsymbol{r}\chi^{*}_{1}(\boldsymbol{r})\chi_2(\boldsymbol{r})\exp[-\alpha(\boldsymbol{n}_1\boldsymbol{r} + \boldsymbol{n}_2\boldsymbol{r})/2$$
$$-\text{i}(\boldsymbol{k} - \boldsymbol{k}_{\text{echo}})\boldsymbol{r}] \propto \delta_{\boldsymbol{k}-\boldsymbol{k}_{\text{echo}}}, \qquad (25)$$

$$T_{12}(t - t_{\text{echo}}) = \int_{-\infty}^{\infty}\text{d}\xi\, f(\xi)\alpha^{*}_{1}(\xi\omega_{31})\alpha_2(\xi\omega_{31})$$
$$\times \exp[\text{i}\xi\omega_{32}(t - t_{\text{echo}})]. \qquad (26)$$

are introduced.

The functions $\chi_{1,2}(\boldsymbol{r})$ and $\exp[-\alpha(\boldsymbol{n}_1\boldsymbol{r} + \boldsymbol{n}_2\boldsymbol{r})/2]$ vary slowly over distances of the order of the radiation wavelength λ; therefore, the function $\gamma_{\boldsymbol{k}-\boldsymbol{k}_{\text{echo}}}$ has a sharp peak near $\boldsymbol{k} \approx \boldsymbol{k}_{\text{echo}}$. The function $T_{12}(t - t_{\text{echo}})$ also has a peak at $t = t_{\text{echo}}$, which corresponds to the emergence of single-photon echo signal (see Fig. 2). Note that the echo signal [see Eqns (24) and (26)] is proportional to the product $\alpha^{*}_{1}(\xi\omega_{31})\alpha_2(\xi\omega_{31})$, where the subscripts 1 and 2 at α correspond to the two trajectories of the photon, for which the time of arrival at the medium differs by τ. Thus, the echo signal is a consequence of the existence of quantum superposition of single-photon states (2).

We will use the normalisation (12) and the approximation $\exp[-\alpha(\boldsymbol{n}_1\boldsymbol{r} + \boldsymbol{n}_2\boldsymbol{r})/2] \approx \exp(-\alpha\boldsymbol{n}_1\boldsymbol{r}) \approx \exp(-\alpha\boldsymbol{n}_2\boldsymbol{r})$, which is satisfactory when the direction $\boldsymbol{K}_1$ does not differ strongly from the direction $\boldsymbol{K}_2$, so that the two wave packets overlap in the medium almost completely. In this approximation, for $\boldsymbol{k} = \boldsymbol{k}_{\text{echo}}$ and $t = t_{\text{echo}}$, we find that $A(0; \boldsymbol{k}_{\text{echo}}, t_{\text{echo}}) \approx 1$. Thus, we can write the following expression for the field amplitude of the echo signal:

$$\langle E^{+}_{\boldsymbol{k}_{\text{echo}}}(t, \boldsymbol{r})\rangle \propto \langle P^{-}_{\boldsymbol{k}_{\text{echo}}}(t)\rangle = -\text{i}\frac{d_{23}}{2}\sin(\vartheta/2)T_{12}(t - t_{\text{echo}})$$
$$\times \exp[-(\gamma_{31} + \gamma_{32})(t + t_{\text{las}} - \tau)/2]$$
$$\times \exp\{-\text{i}[\omega_{32}(t - t_{\text{echo}}) - \boldsymbol{k}_{\text{echo}}\boldsymbol{r} + \varphi_{\text{las}}]\}. \qquad (27)$$

Expression (26) shows that if the inhomogeneous line broadening prevails, the time evolution of echo (27) is determined by the single-photon wave function (i.e., by the spectrum of the function $\alpha^{*}_{1}(\xi\omega_{31})\alpha_2(\xi\omega_{31})$). For a reliable detection of the echo signal in the case of accumulation of vast body of experimental data, it is necessary that all the three instants t_1, t_{las} and t_2 (see Fig. 2) be coupled, because the phase and the instant $t_{echo} = \tau + (\tau - t_{las})\omega_{31}/\omega_{32}$ of the emergence of the signal are shifted upon a change in the time t_{las} of laser pulse action. Such a coupling to laser pulses in the case of generation of single-photon wave packets was realised, for example, in the recent experiments on teleportation of single-photon states [8].

5. Radiation intensity $\langle I_{\omega_{32}\boldsymbol{k}}(t,\boldsymbol{r})\rangle$

The radiation intensity can be expressed in terms of atomic operators [22]:

$$\langle I_{\boldsymbol{k}}\rangle \sim \frac{\omega_{32}^4}{r^2}\left|\frac{[\boldsymbol{k}[\boldsymbol{k}\boldsymbol{d}_{32}]]}{c^2k^2d_{32}}\right|^2\langle P_{\boldsymbol{k}}^+P_{\boldsymbol{k}}^-\rangle. \tag{28}$$

To study nonclassical features of radiation, it would be interesting to compare the behaviour of the field amplitude (27) with the behaviour of the intensity $\langle I_{\boldsymbol{k}_{echo}}(t)\rangle$ of the field emitted also in direction $\boldsymbol{k}_{echo}$ after the interaction of the medium with the second wave packet of the photon. In this case, we can find $\langle P_{\boldsymbol{k}}^+P_{\boldsymbol{k}}^-\rangle$ by using again the wave function (18):

$$\langle P_{\boldsymbol{k}}^+P_{\boldsymbol{k}}^-\rangle = \langle\Psi_a(t)|P_{\boldsymbol{k}}^+(t)P_{\boldsymbol{k}}^-(t)|\Psi_a(t)\rangle = \frac{d_{32}^2}{2}\{\cos^2(\vartheta/2)$$
$$\times\exp[-(\gamma_{31}+\gamma_{32})t]\rho_{11}(t;\delta t,\delta t) + \exp[-(\gamma_{31}+\gamma_{32})$$
$$\times(t-\tau)]\rho_{22}(t;\tau,\tau) + \cos(\vartheta/2)\exp[-(\gamma_{31}+\gamma_{32})$$
$$\times(t-\tau/2)][\rho_{12}(t;\delta t,\tau) + \rho_{21}(t;\tau,\delta t)]\}, \tag{29}$$

where

$$\rho_{nm}(t;t',t'') = \langle\Psi_{an}^{(3)}(t')|P_{\boldsymbol{k}}^+(t)P_{\boldsymbol{k}}^-(t)|\Psi_{am}^{(3)}(t'')\rangle. \tag{30}$$

Using expressions (8)–(12) for functions $|\Psi_{a1}^{(3)}(0)\rangle$ and $|\Psi_{a2}^{(3)}(\tau)\rangle$ and taking into account expressions (25) and (26), we obtain for the functions $\rho_{nm}(t;t',t'')$ the following relations:

$$\rho_{11}(t;\delta t,\delta t) \simeq T_{11}(0) \approx 1, \tag{31}$$

$$\rho_{22}(t;\tau,\tau) \simeq T_{22}(0) \approx 1, \tag{32}$$

$$\rho_{12}(t;0,\tau) = \exp(i\omega_{31}\tau)\gamma_{\boldsymbol{k}_1-\boldsymbol{k}_2}T_{12}(\tau)\Big|_{t>\delta t_{ph},\tau} \simeq 0, \tag{33}$$

$$\rho_{21}(t;\tau,0) = \exp(-i\omega_{31}\tau)\gamma_{\boldsymbol{k}_2-\boldsymbol{k}_1}T_{21}(\tau)\Big|_{t>\delta t_{ph},\tau} \simeq 0. \tag{34}$$

Here, functions T_{mn} are formed in the same way as functions T_{12} (26).

Relations (29) and (31)–(34) show that the intensity of the emitted field is isotropic and decays with the characteristic time $(\gamma_{32}+\gamma_{31})^{-1}$ without the emergence of a spike corresponding to echo signal. Thus, the relation between the dynamics of the field intensity $\langle I_{\omega_{32}\boldsymbol{k}_{echo}}(t,\boldsymbol{r})\rangle$ and the field amplitude do not satisfy classical concepts; i.e.,

$$\langle I_{\omega_{32}\boldsymbol{k}_{echo}}(t,\boldsymbol{r})\rangle \neq \langle E^-_{\omega_{32}\boldsymbol{k}_{echo}}(t,\boldsymbol{r})\rangle\langle E^+_{\omega_{32}\boldsymbol{k}_{echo}}(t,\boldsymbol{r})\rangle. \tag{35}$$

The reason behind the disagreement in the behaviour of the field amplitude and intensity is associated with the single-atom nature of the state $|\Psi_a(t)\rangle$ (18) of the medium excited by a single photon. In contrast to the amplitude $\langle E^+_{\boldsymbol{k}_{echo}}(t,\boldsymbol{r})\rangle$ (27), the behaviour of the field intensity $\langle I_{\boldsymbol{k}_{echo}}(t)\rangle$ is determined by independent spontaneous emission of individual atoms. This is just reflected in the time dependence (29). In this case, the radiation intensity (29) is the sum of only two terms including (31) and (32) and exponentially decaying in time in accordance with the decay of an individual atom. Each of these two terms is associated with the arrival of one of the two photon wave packets in the medium. In this case, any interference of the fields associated with different trajectories of photons is suppressed in the intensity $\langle I_{\boldsymbol{k}_{echo}}(t)\rangle$, so that the corresponding terms containing expressions (33) and (34) become close to zero.

The emergence of coherent collective properties in the radiation emitted by a system of atoms is possible only in the presence of two-atom excitations in the medium since the radiation intensity $\langle I_{\boldsymbol{k}}(t)\rangle$ can be expressed in terms of two-atom operators [see Eqn (28)]. The classical relation between the behaviour of the field amplitude and the field intensity, when $\langle I_{\omega_{32}\boldsymbol{k}_{echo}}(t,\boldsymbol{r})\rangle \approx \langle E^-_{\omega_{32}\boldsymbol{k}_{echo}}(t,\boldsymbol{r})\rangle\langle E^+_{\omega_{32}\boldsymbol{k}_{echo}}(t,\boldsymbol{r})\rangle$, is restored if the initial quantum state $|\Psi_{in}\rangle$ contains a two-photon component in addition to the single-photon component. For instance, this is observed when the medium interacts not with the single-photon wave packet, but with the field in the Glauber state $|\alpha\rangle$ even if the average number of photons in it is $|\alpha|^2 = 1$. In the present case, this result appears as natural (see, for example, Ref. [19]) since the state $|\alpha\rangle$ usually reproduces the results of the classical approach to the description of an electromagnetic field.

6. Conclusions

The first version of the experiment on the observation of the TDSP interference, which is based on the use of a resonance two-level system, was proposed in Ref. [14]. Recently, this version has attracted attention again in connection with the estimate of the observed signal for a number of substances exhibiting photon echo and the use of accumulated single-photon echo [18]. This version of observation also stimulated the study of nonclassical dynamics of photon echo [19] predicted and studied most comprehensively in Refs [15, 16] in the case of single-photon echo.

In this work, we studied the formation of the TDSP interference in a three-level medium with the Λ configuration of allowed transitions in atoms. To obtain such interference, it is proposed to use additional excitation of the medium by a laser pulse, which transfers the excitation of the medium caused by the first photon wave packet from the third to the second atomic level. An important aspect of the interference under study is that a laser pulse acts on the medium in the time interval between the action of two wave packets of the photon. In this case, the laser pulse will probe not the result of interference of single-photon states in the medium, but the intermediate entangled quantum state $|\Psi_{tr}(t)\rangle$ (also called the Schrödinger cat state or the hybrid state) combining the states of the photon and the macroscopic medium.

Such an action of a laser pulse on the medium cannot be reduced to the ordinary procedure of reading out the interference pattern (even a single-photon pattern) in the medium because the latter is formed only after the interaction of the second wave packet of the photon with the atoms of the medium. The interaction of a photon and a laser pulse with the medium can lead to the emergence of single-photon echo. It is important that the echo signal is a direct manifestation of the TDSP interference in the medium. It should be noted that the radiation consists exclusively of the echo signal $\langle E^{+}_{k_{\rm echo}}(\boldsymbol{r}, t)\rangle$ (27), which has the maximum value for $\vartheta = \pi$, when the laser pulse completely transfers excitation from the third to the second atomic level.

The interest in single-photon echo in itself can be associated with the study of nonclassical properties of the field reemitted by the medium because the observation of single-photon echo in the radiation intensity [see relations (28)–(34)] is impossible in view of the quantum origin of a photon. In analogy with the well-known amplitude interference [6], note that the classical properties of the amplitude interference are reproduced during the observation of single-photon echo in the field amplitude, while no such reproduction is observed for the field intensity.

The nonclassical properties of the field reemitted by coherent resonance media during their interaction with photons have been studied insufficiently so far. For instance, in the case of correlation between photons with different wave vectors and frequencies in the two-photon state, the behaviour of the amplitude and intensity of the echo-signal field acquires other unexpected features [29]. Note in this connection that new quantum features in the behaviour of radiation were considered in Ref. [20]. It should be emphasised that the experimental study of these questions is required for understanding the nonclassical dynamics of fields associated with photons.

It is important that the detection of the echo signal reflects the existence of the hybrid state $|\Psi_{\rm tr}(t)\rangle$ of the photon and the medium. In addition, it should be emphasised that the echo signal becomes possible in the case of determinate evolution of the interaction between the medium and the photon whose initial state is delocalised on the time interval including the action of a negligibly short laser pulse on the medium. Consequently, it can be expected that the experimental study of the single-photon echo will permit a detailed study of fundamental problems in the quantum dynamics of spatially delocalised states of the $|\Psi_{\rm tr}(t)\rangle$ type. In particular, it will clarify the mechanisms of evolution of such states over time periods much longer than the time of absorption of a photon by the medium possessing an inhomogeneously broadened resonance line. Such experiments might provide valuable information on latent features in the interaction between a photon and the substance, which are determined by the features of irreversible absorption and decay of excitations in a polyatomic macroscopic system.

Finally, note that the problems associated with the control of the evolution of delocalised states of the $|\Psi_{\rm tr}(t)\rangle$ type and the prospects of their practical application in quantum informatics may attract considerable interest. The above scheme of the interaction of single-photon fields with macroscopic three-level media may be used for storing information on the dynamics of quantised fields. The proposed scheme is interesting because a photon experiences 100 % absorption, and the quantum information transferred to the medium can be subsequently extracted upon passing to another frequency range which does not include the frequency of laser radiation and, hence, does not affect the behaviour of unexcited atoms.

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