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Generation regimes in a laser with annular aperture

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Abstract. **The basic properties of fields emitted by a laser with an annular aperture and a plane – parallel and stable resonator configuration are studied theoretically. It is shown that multipass modes also emerge in a system with a stable resonator, which leads to the formation of combined light beams with a helical wavefront and an autowave profile.**

Keywords*: laser with annular aperture, saturable absorber, autowave, multipass mode*

1. Introduction

The dynamics of optical fields in wide-aperture lasers is usually quite complicated and random in most cases. However, a periodic generation [1] or generation of laser autosolitons [2] is possible in the presence of an intracavity saturable absorber. Under certain conditions, the profile of the laser optical field has the form of a travelling sinusoid, which can naturally be called an autowave.

The main dependences for the case of plane autowaves were obtained in Ref. [3] using linear approximation under conditions close to the bifurcation of the steady-state solution. It was shown in Ref. [4] that autowave structures may appear in existing lasers. Calculations of autowave lasing in a laser with an annular aperture were made in Ref. [5], where it was shown that the optical field profile in a laser with a Fabry – Perot resonator assumes a steady-state harmonic shape with time and turns around the annular aperture of the laser. This leads to the generation of helical light beams tilted towards the resonator axis. In this work, we continue our study of rotating laser beams and determine their basic features.

Similar radiation is also observed in lasers with stable resonators in which the multipass mode (M-mode) is generated. In this case, the light field on the mirror consists of spots forming a circle or an ellipse. The question arises of whether the properties of autowaves and multipass light fields can be combined to create new possibilities for controlling laser radiation. To elucidate this question, we also studied in this paper a system in which autowave lasing occurs in the presence of a stable resonator.

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2. Autowave generation in a Fabry – Perot resonator

We used below the equations describing the dynamics of a laser optical field, peculiarities of the theoretical model and the computational method described in Refs [5, 6]. It was found that a parameter of the system like the relaxation time of the absorbing medium virtually does not affect the qualitative features of the system. Therefore, we can assume that the nonlinear system is inertialess and its population is $N_{\rm f} = N_{\rm fe}(1-\delta)^{-1}$. In this approximation, the system of equations in dimensionless variables takes the form

$$\frac{\partial E}{\partial t} - {\rm i}\,\frac{\partial^2 E}{\partial x^2} = \frac{v}{2}\,E\left(N - \frac{N_{\rm fe}}{1+I\delta} - 1\right), \qquad (1)$$

$$\frac{{\rm d}N}{{\rm d}t} = N_{\rm e} - N(1+I). \qquad (2)$$

Here, E is the amplitude of the light field, $I = |E|^2$; N, $N_{\rm f}$ are the populations in the active and absorbing media; and $N_{\rm e}$ and $N_{\rm fe}$ are their unsa-turated values, respectively; v is the ratio of the relaxation time of the active medium to the photon lifetime in the resonator, and δ is the ratio of the saturation intensities for active and absorbing media.

Eqns (1) and (2) are applicable in the case of a small variation in the optical field during a round trip in the resonator. This means that all the factors determining laser radiation (amplification, absorption, and diffraction) only weakly change the light field profile for a round trip in the resonator.

An analysis of these equations was carried out in papers [3 – 6] for the one-dimensional case, i.e., for plane autowaves. Some results pertaining to laser with an annular aperture are also presented in these works. Here, we continue to study the latter case. It is assumed that one of the resonator mirrors with reflectivity $r = 1$ has an infinite size. The second mirror is partially transparent and is annular in shape with the outer radius $\rho_{\rm out}$ and the inner radius $\rho_{\rm in}$. The edges of this mirror are assumed to be rounded so that the reflectivity r decreases with the distance $\Delta\rho$ from the mirror edge as $r \sim \exp[-(\Delta\rho/d)^2]$, where $d = 0.03a$. Here, $2a$ is the transverse dimension of the computational domain, and other transverse sizes will be presented in terms of this quantity for the sake of convenience. The length of the resonator is assumed to be $L/2$ and the Fresnel number $N_{\rm F} = a^2/\lambda L = 40$. The cross sections of the active and absorbing media are annular in shape.

It was shown in Ref. [5] that for a certain range of laser parameters, a field with a profile running around the annular mirror is formed, i.e., the light field rotates around the resonator axis. The shape of this field can be presented in details by analysing its cross section along the central line of the annular mirror, i.e., along a ring of radius $\rho_c = (\rho_{in} + \rho_{out})/2$. Fig. 1a shows such cross sections calculated at the instants of times that are multiples of the round-trip transit time in the resonator. One can see that the field acquires a stationary periodic form. Fig. 2 shows a field of this kind, i.e., the phase and intensity profiles at the last stage of computations.

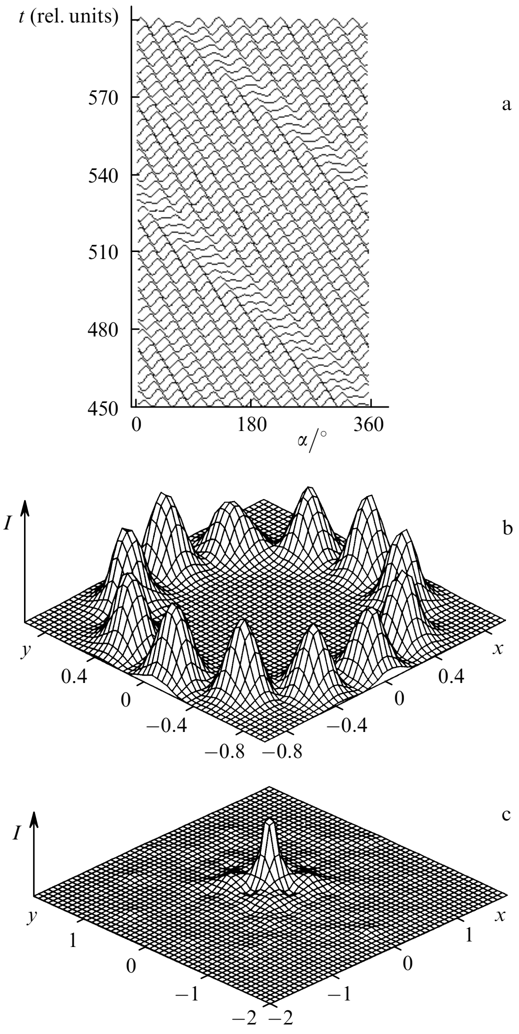

Figure 1. Optical field profile at the central circumference of an annular mirror at various instants of time (a) and the field distribution in near- (b) and far-field (c) zones at the final stage of computations for $N_e = 20$, $N_{fe} = 14$, $\delta = 3$, $v = 4$, $\rho_{out} = 0.94a$, $\rho_{in} = 0.66a$ (in rel. units).

Within each autowave period, the phase of the optical field grows linearly by a certain amount (less than 2π), and drops almost to its minimum value between the periods at the point corresponding to the minimum light intensity. Judging by Fig. 2a, these qualitative conclusions are also valid for the case when the harmonicity of the autowave is slightly violated, i.e., the properties of the light field are topologically stable. The slope of the phase profile affects the divergence of the radiation, but the diffraction effects are more significant in this case. Figs. 1b and c show the field pattern in the near- and far-field zones at the end of computations. One can see that the divergence pattern is similar to the pattern obtained from a uniformly illuminated annular orifice [7].

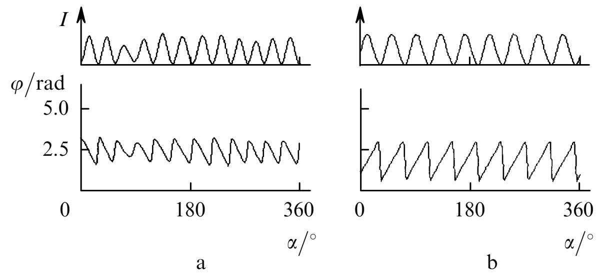

Figure 2. Intensity and phase profile of the optical field corresponding to Fig. 1b (a), and in the case when the initial approximation has the nonagonal symmetry (b), for $N_e = 20$, $N_{fe} = 14$, $\delta = 3$, $v = 4$.

The calculated autowave pattern may depend on the initial approximation. If the initial field profile is assumed to be close to the uniform distribution with a slight random variation, the obtained result almost always corresponds to Fig. 1. However, if the initial field is symmetric relative to a rotation by an angle $2\pi/n$ around the resonator axis, the established light field profile usually has the same symmetry. Depending on the form of the initial approximation, a field with the number of periods ranging from 8 to 20 may be established under the conditions corresponding to Fig. 1c. One of these realisations is shown in Fig. 2b. Therefore, the system under consideration has a multiple stability.

Obviously, the autowaves can propagate along the annular aperture in both directions. Calculations show that for the conditions being considered here, one of the counterpropagating waves is suppressed and in this sense the system is bistable. If, however, an asymmetry is created in the resonator relative to the opposite directions, one of these will become the preferred one and a light field propagating in the desired direction may be obtained.

Even if the shape of the annular mirror is slightly non-circular, the above autowave properties of the optical field are mainly preserved. They are also preserved if the axial symmetry of the resonator is violated by small amplitude or phase perturbations (within a few percent).

The phase distribution shown in Fig. 2 is such that the phase shift for after a round trip in the annular mirror is equal to zero. For certain initial field distributions, however, optical structures with a phase shift multiple of 2π may appear after a round trip in the annular aperture. Such phase shifts can affect the divergence of radiation only if they are comparable with the phase gradients of the same order of magnitude as shown in Fig. 2.

3. Dynamics of autowave fields in a stable resonator

It is well known that the optical field in a stable resonator may be generated in the form of a set of oblique beams called the M modes [8]. Such beams are reproduced only after a certain number N_M of round trips, one can say that the light field rotates inside the cavity in this case. In this respect, some properties of the M modes are similar to those of the autowave generation regime.

The physical origin of these two generation regimes is different, and hence their wave patterns are also significantly different. The M modes usually form a steady-state pattern in time. In the case of axial symmetry, the resultant field of a set of the M modes is also symmetric. To single out a single mode from the complete set of multipass modes, mirrors with holes and auxiliary reflectors are usually employed. Obviously, the light field of the M modes has a somewhat irregular profile in this case, but is time-independent. On the other hand, although the autowave field varies in time and space, it can still be regarded as a steady-state structure moving uniformly in space. If both lasing regimes exist simultaneously, new peculiarities may be observed in the laser radiation.

It follows from the cases considered by us that during the rotation of an autowave around the axis by an angle 2π, approximately $N_A \approx 150$ round trips occur along the resonator axis. The M mode with $N_M \approx 150$ arises under the condition

$$\frac{1}{N_M} = \frac{\arccos g_1^{1/2}}{\pi}, \quad g_1 = 1 - \frac{L}{2R_M},$$

which is satisfied if the radius of curvature of the annular concave mirror is $R_M \approx 1000L$.

The condition $N_M > N_A$ (i.e., $R_M > 1000L$) indicates that the rotational velocity of the autowave is higher than the velocity of transverse displacement of the M-mode beams. According to calculations, the autowave profile of the laser field, which is similar to those presented in Figs. 1 and 2, is preserved (the qualitative and quantitative differences are insignificant).

For $N_M \approx N_A$, i.e., when the rotational velocities of the autowaves and the M mode are equal, the field distribution still corresponds to an autowave (Fig. 3). The optical field shown in Fig. 3 differs from the previous fields only in phase (see Figs 1 and 2): at the beginning of each period of the autowave, the phase shift now increases sharply, while its variation during each period remains virtually linear, as before. A higher phase gradient means an increase in the rotational velocity of autowaves, which is now approximately equal to the rotational velocity of the M mode. It can be stated that the autowave properties of the system form the profile of the light field in this case, while the configuration of the resonator determines the rotational velocity of the field. In the far-field zone, however, the light field is still mainly concentrated at the central spot and some small side spots.

For $N_M < N_A$, calculations reveal the existence of a rotating field with a wave-like profile. Fig. 4 shows a version of the optical field whose rotational velocity is determined almost entirely by N_M. One can see (cf. Fig. 1) that the field profile is not strictly harmonic in this case, and the spatial period of the autowave has increased significantly. It is important that the light field in the far-field zone has no longer a single maximum, but fills a certain annular region, this filling being nonuniform.

If the curvature of the mirror of a stable resonator is increased further, the value of N_M decreases and the rotational velocity of the light field increases. In this case, the harmonic dependence of the field is violated more and more and the spatial period increases noticeably, so that only a few light spots are left on the mirror. Considerable qualitative variations in the field occur in the far-field zone. One

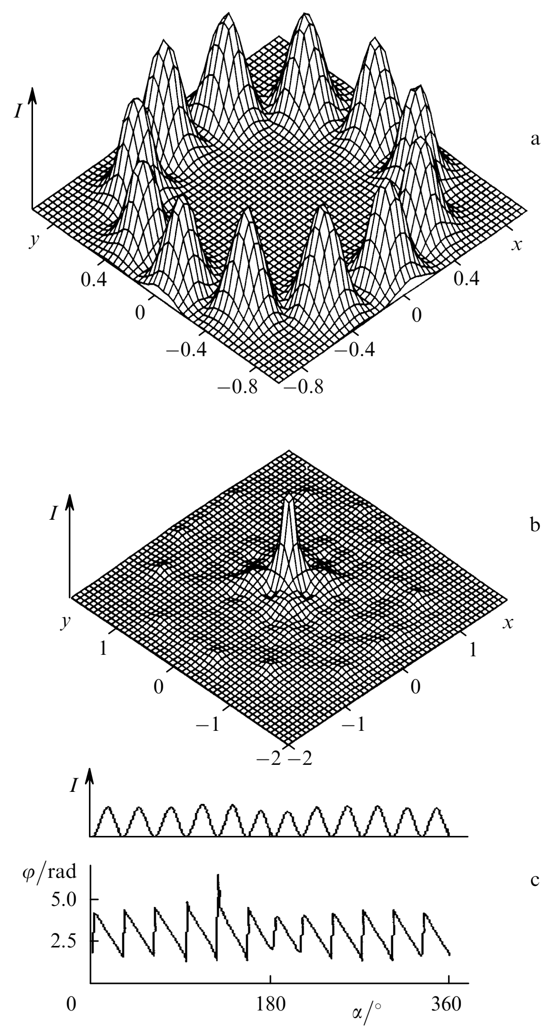

Figure 3. Steady-state optical field in the near- (a) and far-field (b) zones of a stable resonator for the radius of curvature $R_M = 1000L$ of the annular concave mirror, and the intensity and phase profiles of the optical field corresponding to Fig. 3a (c) for $N_e = 20$, $N_{fe} = 14$, $\delta = 3$, $v = 4$.

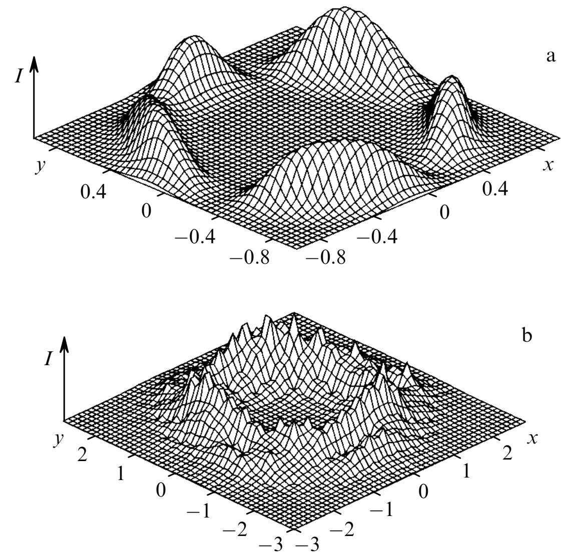

Figure 4. Field in the near- (a) and far-field (b) zones for $R_M = 300L$, $N_e = 20$, $N_{fe} = 14$, $\delta = 3$, $v = 4$.

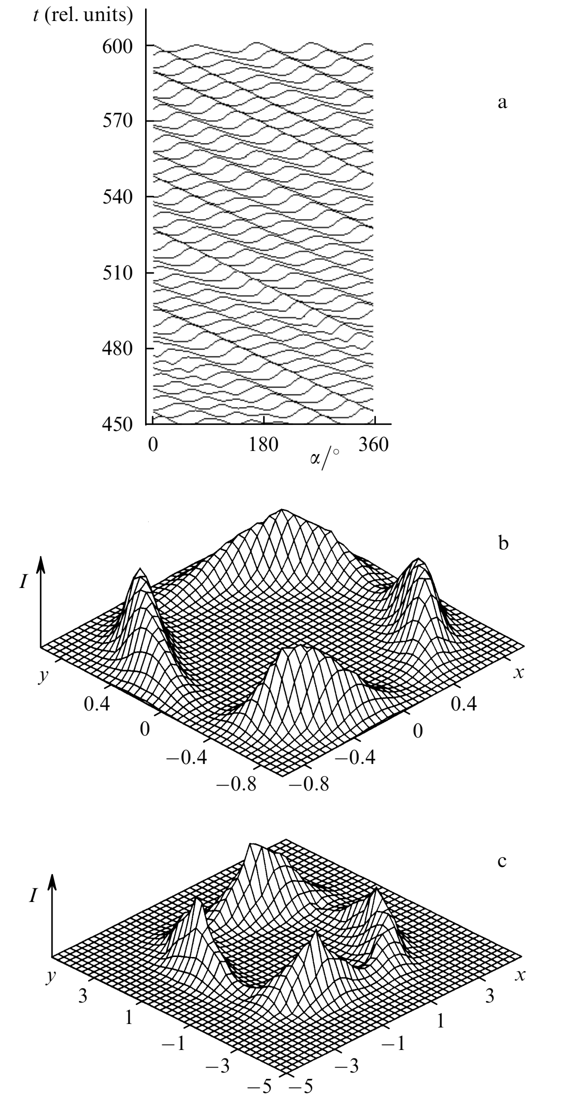

Figure 5. Annular profile of the optical field at various instants of time for $R_M = 100L$, $N_e = 20$, $N_{fe} = 14$, $\delta = 3$, $v = 4$ (a) and the field distribution in near- (b) and far-field (c) zones at the final stage of computations.

can be see from Fig. 5 that the field in the far-field zone now consists of individual spots.

Thus, the rotational velocity of autowave structures in the optical field profile can be increased considerably, and their shape can be modified, by varying the shape of the resonator. Note that the field in the far-field zone suffers the most significant changes in this case.

4. Change of the pulse regime into the autowave one in a stable resonator

It is known that under certain conditions, a laser with a saturating absorber can emit periodic pulses [1]. If the ratio v is chosen larger than a certain critical value, the autowave lasing is replaced by pulsed lasing [6]. This conclusion was reached both by calculations and from an analysis of equations in the linear approximation near the bifurcation point of the steady-state homogeneous solution for a one-dimensional model of a laser with a Fabry–Perot resonator. However, the calculations of a two-dimensional profile of the optical field in a stable resonator give a new result.

Fig. 6 shows a typical profile of the optical field for $v = 20$. According to the results obtained by solving the dispersion equation [5], pulsed lasing should be anticipated in the Fabry–Perot resonator. However, light is emitted as two separate bright spots rotating in the laser aperture. As a result, any region of the laser cross section performs pulsed laser action in time, while the laser as a whole emits individual beams propagating continuously along a circle. This suggests that although the pulsed lasing regime is preserved for such a geometry of the resonator (large cross section and stable configuration), the light pulses, discrete in time and propagating along the longitudinal axis of the resonator are transformed into discrete light spots travelling in the transverse direction.

The number of such spots may be quite large if the aperture is increased in such a way that N_F exceeds 40. The main properties of the light field in this case are almost the same as for the M mode corresponding to the given resonator configuration.

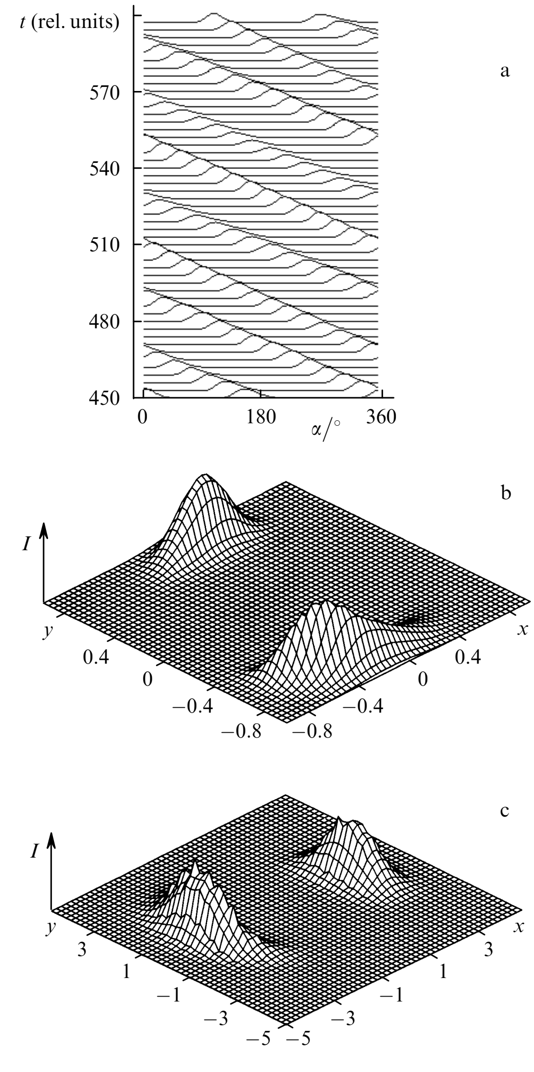

Figure 6. Annular profile of the optical field at various instants of time for $N_e = 10$, $N_{fe} = 7$; $\delta = 3$, $v = 20$, $R_M = 100L$ (a) and the field distribution in near- (b) and far-field (c) zones at the final stage of computations.

The obtained local light spots are outwardly similar to the objects like laser autosolitons in lasers with a saturating absorber [2], but have different origin and properties.

5. Conclusions

Thus, an analysis of the results obtained shows that for an appropriate choice of parameters, lasers with an annular aperture may generate a field with a steady-state rotating nearly sinusoidal wave profile. Such a form of the field is structurally stable relative to small perturbations of the system parameters. Depending on the initial conditions, this autowave structure may have different spatial periods, thus indicating the multistability of the system. The influence of the autowave nature on the light radiation profile does not exceed the influence of diffraction effects, so that the properties of the field in the far-field zone are mainly determined by diffraction phenomena.

The use of a stable resonator results in qualitatively new features of the optical field. The velocity of autowaves increases and becomes equal to the velocity of transverse motion of the multipass mode, which is determined by the resonator configuration. On the other hand, the results of our study show that the frequency of autowaves remains unchanged, so that the spatial period of autowaves increases proportionally to their velocity. As a result, the optical field in the far-field zone undergoes the strongest variation, its shape changing from a spot to a ring and further to an annular spotted structure. Significantly, these variations are the result of only an increase in the curvature of the resonator mirror, which can be varied with the help of an adaptive mirror.

A combination of the autowave properties and the properties of a stable resonator provides another qualitatively new picture – a light field consisting of individual light spots travelling in the annular aperture both in near- and far-field zones. However, the influence of resonator alone is not sufficient for such an effect, and the active medium parameters should also be changed in such a way that they correspond to passive Q-switching.

It can be hoped that the lasing regimes described in this work can be used for controlling the spatial characteristics of laser radiation and applied for optical processing of information as well as laser processing of materials. Light beams with structural peculiarities can also be used for controlling the motion of microparticles [9].

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