Quantum Electronics **31** (7) 639 – 642 (2001)

NONLINEAR OPTICAL PHENOMENA

PACS numbers: 42.65.Tg; 42.70.Nq
DOI: 10.1070/QE2001v031n07ABEH002019

Formation and interaction of nondiffracting beams in a photorefractive medium with diffusion nonlinearity

V A Aleshkevich, V A Vysloukh, Ya V Kartashov

***Abstract*. The profiles of (1 + 1)D soliton-like beams are calculated numerically in a photorefractive medium with diffusion nonlinearity. The features of the propagation of nondiffracting beams are considered and it is shown that their interaction in the medium with diffusion nonlinearity is accompanied by the energy redistribution resulting in the fusion of the beams.**

Keywords: *nondiffracting beams, photorefraction, solitons.*

1. Introduction

Studies of spatial solitons in photorefractive crystals (PRCs) have attracted great recent attention. These crystals exhibit strong nonlinearity already at the radiation intensity of the order of several microwatts per square centimetre, thus being promising media for nonlinear optical devices for data processing. From practical point of view, of most interest is the interaction between soliton beams, which is inelastic, as a rule [1]. In the presence of a strong external static field, the regime of considerable drift nonlinearity can be realised in a PRC in which solitons of three types can exist: quasi-stationary solitons, which are formed during slow screening of the external static field [2, 3]; stationary solitons, which are observed in the nonuniformly screened external field [4 – 7]; and photovoltaic solitons, which can be observed in PRCs with high photovoltaic currents [8, 9].

The features of the interaction of coherent [10 – 13] and incoherent [14, 15] quasi-stationary and stationary solitons in PRCs with drift nonlinearity are well studied. However, the nonlinear response of PRCs is nonlocal due to the presence of the natural diffusion component [16 – 18]. The diffusion nonlinear component in the presence of a significant drift component causes the self-bending of a beam during its propagation [19 – 26] and also is manifested in the additional energy redistribution between the interacting beams [24]. The possibility of the experimental realisation of solitons in a PRC with diffusion nonlinearity was discussed for the first time in Ref. [20].

V A Aleshkevich, V A Vysloukh, Ya V Kartashov Department of Physics, M V Lomonosov Moscow State University, Vorob'evy gory, 119899 Moscow, Russia

Received 25 December 2000; revision received 3 April 2001
Kvantovaya Elektronika **31** (7) 639 – 642 (2001)
Translated by M N Sapozhnikov

In this paper, we perform for the first time numerical simulations of spatial solitons in a PRC with diffusion nonlinearity. Special attention is given to the study of the interaction between nondiffracting soliton beams in PRCs.

2. Theoretical model

The material response of a photorefractive medium with diffusion nonlinearity in the 1D case is described by the system of equations for an internal electric field $E_{sc}(x, y, t)$ produced by the photoinduced distribution of a spatial charge [27]:

$$\frac{\partial n_e}{\partial t} = \frac{\partial n_d^+}{\partial t} - \frac{1}{e}\frac{\partial j}{\partial x},$$

$$\frac{\partial n_d^+}{\partial t} = \sigma(I + I_{dark})\left(n_d - n_d^+\right) - \gamma_r n_e n_d^+, \qquad (1)$$

$$j = e\varsigma n_e E_{sc} - \varsigma k_B T\frac{\partial n_e}{\partial x},$$

$$\frac{\partial E_{sc}}{\partial x} = \frac{4\pi e}{\varepsilon}\left(n_e + n_a - n_d^+\right).$$

Here, n_e, n_d, n_d^+ and n_a are concentrations of free charge carriers, donors, ionised donors, and acceptors, respectively; j is the current density; σ is the photoionisation cross section; I is the incident radiation intensity; I_{dark} is the effective dark intensity; γ_r is the two-body recombination constant; e and ς are the charge and mobility of free carriers, taking their sign into account (negative for electrons and positive for holes); ε is the static dielectric constant of the PRC; k_B is the Boltzmann constant; and T is the temperature of the medium. Optical radiation propagates along the z axis.

The system of constitutive equations (1) is solved together with the standard reduced wave equation for the complex field amplitude $A(x, z, t)$

$$\mathrm{i}\frac{\partial A}{\partial z} = -\frac{1}{2k_0}\frac{\partial^2 A}{\partial x^2} - \frac{k_0}{n}\delta n A, \qquad (2)$$

written in the paraxial approximation. In equation (2), $k_0 = \omega n/c$ is the wave number; $\delta n = (1/2)r_{eff}n^3 E_{sc}(x, z, t)$ is the nonlinear addition to the unperturbed refractive index n of the PRC, which appears in the field $E_{sc}(x, z, t)$ due to linear electrooptical effect; r_{eff} is the effective electrooptical coefficient. Equations (1) and (2) represent a closed self-consistent system, which adequately describes the relation between the spatial distribution of the incident beam intensity and the internal electric field E_{sc} in the photorefractive medium.

We consider the system (1) in the stationary state when $\partial/\partial t \to 0$. Taking into account that $n_a \gg n_e$ for a typical PRC, the system (1) can be solved for $E_{sc} = (k_B T/e) \times(\partial I/\partial x)(I + I_{dark})^{-1}$. By substituting this expression into (2), we obtain the reduced wave equation for the normalised complex amplitude $q(\eta, \xi)$ of the light field:

$$\mathrm{i}\frac{\partial q}{\partial \xi} = -\frac{1}{2}\frac{\partial^2 q}{\partial \eta^2} - \frac{q}{1+S|q|^2}\frac{\partial |q|^2}{\partial \eta}. \qquad (3)$$

Here, $q(\eta, \xi) = (L_{dif}/L_{ref})^{1/2} A(\eta, \xi) I_{dark}^{-1/2}$ the dimensionless amplitude of the light field; $\eta = x/x_0$ is the normalised transverse coordinate; x_0 is the characteristic transverse scale (for example, the input beam radius); $\xi = z/L_{dif}$ is normalised longitudinal coordinate; $L_{dif} = k_0 x_0^2$ is the diffraction length corresponding to x_0; $L_{ref} = 2ex_0/(k_0 n^2 r_{eff} \times k_B T)$ is the nonlinear refraction length; $S = L_{ref}/L_{dif}$ is the parameter determining the relative role of diffusion effects. The typical values of the parameter S in a SnBaNb crystal for 633-nm He – Ne laser beams of intensity of several microwatts per square centimetre, the initial laser beam diameter $x_0 \sim 50$ μm, the effective electrooptical coefficient $r_{eff} = 10^{-9}$ m V^{-1}, and the unperturbed refractive index $n = 2.3$ are $\sim 1.0 - 2.0$.

3. Profiles of nondiffracting soliton-like beams

Note first of all that, using the known solution $q(\eta, \xi, S)$ of equation (3) and the transformation $q_{new}(\eta, \xi, S_{new}) = u^{1/2} \times q(u\eta, u^2\xi, S)$, where u is an arbitrary scaling coefficient, we can find a solution for the new parameter $S_{new} = S/u$. By using the known method of transformation to a curved coordinate system in an arbitrary reduced wave equation [28], we will seek the stationary solutions of equation (3) in the form of a beam with the profile

$$q(\eta, \xi) = \rho\left(\eta + \frac{a\xi^2}{2}\right)\exp\left(\mathrm{i}b\xi - \mathrm{i}a\eta\xi - \frac{\mathrm{i}a^2\xi^3}{3}\right), \qquad (4)$$

which is invariable along the parabolic trajectory $\eta = -a\xi^2/2$, where a is the curvature of the parabolic trajectory; b is the propagation constant; $\rho(\eta + a\xi^2/2)$ is the real envelope. By substituting the field in this form into equation (3) and introducing the parabolic coordinate $\zeta = \eta + a\xi^2/2$, we obtain that the beam envelope, as a function of the variable ζ, satisfies the ordinary differential equation

$$\frac{\mathrm{d}^2\rho}{\mathrm{d}\zeta^2} = 2(b - a\zeta)\rho - \frac{4\rho^2}{1+S\rho^2}\frac{\mathrm{d}\rho}{\mathrm{d}\zeta}. \qquad (5)$$

Note that the propagation constant b can be eliminated from equation (5) with the help of a linear shift along the ζ axis.

An analytic solution of equation (5) can be obtained in two limiting cases: for small amplitudes ($\rho \ll 1$), when nonlinear terms in the right-hand side of (5) can be neglected compared to linear terms, and for large amplitudes ($\rho \gg 1$), when the last term in the right-hand side of (5) can be linearised. In these limiting cases, the solutions of equation (5) has the form

$$\rho(\zeta) = m\mathrm{Ai}[(2a)^{-2/3}2(b - a\zeta)] \quad (m \ll 1),$$

$$\rho(\zeta) = m\mathrm{Ai}\{(2a)^{-2/3}[2(b - a\zeta) + 4S^{-2}]\} \qquad (6)$$

$$\times \exp[2(aS)^{-1}(b - a\zeta)] \quad (m \gg 1),$$

where m is an arbitrary constant. The first of the solutions (6) describes the profile of a beam that does not diffract in a linear medium [29] and has, generally speaking, the infinite energy

$$w = \int_{-\infty}^{\infty} \rho^2(\zeta)\mathrm{d}\zeta, \qquad (7)$$

because the Airy function is not square integrable because of the presence of a slowly decaying oscillating tail. The diffusion nonlinearity in the limit of large amplitudes causes the compensation of the oscillating tail of the nondiffracting beam and its localisation [the second of the solutions of (6)]. As S decreases (i.e., the role of nonlinear effects increases), the beam localisation increases.

Because analytic solutions of equation (5) cannot be obtained in the general case, the numerical integration is required. We sought spatially localised soliton-like solutions of (5) by the shooting method, which allows one to reduce the two-point boundary problem to the Cauchy problem. The initial conditions were chosen based on the fact that for $\zeta \to -\infty$, when the amplitude ρ is sufficiently small, nonlinear terms in (5) can be neglected. In this case, the initial conditions are specified by the asymptotics of the Airy function and have the form

$$\rho|_{\zeta \to -\infty} = m\mathrm{Ai}\left[2(2a)^{-2/3}(b - a\zeta)\right],$$

$$\left.\frac{\mathrm{d}\rho}{\mathrm{d}\zeta}\right|_{\zeta \to -\infty} = m\frac{\mathrm{d}}{\mathrm{d}\zeta}\mathrm{Ai}\left[2(2a)^{-2/3}(b - a\zeta)\right]. \qquad (8)$$

Fig. 1 shows typical profiles of nondiffracting beams with different energies w. In accordance with linear asymptotics (7), the approximate expression for the beam profile for $\zeta \to -\infty$ has the form $(1/2)m\pi^{-1/2}x^{-1/4}\exp[-(2/3)x^{3/2}]$, where $x = 2(2a)^{-2/3}(b - a\zeta)$.

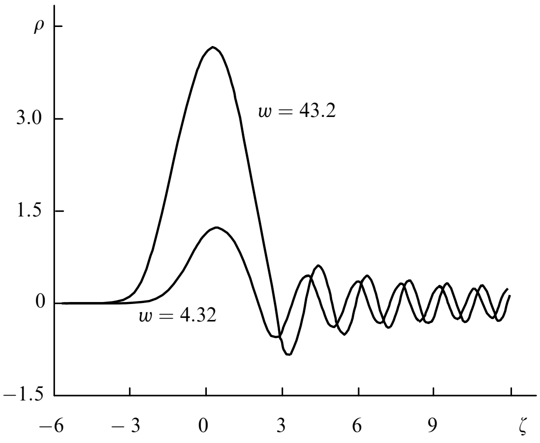

Figure 1. Profiles of nondiffracting beams with different energies w for $a = 1.0$ and $S = 1.0$.

For $\zeta \to +\infty$, however, the nonlinear term in equation (5) can no longer be neglected, because the derivative $d\rho/d\eta$ increases with decreasing amplitude ρ due to the increase in the frequency of oscillations at the right wing of the beam. The influence of diffusion nonlinearity of the PRC is finally manifested in the decrease in the light field amplitude at $\zeta \to +\infty$ that occurs faster than $\zeta^{-1/2}$ (in contrast to the Airy function), so that the soliton-like beams with an arbitrary amplitude have a finite energy w and are localised in space. Therefore, the diffusion mechanism of the spatial charge redistribution in the PRC, as the drift mechanism, can cause the formation of specific soliton structures.

4. Formation, stability, and interaction of nondiffracting beams

Of practical interest is the problem of generation of nondiffracting soliton beams by beams with an arbitrary transverse distribution of the light field intensity due to photoinduced scattering. It is known that the evolution of a beam with an arbitrary input distribution of the light field in a medium with a purely Kerr nonlinearity or local saturating nonlinearity (except some special cases facilitating the formation of bound states) leads to the formation of one or several diverging soliton beams. A medium with diffusion nonlinearity has essentially new properties in this respect.

Fig. 2 shows the typical evolution of a super-Gaussian beam in a medium with diffusion nonlinearity. The beam propagation is accompanied by its gradual spreading and energy scattering to the region of negative η at certain angles. The scattering intensity and the number of maxima in the scattered radiation substantially increase with increasing width (energy) of the initial super-Gaussian beam (cf. Figs 2a and 2b). Note that scattering occurs into the region of negative η, whereas the nondiffracting beam has an oscillating tail at $\eta \to +\infty$. This means that nondiffracting beams do not evolve from the beams with an arbitrary spatial distribution of the light field, and to perform experiments with them, one should produce by holographic methods the initial conditions that are similar to those shown in Fig. 1.

The study of the stability of soliton-like solutions obtained above involves certain problems because of the absence of an analytic expression for the spatial distribution of the field. The criterion $\partial w/\partial b > 0$ for the stability of solitons in media with a local nonlinear response cannot be applied for a medium with diffusion nonlinearity. For this reason, we studied the propagation dynamics of solitons with the perturbed input profile by numerical simulations using the method of splitting over physical factors. The results of numerical simulations demonstrated the stability of nondiffracting beams both to small (up to 10 % in intensity) harmonic and noise perturbations of the input profile. During the beam propagation, a small perturbation experiences decaying oscillations with period gradually increasing along the ξ axis, while the energy excess produced by the perturbation is gradually scattered into the region of negative values of η.

The nondiffracting beams in a medium with diffusion nonlinearity exhibit a quite unusual behaviour upon collisions. We studied collisions of the beams by specifying the initial condition at the input to the nonlinear medium in the form

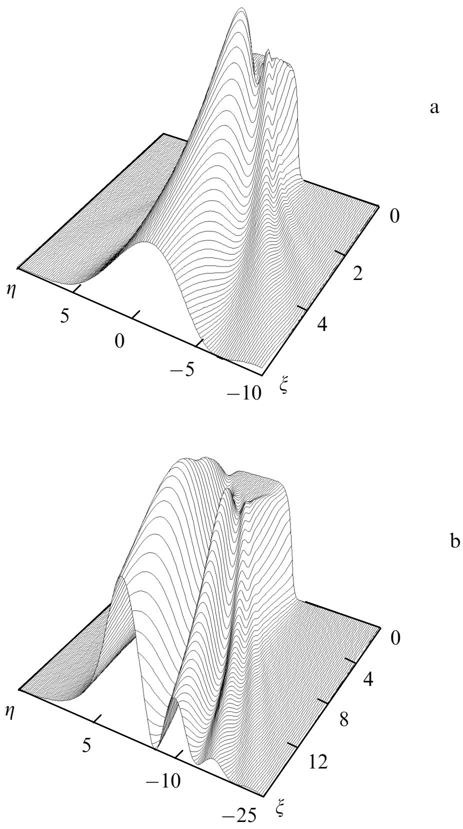

Figure 2. Propagation dynamics of a super-Gaussian beam with the initial profile $\rho(\eta) = (1/2)\exp[-(\eta/\eta_0)^8]$ and the width $\eta_0 = 3.0$ (a) and 8.0 (b) in a PRC with diffusion nonlinearity for $S = 1.0$.

$$q(\eta, \xi = 0) = \rho(\eta + \eta_c)\exp(-i\eta + i\psi) + \rho(\eta - \eta_c)\exp(i\eta), \tag{9}$$

where $\rho(\eta)$ is the beam profile; $2\eta_c$ is the distance between centres of the beams at the input to the medium; υ is the intersection angle of the beams; ψ is the initial phase difference. Fig. 3a shows the free propagation of a nondiffracting beam. The collision dynamics for two nondiffracting beams are presented in Figs 3b and 3c. The specific feature of the interaction is that, irrespective of the phase difference and collision angles, only one stable *nondiffracting* beam is always formed.

The fraction of the energy scattered during collisions at any angles is negligible compared to the total energy of the interacting beams. The curvature of the trajectory of the beam formed in the medium with the given value of S is determined by its energy (which is in fact equal to a sum of energies of the colliding beams). The phase difference of the beams determines the dynamics only in the overlap region of the fields (cf. Figs 3b, 3c). Note that the interaction of two localised beams (for example, Gaussian beams) in the PRC with diffusion nonlinearity at sufficiently large intersection angles also results in the elimination of one of the beams.

5. Conclusions

Thus, the diffusion mechanism of nonlinearity of a PRC can lead to the formation of specific soliton-like beams, which have a finite energy and steadily propagate along the

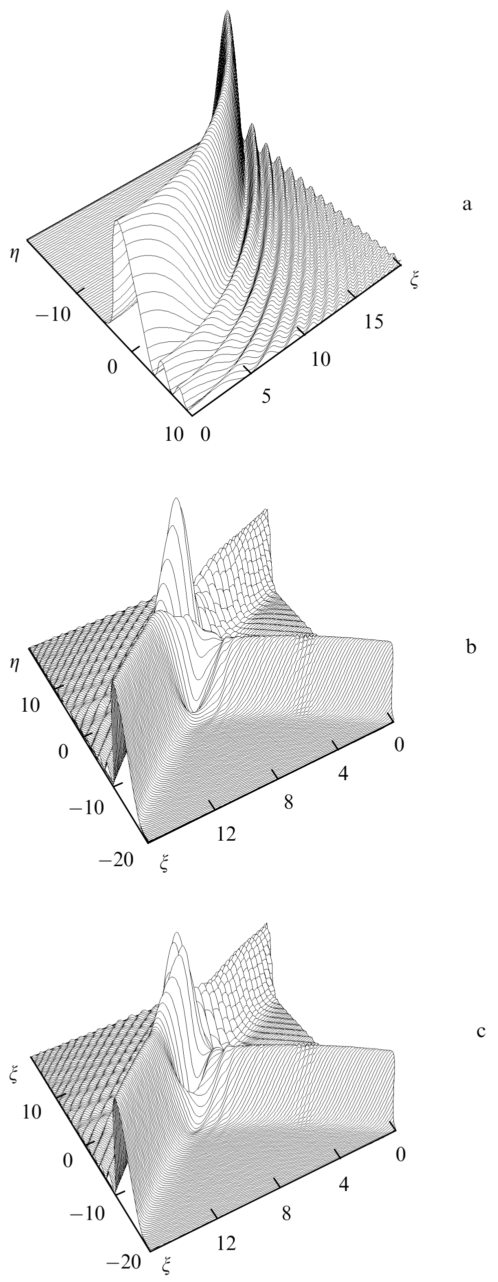

Figure 3. Propagation dynamics of a single nondiffracting beam in a PRC with diffusion nonlinearity (a) and the collision dynamics for inphase (b) and out-of-phase (c) nondiffracting beams for the beam energy $w = 3.88$, the collision angle $v = 1.5, S = 1.0$, and the parabolic parameter of the beam profile $a = 0.1$.

parabolic trajectory in the PRC. Such beams do not evolve from the beams with an arbitrary distribution of the lightfield intensity. Irrespective of the collision angle and the initial phase difference, the interaction of two nondiffracting beams results in the elimination of one of them and the formation of one beam with the energy that is almost equal to a sum of energies of the interacting beams. Such behaviour of the nondiffracting beams upon their interaction is interesting from the practical point of view and can be used in optical switches.

References

1. Snyder A, Kivshar Y *J. Opt. Soc. Am. B: Opt. Phys.* **14** 3025 (1997)
2. Crosignani B, Segev M, Engin D, Di Porto P, Yariv A, Salamo G *J. Opt. Soc. Am. B: Opt. Phys.* **10** 446 (1993)
3. Duree G, Morin M, Salamo G, Segev M, Crosignani B, Di Porto P, Sharp E, Yariv A *Phys. Rev. Lett.* **74** 1978 (1995)
4. Iturbe-Castillo M, Marques-Aquilar P, Sanchez-Mondragon J, Stepanov S, Vysloukh V *Appl. Phys. Lett.* **64** 408 (1994)
5. Christdoulides D, Carvalho M *J. Opt. Soc. Am. B: Opt. Phys.* **12** 1628 (1995)
6. Segev M, Shih M, Valley G *J. Opt. Soc. Am. B: Opt. Phys.* **13** 706 (1996)
7. Chen Z, Segev M, Singh S, Coskin T, Christodoulides D *J. Opt. Soc. Am. B: Opt. Phys.* **14** 1407 (1997)
8. Bian S, Frejlich J, Ringhofer K *Phys. Rev. Lett.* **78** 4035 (1997)
9. Jinsong L, Keqing L *J. Opt. Soc. Am. B: Opt. Phys.* **16** 550 (1999)
10. Garcia-Quirino G, Iturbe-Castillo M, Vysloukh V, Stepanov S *Opt. Lett.* **22** 154 (1997)
11. Krolikowski W, Luther-Davies B, Denz C, Tshudi T *Opt. Lett.* **23** 97 (1998)
12. Krolikowski W, Holmstrom S *Opt. Lett.* **22** 369 (1997)
13. Mamaev A, Saffman M, Zozulya A *J. Opt. Soc. Am. B: Opt. Phys.* **15** 2079 (1998)
14. Shih M, Segev M *Opt. Lett.* **21** 1538 (1996)
15. Krolikowski W, Saffman M, Denz C *Phys. Rev. Lett.* **79** 3240 (1998)
16. Vysloukh V, Kutuzov V, Petnikova V, Shuvalov V *Zh. Eksp. Teor. Fiz.* **111** 705 (1997)
17. Garcia-Quirino G, Iturbe-Castillo M, Sanchez-Mondragon J, Stepanov S, Vysloukh V *Opt. Commun.* **123** 597 (1996)
18. Gomes-Sarabia C, Marquez-Aquilar P, Sanchez-Mondragon J, Stepanov S, Vysloukh V *J. Opt. Soc. Am. B: Opt. Phys.* **12** 2767 (1996)
19. Christodoulides D, Carvalho M *Opt. Lett.* **19** 1714 (1994)
20. Christodoulides D, Coskun T *Opt. Lett.* **21** 1220 (1996)
21. Christodoulides D, Coskun T *Opt. Lett.* **21** 1460 (1996)
22. Krolikowski W, Luther-Davies B, McCarthy G, Bledowski A *Phys. Rev. E* **61** 2010 (2000)
23. Krolikowski W, Akhmediev N, Luther-Davies B, Cronin-Colomb M *Phys. Rev. E* **54** 5761 (1996)
24. Aleshkevich V, Vysloukh V, Kartashov Ya *Kvantovaya Elektron.* **28** 64 (1999) [*Quantum Electron.* **29** 621 (1999)]
25. Aleshkevich V, Vysloukh V, Kartashov Ya *Kvantovaya Elektron.* **30** 905 (2000) [*Quantum Electron.* **30** 905 (2000)]
26. Aleshkevich V, Vysloukh V, Kartashov Ya *Opt. Commun.* **173** 277 (2000)
27. Kukhtarev N, Markov V, Odulov S, Soskin M, Vinetskii V *Ferroelectrics* **22** 949 (1979)
28. Gagnon L, Belanger P *Opt. Lett.* **15** 466 (1990)
29. Berry M, Balazs N *Am. J. Phys.* **48** 264 (1979)