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Effect of a magnetic field on self-modulation oscillations in a ring chip laser

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***Abstract*. The phase shift of self-modulation oscillations induced in a solid-state ring laser by an external magnetic field is studied theoretically and experimentally. It is found that the phase shift is caused by the amplitude nonreciprocity of the ring laser due to the Faraday effect in an active medium.**

Keywords: *solid-state ring laser, self-modulation oscillations, magnetooptics of solid-state lasers.*

1. Introduction

Magnetooptics of solid-state lasers is one of the promising fields in laser physics. Magnetooptical properties of active elements of solid-state ring lasers are responsible for a number of interesting effects in the nonlinear dynamics of these lasers, which depend on the magnetic field.

Among these effects are the appearance of the frequency, amplitude, or polarisation reciprocity of a ring laser, the change in the phase difference of self-modulation oscillations, the change in the coupling between counterpropagating waves (both in the linear coupling caused by backscattering and in the nonlinear coupling related to the presence of population inversion gratings) caused by a change in their polarisation in a magnetic field. A magnetic field can also change the conditions of the emergence of the dynamic chaos and its properties in a solid-state ring laser (SRL).

The use of magnetooptical properties of an active medium opens up new possibilities for controlling output parameters of a SRL. This is especially important for miniature monolith ring chip lasers. Despite the fact that the dependence of the properties of SRLs on an external magnetic field has been studied in many papers, magnetooptics of SRLs has not been adequately investigated (especially in the case of two-directional lasing). Indeed, until very recently, only one-directional lasing (the travelling wave regime) has been studied in these lasers [1 – 5]. Only few papers were devoted to the analysis of the dependence of the nonlinear dynamics of a SRL on the external magnetic field.

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Among these papers are papers [6, 7] where the effect of a magnetic field on the spectrum of relaxation frequencies was studied, and papers [8, 9] where the development of the dynamic chaos depending on the magnetic field was analysed. In a recent paper [10], a new effect of a magnetic-field-dependent phase shift of self-modulation oscillations in counterpropagating waves was experimentally observed in a SRL.

In this paper, we present the results of a more detailed theoretical and experimental study of this effect.

2. Experiment

Fig. 1 shows the scheme of the experimental setup. We studied a monolith ring chip laser with a nonplanar resonator operating in self-modulation regime. The laser resonator was formed by the spherical face of the monolithic block and its three faces, which possessed total internal reflection. The resonator contour represented two isosceles triangles with heights 9.32 and 1.98 mm. The geometrical perimeter of the ring resonator was 2.6 cm and the angle of the nonplanar resonator was 80°. The laser was pumped by a 0.810-μm semiconductor diode using the longitudinal scheme. The design and parameters of the ring chip laser are described in detail in Ref. [11].

The magnetic field of strength H up to 500 Oe was produced with the help of permanent micromagnets or an electromagnet. In experiments, both the magnetic field

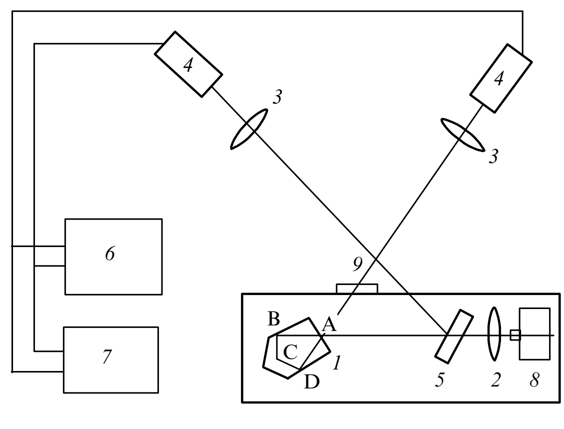

Figure 1. Scheme of the experimental setup: (*1*) monolithic ring laser; (*2*) objective; (*3*) focusing lenses; (*4*) photodetectors; (*5*) deflecting selective mirror; (*6*) two-beam oscilloscope; (*7*) ASK-3151 digital oscilloscope; (*8*) laser diode with a thermostat; (*9*) optical filter.

strength and its orientation with respect to the ring resonator contour were varied. In the absence of the magnetic field, the ring chip laser exhibits self-modulation sinusoidal antiphase (i.e., with the phase shift equal to π) 200-kHz oscillations of the intensity of counterpropagating waves (self-modulation regime of the first kind).

We performed experiments at pump powers above the lasing threshold η, which provided lasing at the fundamental longitudinal TEM_{00q} mode ($\eta < 1.15$). Because the laser resonator was nonplanar, the counterpropagating waves had elliptic polarisation (the axial ratio of the polarisation ellipse was 1:2). The difference of azimuths of polarisation ellipses in the absence of a magnetic field was 90°.

The imposition of a magnetic field on the active medium of the chip laser resulted in changes in the average intensity, frequency, and the modulation degree and in the appearance of the magnetic-field-dependent phase shift of self-modulation oscillations of the counterpropagating waves. Recall that in the absence of a magnetic field, the counterpropagating should be modulated in the antiphase.

The dependence of the frequency of self-modulation oscillations on the magnetic field strength H in the first approximation is described by the expression [12]

$$\omega_{\mathrm{m}} = \left(\omega_{\mathrm{m}0}^2 + \Omega^2\right)^{1/2} = \left[\omega_{\mathrm{m}0}^2 + \left(k_1 H\right)^2\right]^{1/2}, \quad (1)$$

where Ω is the frequency nonreciprocity of the resonator; $\omega_{\mathrm{m}0}$ is the self-modulation frequency at $\Omega = 0$; and k_1 is a numerical coefficient.

The study of the dependence of ω_{m} on H allows us not only to calculate the magnetic-field-induced frequency nonreciprocity but also to measure accurately the magnetic field during experiments. In our case, $k_1 = 2.6$ kHz Oe^{-1}, and the maximum nonreciprocity achieved ~ 750 kHz.

As was mentioned in Ref. [10], a magnetic field induces the magnetic-field-dependent phase shift $\Delta\varphi$ between self-modulation oscillations of the counterpropagating waves. Fig. 2 shows oscillograms of the intensities of counterpropagating weaves obtained for different H. The phase shift depends on the magnetic field strength and its orientation with respect to the resonator contour. It is maximal when the vector H is parallel to the resonator arm AB (Fig. 1).

The sign of the phase shift depends on the magnetic field direction and changes to the opposite when the polarity of an electromagnet (magnet) is changed. Fig. 3a shows the dependence $\sin\Delta\varphi$ on the strength of a magnetic field whose direction coincides with the arm AB. One can see that this dependence is slightly asymmetric with respect to the point $H = 0$: the maximum phase shift in our experiments was $-17°$ and $9°$ for $H = -313$ and 250 Oe, respectively. The positive direction of the magnetic field is conventionally taken to be coincident with the direction of the pump beam.

The magnetic-field-induced phase difference strongly depends on the magnetic field orientation with respect to the resonator contour. Indeed, when the applied magnetic field is oriented along the interval DB (when the area of the effective region of interaction of the magnetic field with light waves is small), the dependence of the phase shift of self-modulation oscillations on the magnetic field is substantially different. This is illustrated in Fig. 3b.

The polarisation study of the counterpropagating waves showed that the difference of azimuths of polarisation of these waves also depend on the magnetic field strength and polarisation. For $H = 450$ Oe and the magnetic field H oriented parallel to the arm AB, the difference between azimuths of polarisation of the counterpropagating waves increases by 1°.

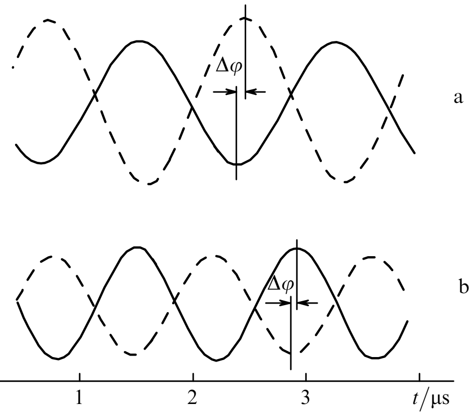

Figure 2. Oscillograms of self-modulation oscillations of counterpropagating waves for $H = -180$ Oe, $\omega_{\mathrm{m}}/2\pi = 602$ kHz, $\Delta\varphi = 9.2°$ (a) and $H = 210$ Oe, $\omega_{\mathrm{m}}/2\pi = 657$ kHz, $\Delta\varphi = 5.6°$ (b).

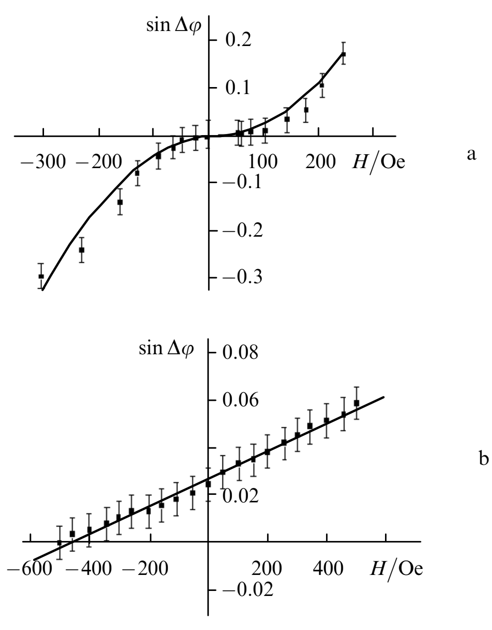

Figure 3. Experimental (points) and theoretical (9) dependences (solid lines) of $\sin\Delta\varphi$ on the magnetic field H oriented along arms AB (a) and DB (b).

3. Theory

The observed effects can be explained as follows. A permanent magnetic field H imposed on the active medium of a ring chip laser causes the rotation of the polarisation plane (the major axis of the polarisation ellipse) due to the Faraday effect by the angle

$$\theta = V l_0 H,$$

where V is the Verdet constant; l_0 is the effective length of interaction between the magnetic field and a light wave in the chip laser resonator. If we assume (which is quite likely) that a mirror on the spherical face of the monolithic block has the anisotropic reflectivity ($r_{\mathrm{s}} \neq r_{\mathrm{p}}$), this will result in the appearance of the amplitude (Δ) and frequency (Ω) nonreciprocity of the resonator.

In this case, the amplitude–frequency parameters of the ring chip laser can be described by the system of equations [13]

$$\frac{dE_{1,2}}{dt} = -\frac{\omega}{2Q_{1,2}}E_{1,2} \mp i\frac{\Omega}{2}E_{1,2} + i\frac{\tilde{m}_{1,2}}{2}E_{2,1}$$
$$+ \frac{\sigma l}{2T}(1 - i\delta)(N_0 E_{1,2} + N_{\mp}E_{2,1}),$$
$$T_1\frac{\partial N_0}{\partial t} = N_{th}(1+\eta) - N_0\left[1 + a\left(|E_1|^2 + |E_2|^2\right)\right] \quad (2)$$
$$-2a\mathrm{Re}(N_+ E_1 E_2^*),$$
$$T_1\frac{\partial N_+}{\partial t} = -N_+\left[1 + a\left(|E_1|^2 + |E_2|^2\right)\right] - aN_0|\boldsymbol{e}_2^*\boldsymbol{e}_1|^2 E_1 E_2^*,$$

where

$$N_0 = \frac{1}{l}\int_0^l N\mathrm{d}z;$$

$$N_{\pm} = \frac{1}{l}\int_0^l \boldsymbol{e}_{2,1}^*\boldsymbol{e}_{1,2}N\exp(\mp 2ikz)\mathrm{d}z; \quad (3)$$

$E_{1,2}$ is the complex amplitudes of the counterpropagating waves; N_0 and $N_{\pm}$ are the spatial harmonics of the inverse population; ω is the laser transition frequency; $Q_{1,2}$ are Q factors of the resonator for the counterpropagating waves; $\tilde{m}_{1,2} = m_{1,2}\exp(\pm i\vartheta_{1,2})$ are the complex coupling coefficients for counterpropagating waves; $T = Ln/c$; L is the resonator perimeter; n is the refractive index of the active medium; N_{th} is the threshold inverse population; η is the pump power excess over the threshold; T_1 is the longitudinal relaxation time for the population difference; $a = \sigma c T_1/8\pi\hbar\omega$ is the saturation parameter; σ is the laser transition cross section; l is the active region length; $\boldsymbol{e}_{1,2}$ are the unit polarisation vectors of the counterpropagating waves. Here, we assume that the detuning δ of the lasing frequency from the gain line centre is small compared to the line width.

Note that system (2) is in fact a system of ordinary differential equations for seven functions: the real and imaginary parts of the complex amplitudes $E_{1,2}$ of counterpropagating waves, the average inverse population N_0, and the real and imaginary parts of the complex amplitude $N_+ = N_-^*$ of the population grating.

It is reasonable to assume that the frequency (Ω) and amplitude (Δ) nonreciprocity of the ring laser are related to its parameters and the magnetic field strength H by the expressions

$$\Omega = \omega_1 - \omega_2 = k_1 H, \quad (4)$$

$$\Delta = (\omega_1/Q_1 - \omega_2/Q_2)/2 = k_2 H + \Delta_0, \quad (5)$$

where ω_1 and ω_2 are the frequencies of lasing in the opposite directions; k_1 and k_2 are some coefficients, which depend on the magnetic field orientation with respect to the resonator contour, the resonator nonplanarity, and other parameters; Δ_0 is the amplitude nonreciprocity of the ring resonator for $H = 0$. Note that the coefficients k_1, k_2 and Δ_0 for a specific resonator can be calculated using the Jones matrices formalism [14].

The imposition of a magnetic field results in some variation in the polarisation of counterpropagating waves, which can be also described by the Jones matrices method. The difference between polarisations of the counterpropagating waves affects [see expressions (3)] the inversion population gratings $N_{\pm}$ induced in the active medium. The influence of the magnetic field on $N_{\pm}$ and the complex coupling coefficients $\tilde{m}_{1,2}$ of the counterpropagating waves is described by the expression proportional to $|\boldsymbol{e}_1\boldsymbol{e}_2|^2$.

However, because the Verdet constant for a Nd:YAG crystal is quite small ($V = 0.034'$ cm^{-1} Oe^{-1}), it is reasonable to assume that a change in the polarisation of the counterpropagating waves induced by magnetic fields up to 500 Oe will be small. Therefore, we can neglect a change in the polarisation and coupling coefficients of the counterpropagating waves in the first approximation.

The expressions for the intensities of counterpropagating waves can be written in the form

$$I_{1,2} = I_{1,2}^0 \pm I_{1,2}^m\cos(\omega_m t + \varphi_{1,2}). \quad (6)$$

The dependence of the phase difference $\Delta\varphi = \varphi_1 - \varphi_2$ on the ring laser parameters was earlier investigated in Refs [13, 15] where the expression

$$\sin\Delta\varphi = \frac{2\omega_m\Delta}{\left[\left(\Omega^2 - \omega_m^2 + \Delta^2\right)^2 + 4\Delta^2\omega_m^2\right]^{1/2}} \quad (7)$$

was obtained for $\Delta\varphi$. When $\Delta \ll \omega_m$, expression (7) can be significantly simplified:

$$\sin\Delta\varphi = 2\omega_m\Delta/\omega_{m0}^2, \quad (8)$$

where ω_{m0} is the self-modulation frequency at $H = 0$. It follows from (8) that

$$\frac{\sin\Delta\varphi}{\omega_m} = \frac{2(\Delta_0 + k_2 H)}{\omega_{m0}^2}, \quad (9)$$

i.e., the ratio $\sin\Delta\varphi/\omega_m$ linearly depends on H.

Taking into account the dependence of ω_m on the frequency nonreciprocity of the resonator [12], we can find the dependence of the phase shift on Ω (note that the frequency nonreciprocity itself also depends on H):

$$\sin\Delta\varphi = 2\left(\omega_{m0}^2 + \Omega^2\right)^{1/2}\left(\Delta_0 + k_2 H\right)\omega_{m0}^{-2}. \quad (10)$$

4. Discussion of results

Let us compare the results of theoretical and experimental studies. Note that all the parameters entering expressions (6)–(10) except Δ were directly measured in experiments. The experimental values of $(\sin\Delta\varphi)/\omega_m$ obtained in different magnetic fields H (the field was oriented along the resonator arm AB) and the theoretical dependence (9) for $\Delta = 1.4$ kHz and $k_2 = 0.07$ kHz Oe^{-1} are shown in Fig. 4. When $H||$BD, the magnetic field almost does not affect ω_m ($\Omega \to 0$), so that $\sin\Delta\varphi \approx 2(\Delta_0 + k_2 H)/\omega_{m0}$, in good agreement with the experiment (Fig. 3b). A comparison of the theoretical dependences of the phase shift on the magnetic field with the experimental data shows that they are in good agreement.

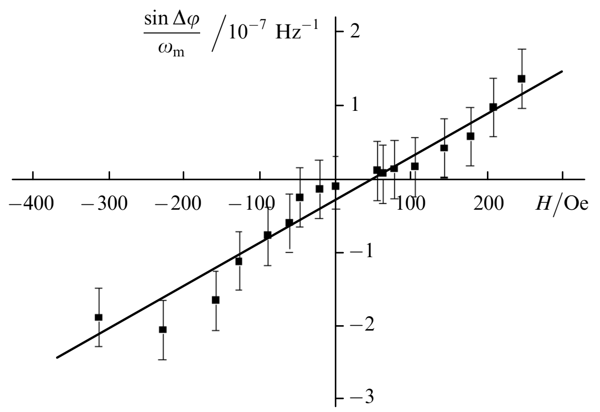

Figure 4. Experimental (points) and theoretical (straight line) dependences of $\sin\Delta\varphi/\omega_m$ on H.

Therefore, we can state that the magnetic-field-induced phase shift is caused by the magnetic-field-induced amplitude nonreciprocity of the ring laser, which depends on the magnetic field strength. Note that $\Delta\varphi$ also depends on the frequency nonreciprocity Ω of the resonator. However, when $\Delta \neq 0$, it follows from expression (8) that $\Delta\varphi$ does not vanish in the absence of the frequency nonreciprocity.

5. Conclusions

Note that we have performed experiments in relatively weak magnetic fields, when the induced amplitude nonreciprocity satisfied the relation $\Delta^2 \ll \omega^2$ and changes in the polarisation planes of the counterpropagating waves were small. The approximations that we used in our calculations corresponded to these conditions.

If these approximations are violated, all the dependences become more complicated. The critical magnetic field, at which the approximations used in our paper are not valid, depends on the Verdet constant of the active medium and the nonplanarity angle of the ring resonator contour.

It is interesting to study the phase shift of self-modulation oscillations in a broader range of the SRL parameters, in particular, in the case of the development of bifurcations in the self-modulation regime.

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References

1. Kane T J, Byer R L *Opt. Lett.* **10** 65 (1985)
2. Trutha W R, Donald D K, Nazarathy M *Opt. Lett.* **12** 249 (1987)
3. Park J R, Yoon T H, Chung M S, Lee H W *Appl. Opt.* **38** 4586 (1999)
4. Garbuzov D Z, Dedysh V V, Kochergin A V, et al. *Kvantovaya Elektron.* **16** 2423 (1989) [*Sov. J. Quantum Electron.* **19** 1557 (1989)]
5. Kravtsov N V, Nanii O E *Kvantovaya Elektron.* **20** 322 (1993) [*Quantum Electron.* **23** 272 (1993)]
6. Garbuzov D Z, Dedysh V V, Kochergin A V, et al. *Izv. Akad. Nauk SSSR, Ser. Fiz.* **54** 2397 (1990)
7. Khandokhin P A, Khanin Ya I *J. Opt. Soc. Am. B: Opt. Phys.* **2** 226 (1985)
8. Mamaev Yu A, Milovskii N D, Turkin A A, Khandokhin P A, Shirokov E Yu *Kvantovaya Elektron.* **27** 228 (1999) [*Quantum Electron.* **30** 505 (1999)]
9. Klimenko D N, Kravtsov N V, Lariontsev E G, Firsov V V, Kochergin A V, et al. *Kvantovaya Elektron.* **23** 438 (1996) [*Quantum Electron.* **26** 428 (1996)]
10. Kravtsov N V, Lariontsev E G, Sidorov S S, Chekina S N, Firsov V V *Kvantovaya Elektron.* **31** 189 (2001) [*Quantum Electron.* **31** 189 (2001)]
11. Boiko D L, Golyaev Yu D, Dmitriev V G, Kravtsov N V *Kvantovaya Elektron.* **24** 653 (1997) [*Quantum Electron.* **27** 635 (1997)]
12. Kravtsov N V, Lariontsev E G *Kvantovaya Elektron.* **30** 105 (2000) [*Quantum Electron.* **30** 105 (2000)]
13. Boiko D L, Kravtsov N V *Kvantovaya Elektron.* **27** 27 (1999) [Quantum Electron. **27** 309 (1999)]
14. Nilson A C, Gustafson E K, Byer R L *IEEE J. Quantum. Electron.* **25** 767 (1989)
15. Zolotoverkh I I, Lariontsev E G *Kvantovaya Elektron.* **23** 620 (1996) [*Quantum Electron.* **26** 604 (1996)]