 

*NONLINEAR OPTICAL PHENOMENA*

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# Formation and stability of optical surface waves at the interface between a metal and a photorefractive crystal with drift and diffusion components of the nonlinear response

V A Aleshkevich, V A Vysloukh, A A Egorov, Ya V Kartashov

*Abstract*. **Peculiarities of the surface wave propagation at the interface between a metal and a photorefractive crystal are considered. The profiles of these waves are determined numerically, and the guiding properties of the interface are studied. The stability of the surface modes is investigated and it is shown that the Vakhitov – Kolokolov criterion can be applied for estimating the stability.**

***Keywords***: *surface waves, photorefraction, diffusive nonlinearity.*

## 1. Introduction

The propagation of a laser beam at the interface between two media with different optical properties is one of the classical problems in nonlinear optics. For practical purposes, the most interesting case is an interface between two media of which at least one is nonlinear. The peculiarities of the laser beam interaction with the interface depend to a considerable extent on the type of nonlinearity of the media forming the interface. Interfaces between the Kerr media (including linear – nonlinear [1, 2] and nonlinear – nonlinear [3] interfaces), materials with a quadratic nonlinearity [4], as well as the interface between a Kerr medium and an absorbing medium [5], have been studied by many authors. From the mathematical point of view, the problem of interaction of a laser beam with an interface is solved by finding a certain class of solutions of the nonlinear Schrödinger equation whose coefficients depend on transverse coordinates. The absence of translational symmetry in the transverse direction results in the generation of nonlinear surface waves (solitons) localised at the inhomogeneities in the medium. In this case, both the stability of solitons and their interaction with moving wave packets are important. The methods of inverse scattering problem cannot be applied for solving such problems because of the absence of translational symmetry.

From the practical point of view, it is also quite interesting to study the interaction of laser radiation with interfaces formed by media with complex nonlinearities. Significant achievements [6, 7] in the field of soliton generation in photorefractive crystals with strong nonlinearities for rather low intensities of laser radiation have stimulated investigations of photorefractive surface waves. The peculiarities of the formation of distributed surface waves (i.e., waves with an infinite energy) near the interface between a photorefractive crystal with a purely diffusive (logarithmic) nonlinearity and a linear medium (dielectric or metal) were studied for the first time in paper [8]. A surface wave is formed in this case due to the interference and energy exchange between waves reflected from the interface and from the Bragg grating formed in the volume of the crystal. Later, the surface propagation of a laser beam in the presence of drift and diffusion components of the nonlinear response of a photorefractive crystal (in the case of a high dark conductivity) was interpreted [9 – 11] as the result of a balance between the self-bending of the beam due to the diffusion component of the nonlinear response and the beam reflection [12].

In this work, we consider localised surface waves at the metal-photorefractive crystal interface with drift and diffusion components of the nonlinear response for arbitrary values of the light field intensity and effective dark conductivity. Surface mode profiles are obtained and waveguide properties of the interface are investigated. Special attention is paid to the stability of photorefractive surface waves, which was not considered in earlier works. The well-known linear method for stability analysis [2, 13 – 24] is generalised to the case of a photorefractive medium with a diffusion component of the nonlinear response.

## 2. Theoretical model

Consider a slit beam (infinite along the $y$ axis and finite in the direction of the $x$ axis) propagating along the $z$ axis near the interface between a metal occupying the half-space $x \geqslant 0$ and a photorefractive medium with drift and diffusion components of the nonlinear response occupying the half-space $x < 0$. The laser beam is polarised along the $y$ axis. The metal is assumed to be perfect, i.e., the skin layer, which actually is of the order of the wavelength, is assumed zero. In this case, the presence of the metal leads to the purely geometrical condition of vanishing of the light wave field in the half-space $x \geqslant 0$, and the propagation of the laser beam is described by the standard truncated wave equation for the normalised complex amplitude $q(\eta, \xi)$ of the field:

$$\mathrm{i}\frac{\partial q}{\partial \xi} = -\frac{1}{2}\frac{\partial^2 q}{\partial \eta^2} - \frac{q|q|^2}{1+S|q|^2} + \mu\frac{q}{1+S|q|^2}\frac{\partial |q|^2}{\partial \eta} \text{ for } \eta < 0, \quad (1)$$
$$q = 0 \text{ for } \eta \geqslant 0,$$

**V A Aleshkevich, V A Vysloukh, A A Egorov, Ya V Kartashov** Department of Physics, M V Lomonosov Moscow State University, Vorob'evy gory, 119899 Moscow, Russia

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where $q(\eta, \xi) = (L_{\rm dif}/L_{\rm ref})^{1/2} A(\eta, \xi) I_{\rm dark}^{-1/2}$ is the dimensionless complex amplitude of the light field; $A(\eta, \xi)$ is the slowly varying envelope of the light field; $\eta = x/x_0$ is the normalised transverse coordinate; $x_0$ is the characteristic transverse scale (for example, the radius of the input beam); $\xi = z/L_{\rm dif}$ is the normalised longitudinal coordinate; $L_{\rm dif} = k_0 x_0^2$ is the diffraction length corresponding to $x_0$, $k_0 = n\omega/c$ is the wave number; $n$ is the unperturbed refractive index of the photorefractive medium, $\omega$ is the carrier frequency of the optical radiation; $L_{\rm ref} = 2(k_0 r_{\rm eff} n^2 E_0)^{-1}$ is the nonlinear refraction length; $r_{\rm eff}$ is the effective electrooptical coefficient; $E_0$ is the strength of the external static field applied to the photorefractive medium; $I_{\rm dark}$ is the effective dark intensity; the parameter $S = L_{\rm ref}/L_{\rm dif}$ defines the relative contribution of the drift component of the nonlinear response; the parameter $\mu = k\Theta/(ex_0E_0)$ defines the relative contribution of the diffusion component of the nonlinear response; $\Theta$ is the temperature of the photorefractive medium; $k$ is the Boltzmann constant; and $e$ is the free carrier charge with consideration for its sign (positive for holes and negative for electrons).

The first term in the right-hand side of the truncated wave equation (1) describes the diffraction-induced spreading of the laser beam, and the second term describes its self-focusing caused by the local drift component of the nonlinear response of the photorefractive medium produced by the linear electrooptical effect in an external static field $E_0$. The last term in the right-hand side of Eqn (1) describes the self-bending of the laser beam caused by the diffusion component of the nonlinear response of the medium as a result of the induced energy transfer from high-frequency spatial harmonics of the beam to the low-frequency harmonics. Surface modes can be formed at the interface between a metal and a photorefractive crystal owing to compensation of the beam reflection from the interface and its self-bending towards the interface. Typical experimental values of parameters $S$ and $\mu$ in a SnBaNb crystal are $\sim 1.0$ and $\sim 0.1$, respectively, for a 633-nm He–Ne laser with an input laser beam radius $x_0 \sim 50$ μm and an intensity of radiation $\sim 3$ mW cm$^{-2}$, for the electrooptical coefficient of the crystal $r_{\rm eff} = 2.5\times10^{-10}$ m V$^{-1}$, the unperturbed refractive index $n = 2.35$, the crystal temperature $T = 300$ K, and the external static field $E_0 = 6 \times 10^3$ V m$^{-1}$.

## 3. Profiles of stationary surface waves

In this section, we shall use the truncated equation (1) to determine the profiles of stationary surface modes (propagating without distortion) at the interface between a metal and a nonlinear photorefractive medium. By representing the wave field amplitude in the form $q(\eta, \xi) = \rho(\eta) \times \exp(ib\xi)$ and substituting this expression into Eqn (1), we arrive at an ordinary second-order differential equation for the real envelope $\rho(\eta)$:

$$\frac{{\rm d}^2\rho}{{\rm d}\eta^2} = 2b\rho - \frac{2\rho^3}{1+S\rho^2} + \frac{4\mu\rho^2}{1+S\rho^2}\frac{{\rm d}\rho}{{\rm d}\eta} \quad \text{for } \eta < 0, \qquad (2)$$

$$\rho = 0 \quad \text{for} \quad \eta \geqslant 0,$$

where $b$ is the real propagation constant. Due to the presence of a diffusion component and saturation of the nonlinear response of the photorefractive medium, Eqn (2) cannot be solved analytically and should be integrated numerically, for example, by the soothing method, which reduces a two-point boundary problem to the Cauchy problem. Since the field in the metal region is equal to zero, it is convenient to choose the initial point of integration at the interface. The initial conditions corresponding to the continuity of the tangential component of the electric field and of the normal component of the magnetic induction vector at the interface have the form $\rho(\eta = -0) = 0$ and ${\rm d}\rho(\eta = -0)/{\rm d}\eta = m$, where $m$ is a variable parameter determining the strength of nonlinear effects. By varying parameters $b$, $m$, $\mu$ and $S$, and integrating Eqn (2), we obtained various profiles of surface waves at the interface between a metal and a photorefractive medium. Note that in the case considered by us, surface waves can be formed only for $\mu > 0$.

Possible classes of solutions of Eqn (2) can be easily obtained from quite general considerations based on the analogy of Eqn (2) for the envelope of the surface wave with the equation describing the motion of a mechanical particle in a potential well with nonlinear dissipation, where the wave envelope $\rho$ plays the role of the displacement, and the transverse coordinate $\eta$ plays the role of time. It can be easily verified that for a photorefractive medium ($\eta < 0$), Eqn (2) can be written in the form

$$\frac{\rm d}{{\rm d}\eta}(U+T) = \frac{4\mu\rho^2}{1+S\rho^2}\left(\frac{{\rm d}\rho}{{\rm d}\eta}\right)^2, \qquad (3)$$

where

$$U = \left(\frac{1}{S} - b\right)\rho^2 - \frac{1}{S^2}\ln\left(1+S\rho^2\right), \quad T = \frac{1}{2}\left(\frac{{\rm d}\rho}{{\rm d}\eta}\right)^2$$

are the potential and kinetic energy of a particle of unit mass, respectively, and the right-hand side of Eqn (3) describes the nonlinear friction force, which is proportional to the square of the particle velocity ${\rm d}\rho/{\rm d}\eta$. Typical forms of the potential well $U(\rho)$ are depicted in Fig. 1 for different values of the parameter $S$ and the propagation constant $b$. The potential well is symmetric relative to point $\rho = 0$, hence, Fig. 1 shows only the part of the well corresponding to positive values of $\rho$. The shape of the potential well changes qualitatively upon reversal of the sign of the propagation constant, which affects the nature of the possible types of motion described by Eqn (3), as well as the profiles of the corresponding surface waves.

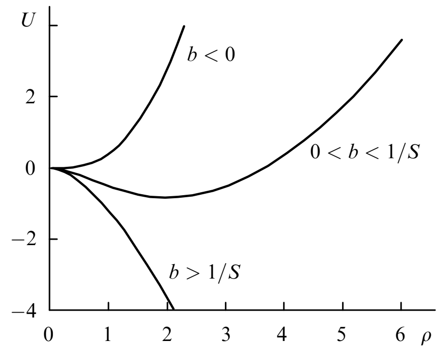


**Figure 1.** Typical shapes of the potential well $U(\rho)$ for various ratios of the parameter $S$ and the propagation constant $b$ for $S = 1$ and $b = -0.1$ ($b < 0$), 0.8 ($0 < b < 1/S$), 1.5 ($b > 1/S$).

For negative values of the propagation constant $b < 0$, the potential well has a single stable stationary point $\rho = 0$ (see Fig. 1). In this case, a particle with a nonzero initial energy $U + T$ performs damped oscillations (as $\eta$ varies from zero to $-\infty$), passing from the region of positive $\rho$ to the region of negative $\rho$ and gradually 'rolling down' to the stable equilibrium position $\rho = 0$, which corresponds to the well-known distributed surface waves [8]. The profiles of such waves are shown in Fig. 2. However, numerical integration and linear analysis of Eqn (2) at small amplitudes $\rho$ show that distributed surface waves generally possess an infinite energy because their amplitude $\rho$ decreases too slowly when $\eta \to -\infty$ or, to be more precise, $\rho\,(\eta \to -\infty) \sim |\mu\eta|^{-1/2}\cos[(-2b)^{1/2}\eta]$. The question of the stability of such oscillating waves with an infinite energy remains open so far.

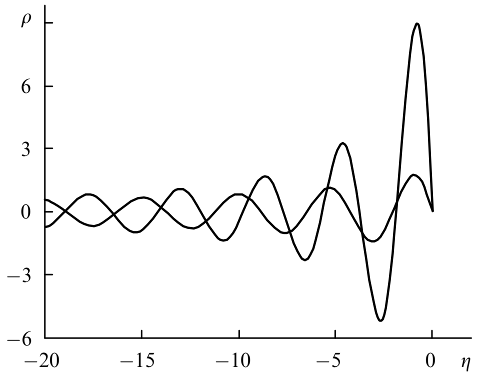


**Figure 2.** Profiles of distributed surface waves with different amplitudes for $b = -0.5$, $S = 1$, $\mu = 0.2$. The interface is located at the point $\eta = 0$.

For positive values of the propagation constant $b$ lying in the interval $0 < b < 1/S$, the potential well has two stable ($\rho = \pm[b/(1 - Sb)]^{1/2}$) and one unstable ($\rho = 0$) stationary points (see Fig. 1). A particle with a nonzero initial energy $U + T$ will 'roll over' from the right side of the potential well (corresponding to positive $\rho$) to the left side of the well (corresponding to negative $\rho$) until it stops at one of the two stable stationary points or at the unstable stationary point due to the energy loss. The former case corresponds to *shock* surface waves with an infinite energy [25], which have a nonzero asymptotic form for $\eta \to -\infty$. Fig. 3 shows typical profiles of surface shock waves of the first three orders (hereafter, the order of a wave is defined by the number of intersections of its envelope with the $\eta$ axis, including the point $\eta = 0$).

One can see that for $\eta \to -\infty$, the shock waves represent damped oscillations superimposed on a constant background. The magnitude of the background is determined by the propagation constant $b$ and is equal to $\pm[b/(1-Sb)]^{1/2}$, while the amplitude of oscillations and the rate of their attenuation are determined by the parameter $\mu$, and increase with $\mu$. Note that shock waves are subjected to modulation instability because a wave having a zeroth harmonic in the spectrum is unstable in a medium with a diffusion component of the nonlinear response for any modulation frequency.

From the practical point of view, the most interesting case is encountered when a particle describing a surface wave stops at the point $\rho = 0$, corresponding to *localised*

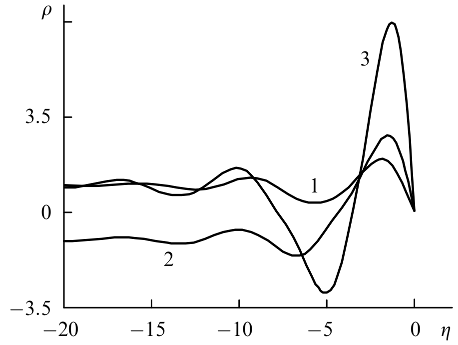


**Figure 3.** Profiles of surface shock waves of the first three orders (1 – 3) for $b = 0.5$, $S = 1$, $\mu = 0.2$. The interface is located at the point $\eta = 0$.

surface waves (as before, we consider the interval $0 < b < 1/S$). Typical profiles of localised surface waves are presented in Fig. 4b. Note the following characteristic feature of localised waves: except for the first half-period closest to the interface, the profile of the $(n+1)$th order wave coincides with that of the $n$th order wave. As the energy of the surface wave

$$w = \int_{-\infty}^{0} \rho^2(\eta)\,\mathrm{d}\eta \qquad (4)$$

increases, the wave profile becomes more and more asymmetric, and the intensity maximum of the first half-period of the wave is displaced towards the interface

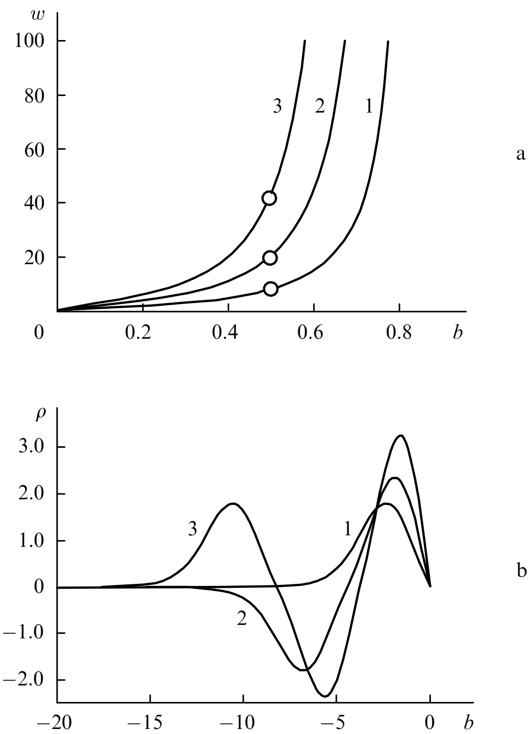


**Figure 4.** (a) Dependence of energy $w$ of localised surface waves (dispersion curves) of first three orders (1 – 3) on the propagation constant $b$, and (b) localised surface wave profiles corresponding to the points on dispersion curves for $S = 1$, $\mu = 0.1$. The interface is located at the point $\eta = 0$.

because the compensation of the beam self-bending (which increases approximately as the fourth power of the amplitude $\rho$) requires an appropriate enhancement of the boundary effects (which are enhanced when the beam approaches the interface).

The energy $w$ of the surface waves increases monotonically with the propagation constant $b$. In this case, the wave amplitude increases, while the transverse size (width) decreases. Fig. 4a shows the dependences $w(b)$ (the so-called dispersion curves) for waves of the first three orders. In the limit of high amplitudes, Eqn (2) can be linearised and has an analytic solution $\rho(\eta) = m\exp(2\mu S^{-1}\eta)\cos[(2S^{-1} - 2b - 4\mu^2 S^{-2})^{1/2}\eta]$, which leads to a more precise expression than $1/S$ for the upper limit of the propagation constant $b$ for which localised surface waves can still exist:

$$b = \frac{1}{S} - \frac{2\mu^2}{S^2}. \tag{5}$$

As in any medium with saturation of the nonlinear response, an increase in the parameter $S$ for fixed values of the energy and the parameter $\mu$ leads to an increase in the characteristic transverse size of the surface mode, which can be determined, for example, by half the peak intensity. As $\mu$ increases, the profiles of the surface waves become more and more asymmetric. Investigation of the stability of localised waves is not a trivial problem and is performed in the next section.

Finally, for positive values of $b > 1/S$, the potential well has a single unstable stationary point $\rho = 0$ (see Fig. 1). In this case, finite motions are not possible and it is meaningless to speak about the surface waves.

## 4. Stability of localised surface waves

We will study the stability of surface waves at the metal-photorefractive medium interface by using the conventional linear analysis, which is valid for the initial stage of evolution of small perturbations. Note that linearisation was used earlier mainly for analysing the stability of one-dimensional solitons in local media, when the nonlinear correction to the refractive index introduced by the light field does not contain derivatives of intensity with respect to the transverse coordinate. In our case, however, the presence of a diffusion component of the nonlinear response proportional to the field intensity derivative is a necessary condition for the existence of surface waves. We will seek the solution of Eqn (1), describing the propagation of a surface wave with a perturbed initial profile in the form

$$q(\eta, \xi) = [\rho(\eta) + u(\eta, \xi) + \mathrm{i}v(\eta, \xi)]\exp(\mathrm{i}b\xi), \tag{6}$$

where $\rho(\eta)$ is the real envelope of the stationary surface wave, $u(\eta, \xi)$ and $v(\eta, \xi)$ are the real and imaginary components of the small perturbation, respectively ($u, v \ll \rho$). The requirement of the smallness of the perturbation amplitude relative to the amplitude of the stationary surface wave, which is a quite general requirement for the first-order surface mode, imposes stringent constraints on the type of perturbation for higher-order modes (thus, restricting the generality of the results for higher-order modes). For example, the perturbation should vanish at the same points where the higher-order mode amplitude vanishes. Hence, we will confine the following analysis to the stability of fundamental surface waves only (first-order modes).

Substituting Eqn (6) into the truncated wave equation (1), linearising the resulting equation and separating the real and imaginary components, we arrive at the following system of linear equations:

$$\frac{\partial u}{\partial \xi} = -\mathscr{L}v, \quad \frac{\partial v}{\partial \xi} = \mathscr{R}u, \tag{7}$$

where the linear operators $\mathscr{L}$ and $\mathscr{R}$, which depend only on the transverse coordinate $\eta$, have the following form in the nonlinear photorefractive medium:

$$\mathscr{L} = \frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}\eta^2} - b + \frac{\rho^2}{1+S\rho^2} - \frac{2\mu\rho}{1+S\rho^2}\frac{\mathrm{d}\rho}{\mathrm{d}\eta},$$
$$\mathscr{R} = \mathscr{L} + \frac{2\rho^2}{(1+S\rho^2)^2} + \frac{4\mu S\rho^3}{(1+S\rho^2)^2}\frac{\mathrm{d}\rho}{\mathrm{d}\eta} - \frac{2\mu\rho}{1+S\rho^2}\frac{\mathrm{d}\rho}{\mathrm{d}\eta} - \frac{2\mu\rho^2}{1+S\rho^2}\frac{\mathrm{d}}{\mathrm{d}\eta}. \tag{8}$$

According to the form of operators (8), the main difference from the case of a medium with a local nonlinear response is that the linear operator $\mathscr{R}$ is not self-adjoint owing to the presence of a first-order derivative with respect to $\eta$ in the last term. This circumstance rules out the possibility of using the standard procedure of deriving the Vakhitov–Kolokolov criterion for the system of equations (7), because the operators $\mathscr{L}\mathscr{R}$ and $\mathscr{R}\mathscr{L}$ may now have complex eigenvalues (indicating the presence of perturbations that experience exponential growth of the amplitude as well as harmonic oscillations along the $\xi$ axis) rather than purely real eigenvalues (which indicates the possibility of existence of either purely exponentially increasing perturbations, or perturbations performing harmonic oscillations along the $\xi$ axis) as in the case of media with a local nonlinear response. We will seek the solution of system (7) in the form of an expansion in all perturbation 'modes'

$$u(\eta, \xi) = \mathrm{Re}\sum_{\delta} C_\delta u_\delta(\eta)\exp(\delta\xi),$$
$$v(\eta, \xi) = \mathrm{Re}\sum_{\delta} C_\delta v_\delta(\eta)\exp(\delta\xi) \tag{9}$$

with different increments. Here, $\delta$ is the complex increment of the perturbation, $C_\delta$ are arbitrary constants, and $u_\delta$ and $v_\delta$ are the initial perturbation profiles, which can be complex now. Substituting (9) into the system of equations (7) and equating terms with identical exponential factors, we finally arrive at the following system of complex equations with real linear operators $\mathscr{L}$ and $\mathscr{R}$:

$$\delta u_\delta = -\mathscr{L}v_\delta, \quad \delta u_\delta = \mathscr{R}u_\delta. \tag{10}$$

This system of equations was solved numerically taking into account the conditions $u_\delta(\eta = 0) = 0$ and $v_\delta(\eta = 0) = 0$ at the metal-photorefractive medium interface. The profile of the stationary fundamental surface wave $\rho(\eta)$ was calculated for a given value of the propagation constant $b$, and was then used in Eqns (8) for obtaining the operators

$\mathscr{L}$ and $\mathscr{R}$. Numerical integration shows that for any values of the propagation constant $b$, there are no solutions of the system of linear equations (10) corresponding to increments with a nonzero real part, i.e., there are no exponentially increasing perturbations. Thus, fundamental surface waves at the metal-photorefractive medium interface are stable to small perturbations of the initial wave profile and preserve their structure during propagation, while an arbitrarily small perturbation will experience harmonic oscillations along the $\xi$ axis.

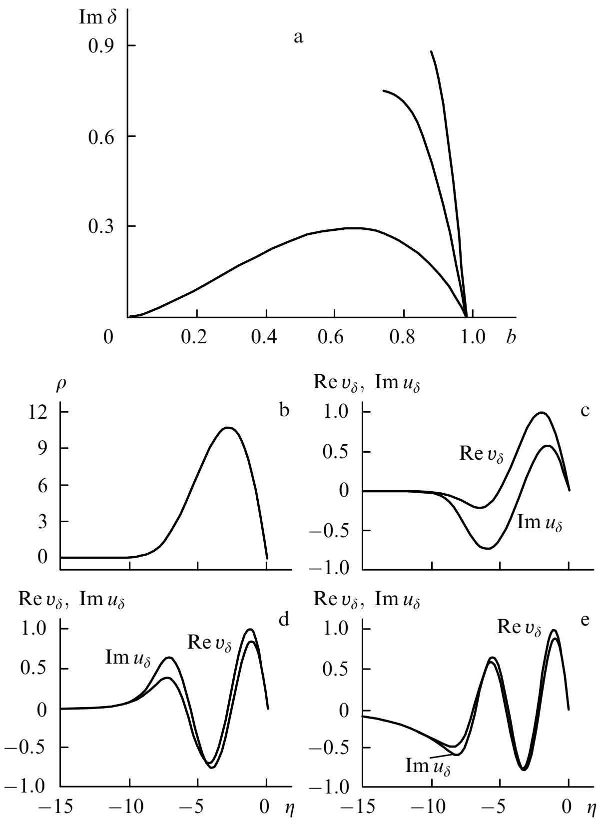


**Figure 5.** (a) Dependence of the imaginary part of the growth increment $\delta$ of small perturbation on the propagation constant $b$ for the fundamental surface wave, (b) its profile, and (c)–(e) profiles of the perturbation modes of the first three orders for $b = 0.88$ corresponding to the lower (c), middle (d) and upper (e) curves in Fig. 5a for $S = 1$, $\mu = 0.1$. The interface is located at the point $\eta = 0$.

Note that exponentially increasing perturbations are also not found for higher-order modes within the framework of model (10), but this still does not mean the stability of higher-order modes due to the above-mentioned stringent constraint imposed on the perturbation profile. Fig. 5a shows the dependence of the imaginary part of increment $\delta$ on the propagation constant $b$ for the first-order surface mode. One can see that several perturbation modes with different increments $\delta$ (several solutions of the system of equations (10)) exist for the same value of $b$. For $b \to S^{-1} - 2\mu^2 S^{-2}$, the number of possible perturbation modes increases infinitely (Fig. 5 shows the dependences $\mathrm{Im}\,\delta(b)$ for the first three perturbation modes only), and the corresponding increments tend to zero. Note that the branches corresponding to various perturbation modes start from the straight line $\mathrm{Im}\delta = \beta$.

Figs 5c–e show the normalised profiles of perturbation modes ($u$ and $\upsilon$ components) for $b = 0.88$, for which three solutions of system (10) are possible. Since increment $\delta$ is purely imaginary, the perturbation component $u$ must be purely imaginary for real $\upsilon$. One can see that, although the envelope of the fundamental surface wave does not have any zeros (Fig. 5b) except for the point $\eta = 0$, the lowest-order mode has one zero (Fig. 5c), the next-order mode has two zeros (Fig. 5d), and so on. With increasing order of the perturbation mode, the profiles of $u$ and $\upsilon$ components become virtually identical (Fig. 5e). Comparing the results of linear analysis of stability with the dispersion curves exhibiting a monotonic growth in the wave energy $w$ with increasing $b$ (Fig. 4), we can conclude that the widely used stability criterion $\mathrm{d}w/\mathrm{d}b > 0$ [24] is also applicable for surface waves at the metal-photorefractive medium interface with a nonlocal diffusion component of the nonlinear response.

## 5. Conclusions

Thus, the fundamental surface waves are stable to small perturbations of the initial profile in the entire range of their existence. The Vakhitov–Kolokolov stability criterion is applicable in the presence of a nonlocal diffusion component of the nonlinear response of the photorefractive medium. One of the practically interesting aspects of surface propagation of a beam is that the energy concentration in a thin layer of the material significantly reduces the characteristic response time of the photorefractive crystal. This makes it possible to considerably increase the operational speed of optical devices fabricated from photorefractive materials without resorting to any auxiliary optical waveguide structures. Combined with the structural stability, the latter property makes surface waves an extremely interesting object from the point of view of constructing nonlinear logic elements and for light control by light devices.

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